

A Tale of Two Platforms: Dealer Intermediation in the European Sovereign Bond Market

Peter Dunne*

Queens University, Belfast

Harald Hau**

INSEAD and CEPR

Michael Moore***

Queens University, Belfast and Harvard University

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Abstract

European sovereign bond trading occurs in a highly liquid interdealer market and a parallel dealer-customer market in which buy-side financial institutions request quotes from primary dealers. Synchronized price data from both market segments allow us to compare market quality. We find that customer transaction (i) are on average priced very favorable relative to the best interdealer quotes, (ii) feature a relatively high price dispersion at any given moment and (iii) are less price sensitive to volatility increases than the best interdealer quotes. We develop a simple dynamic model of dealer intermediation which can account for these findings. The dealers' inventory management concerns are shown to be an important determinant of customer transaction quality both in the model and in the data.

Keywords: Dealer Intermediation, Spread Determination, Adverse Selection, Market Segmentation

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*School of Management and Economics, Queens University, Belfast, BT7 1NN, Northern Ireland, United Kingdom. Telephone: (+44) 28 9097 3310. E-mail: p.g.dunne@qub.ac.uk. Web page: www.qub-efrg.com/staff/pdunne/

**Department of Finance, Boulevard de Constance, 77305 Fontainebleau Cedex, France. Telephone: (+33) 1 6072 4484. Fax: (33)-1 6074 5500. E-mail: harald.hau@insead.edu. Web page: <http://faculty.insead.edu/hau/>

***School of Management and Economics, Queens University, Belfast, BT7 1NN, Northern Ireland, United Kingdom. Telephone: (+44) 28 9097 3208. E-mail: m.moore@qub.ac.uk. Web page: www.qub-efrg.com/staff/mmoore/

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1 Introduction

The European sovereign bond market is the world's largest market for debt securities. The interdealer segment of the market comes close to an 'ideal market' with high liquidity in many bond issues. Price transparency is also high as interdealer trading occurs through centralized modern electronic trading systems. Transaction spreads are therefore generally small in the interdealer market. But do these favorable market conditions in the interdealer market also translate into favorable trading conditions in the customer segment of the bond market, in which smaller banks and other financial institutions request quotes from the primary dealers? As with many other markets, these wholesale customers do not have access to the interdealer trading platforms. Does dealer intermediation impose considerable costs for the clients? What determines the quality of customer quotes and their dispersion?

This paper addresses these questions based on new data which combine interdealer price data from the largest European bond trading platform MTS with customer price data from the 'BondVision' customer quote request system, which is also owned by MTS. For simplicity, we refer to the interdealer segment of the bond market as the B2B market and the customer segment as the B2C market. Electronic recording of all accepted B2C quotes allows a direct comparison of the customer prices to the prevailing interdealer prices on both the ask and bid side of the market. The price difference of the B2C quote relative to the best B2B quote is referred to as the '*cross-market spread*'. We study the cross-market spread for different bonds and different levels of market volatility. Three empirical findings can be highlighted:

1. B2C transactions occur at very favorable prices in the European bond market. The cross-market spread as a measure of B2C price quality is *on average* negative, which shows that B2C transactions occur at prices which are on average more favorable than the best simultaneous quote in the interdealer (B2B) segment of the market.
2. The cross-market spread is characterized by high price dispersion. Its dispersion measured by the difference between the (average of the) 25 percent best and worst trades is 3.30 cents on the ask side and 2.58 cents on the bid side. This is large relative to an average interdealer (B2B) spread of approximately 3.08 cents.
3. The interdealer (B2B) spread is increasing in market volatility, while the cross-market spread is either constant (bid side) or even decreasing (ask side) in volatility. The spread deterioration of the B2B market under higher volatility is therefore not fully passed on to the B2C segment of the market. More interest sensitive long-run bonds generally have lower cross-market spreads

and therefore more favorable B2C transaction prices.

Studies of customer price quality are rather rare even though most investors do not have direct access to an interdealer market. Recently, work on retail prices in the U.S. municipal bond market met considerable interest (Harris and Piwowar (2006), Green et al. (2007)). This over-the-counter market lacks the price transparency of the European bond market and liquidity is dispersed over a large number of bonds. Dealer intermediation in the U.S. municipal bond market results in a large retail price dispersion and very unfavorable retail prices for many small investors. Green et al. (2007) explain the retail price dispersion in the U.S. bond market by reference to dealer price discrimination against uninformed small retail customers.¹ Our B2C data on European sovereign bonds concerns larger financial investors with access to the electronic quote request system. It is important to emphasize that our B2C market is a market between dealers and sophisticated financial customers rather than a ‘retail’ market in which private households transact. A second difference to the studies on U.S. municipal bonds lies in a different market structure. The European sovereign bond market has a large number of highly liquid bonds with benchmark status and high price transparency. Nevertheless, we find considerable customer price dispersion relative to the best interdealer quote. What explains the large dispersion of the customer quotes?

We argue in this paper that dealer inventory management concerns are important for explaining the B2C price behavior in the European bond market. Under inventory constraints, dealers find it optional to quote inventory contingent B2C price provided that their dealer-client relationship grants them some degree of market power. Inventory dispersion among dealers can thus explain the observed cross sectional B2C price dispersion. Dealer market power can also explain the volatility puzzle for the cross-market spreads. A dealer’s monopolistic pricing power is counterbalanced by an adverse selection effect if the volatility of the customer demand increases. A competitive interdealer market should fully reflect increased adverse selection risk through higher B2B spreads, while B2C spreads buffer this volatility related spread increase through diminished dealer profits. Higher volatility may therefore decrease dealer rents from market making. This latter aspect explains why the cross-market spread decreases in volatility.

To structure the discussion, we develop a new dynamic market model of dealer intermediation. The model characterizes the dealers’ optimal customer quotes for sequentially arriving customers. Dealers face inventory constraints and use the B2B market to rebalance. The B2B spread is determined under perfect competition. Dealers provide to each other limit orders which reflect their reservation price for

¹Evidence that higher post-trade transparency lowers trading costs is found for the corporate bond market in a variety of studies (Bessembinder et al. (2006), Edwards et al. (2007), Goldstein et al. (2007)).

buying (bid price) or selling (ask price) one unit of the asset. No trading profits are earned in the B2B segment of the market; its sole purpose is to facilitate inventory management. In contrast, the B2C relationship is characterized by monopolistic quote setting under uncertainty about the customer's reservation price. The model abstracts from all dealer competition for the clientele of other dealers. The latter assumption grants maximal market power to a dealer in his dealer-client relationship. While somewhat extreme, such a set-up brings out the role of B2C market power most clearly and is also most tractable. The dynamic setting allows us to study how increased levels of price volatility and adverse selection erode a dealer's market power and generate very favorable B2C quotes relative to the B2B benchmark spreads.

A more direct test of the model would relate the B2C quote behavior directly to the inventory state of the quoting dealer. Unfortunately, dealer inventory data is rarely available in multi-dealer markets. But aggregate inventory imbalances can be indirectly inferred from the limit order book of the interdealer market. According to our inventory model of dealer intermediation, the best B2B ask quotes are provided by dealers with positive inventory imbalances and the best B2B bid quotes come from dealers with negative imbalances. The difference in market depth at the best ask and bid quotes measures therefore aggregate dealer imbalances. Under inventory contingent customer pricing, such differences in B2B market depth should be related to the average quality of B2C trade at the opposite side of the market. Positive imbalances deteriorate the average B2C bid side quote and negative imbalances deteriorate the B2C ask side quote. We test if these model predictions are confirmed by the data and find strong empirical support for inventory effects determining customer transaction quality.

A theoretical contribution of this paper is to examine the role of both adverse selection and dealer inventory constraints within the market segmentation of the European bond market. The early microstructure literature on dealer behavior has recognized the importance of both adverse selection (Glosten and Milgrom (1985), Kyle (1985)) and inventory management concerns (Stoll (1978), Amihud and Mendelson (1980)) for quote determination. Subsequent work integrated both aspects into dynamic models with a (single) value optimizing dealer (O'Hara and Oldfield (1986), Madhavan and Smidt (1993)). In Madhavan and Smidt (1993), a 'specialist' sets quotes to trade with informed and liquidity traders and simultaneously faces inventory costs. A single market serves both the purpose of customer intermediation and inventory management. Our theoretical set-up differs from this literature. First, modern electronic markets do not have a monopolistic specialist, but typically feature many dealers. The interdealer spread should therefore be determined competitively as in our set-up. Secondly, customer intermediation and inventory management do not need to take place in the

same market, but may occur in separate market segments. The electronic interdealer platform in the European bond market, for example, is not accessible to customers who have to directly interact with dealers. This renders a certain market power to a dealer in her dealer-client relationship. The interdealer market on the other hand serves as a trading venue to mediate inventory imbalances from dealer-client transactions. Both aspects are captured in our model and provide a better fit with the institutional aspects of the European bond market than previous theoretical frameworks.²

The following section provides an overview of the European bond market and establishes some stylized facts about the behavior of retail spreads relative to interdealer spreads. Section 3 presents the model of demand intermediation under inventory constraints. Section 4 develops the empirical implications. We define aggregate dealer inventory imbalances, discuss their role for the average B2C transaction quality on either side of the market, and test the respective predictions. Conclusions follow in section 5.

2 Overview of the European Bond Market

2.1 Market Structure

The European sovereign bond market is the world's largest market for debt securities. With an outstanding aggregate value of approximately 4,395.9 billion Euros in 2006, it exceeds the size of the U.S. sovereign bond market with an aggregate value of roughly 4,413.5 billion Dollars (around 3 trillion Euros). The European market has as many issuers as countries and the outstanding value differs greatly across issuers. Table 1 provides an overview of the outstanding value by issuing country. The largest issuer is the Italian treasury with an outstanding sovereign debt of 1,213 billions Euros in 2005 followed by Germany and France.³

The market participants can be grouped into primary dealers, other dealers and customers. Customers are typically other financial institutions like smaller banks or investment funds. Dealers have access to electronic interdealer platforms, of which the most important is MTS. MTS has different shares of the interdealer market in different countries. The highest market share is reached for Portugal and Italy, where MTS has a market share of close to 100 percent. This dominant position of MTS is explained by market regulation in the case of Italy, which stipulates that all interdealer trades for monitoring purposes, have to occur on the MTS platform. In other countries MTS has a lower market share as shown in the last column of Table 1. But overall, approximately half of all interdealer trades

²A similar market structure is also observed for the foreign exchange market. For recent empirical work see Bjønnes and Rime (2005).

³For more institutional background, see also Dunne et al. (2006, 2007).

are transacted through MTS.

Trading in the MTS interdealer platform is similar in operation to any electronic limit order book market. It is dedicated to interdealer trading and customers do not have access. We therefore refer to MTS trades as B2B transactions. MTS dealers are mostly so-called ‘primary dealers’, which means that they face two-sided quoting obligations in exchange for privileged consideration when it comes to new bond issues. Primary dealers are allowed a maximum spread size in long maturity bonds of usually 7 basis points. However, this seems quite large when compared to the average inside spread of approximately 3 basis points.

Trading in the dealer customer segment of the market may also occur electronically. An important customer trading system in the European bond market is ‘BondVision’. It allows customers to electronically request quotes from a dealer. Dealers are not required to provide quotes when requested and neither are customers obliged to accept any submitted quote. The customer option to transact on any dealer quote expires after 90 seconds. Customers may have trading relationships with more than one of the many registered dealers on BondVision.⁴

The segmentation of European bond trading into the interdealer (B2B) and dealer-customer (B2C) market raises interesting questions with respect to market quality. Dealer intermediation in the European bond market is intermediation between these two market segments. Does this give rise to important differences in execution quality across the markets? How do changes in volatility and adverse selection affect transaction quality in both markets? These questions are addressed based on new micro data from both market segments.

2.2 MTS and BondVision Data

We explore a new data set which combines both interdealer (B2B) and dealer-customer data (B2C). The dealer-to-dealer data is sourced from the MTS inter-dealer electronic platform while the dealer-to-customer data comes from the BondVision request-for-quote system.⁵ The BondVision system is also owned by MTS. The data covers the last three quarters of 2005. It is reliably time stamped and trade initiation is electronically signed in both markets.

The total volume traded for the last 3 quarters of 2005 in the B2C BondVision platform was 404.4 billion Euros spread over 84,429 trades or roughly 4.5 billion Euros per day. Volume in the B2B segment

⁴For example, there are 35 dealers authorised to trade Italian bonds.

⁵The MTS B2B platform operates on a country-specific basis as well as at a pan-euro-area level where only the euro-benchmark bonds are traded. This introduces the possibility of fragmentation since some bonds can be traded on both platforms. However the analysis by DeJong et al. (2004) did not find any significant fragmentation from this source and in our analysis we do not distinguish between trading or quoting that takes place simultaneously on parallel MTS platforms.

was 2,368 billion Euros spread over 365,335 trades. Volume in the B2B was therefore about 4.3 times B2C volumes. The smaller B2C volume may largely reflect the fact that a significant proportion of B2C activity occurs in the OTC market or on other electronic platforms such as Tradeweb and Bloomberg Bond Trader (BBT). Despite the fragmentation of the market the BondVision platform represents a significant proportion of B2C electronic request for quote (RFQ) trading. This is particularly true for Italian issues, where anecdotal evidence suggests that a particularly high proportion of B2C trading occurs on BondVision. Given the dominant market position of MTS in the B2B segment, it is natural to focus the empirical analysis on Italian bonds.

Table 2 provides summary statistics on the B2B and B2C segment of the Italian bonds for the last 3 quarters of 2005. Over this period 72 different Italian bonds were traded on both MTS and BondVision. We group the bonds into three different maturity groups. Short-medium bonds have a maturity of 1.5 to 7.5 years, long bonds of 7.5 to 13.5 years and very long bonds feature maturities beyond 13.5 years. Each maturity group from the same issuer (Italian treasury) represents bonds which are presumably close substitutes so that they can be pooled for the purpose of our transaction cost analysis.⁶

In order to reduce data processing costs, we focus the analysis on the subsample of 13 highly liquid bonds. Overall, these 13 bonds account for 40 percent of the B2B volume and 35 percent of the B2C volume in Italian bonds. For the 6 selected long maturity bonds, the market representation of the subsample is even higher. They represent 81 percent of the B2B volume and 82 percent of the B2C volume.

One of the selected bonds was issued within the first month of our sample, while all others have been issued at least a year prior to the start of the sample period. In terms of the number of trades per month, we detected only a slight ‘on-the-run’ effect for the most recently issued bond. This contrasts with the pronounced ‘on-the-run’ liquidity effects observed by Barclay et al. (2006) in the U.S. Treasury market.⁷

The liquidity in all of the selected bonds is very high and relatively constant over the nine months of the sample. High liquidity at the inside spread justifies why we ignore market depth as an additional measures of B2B market quality. There is virtually no difference between the quoted and transacted spread as the available liquidity at the inside spread almost always exceeds any market order size.

⁶An exception here are the very-long maturity bonds. Pooling those is problematic because of considerable variation in coupon rates, maturity dates, and liquidity. We therefore select only one very long bond for the analysis.

⁷For additional work on the liquidity in the U.S. Treasury market see Fleming and Remolona (1999) and Brandt and Kavajecz (2004).

2.3 Transaction and Quote Quality in the B2C Market

The unique feature of our data is that it combines interdealer and dealer-customer prices data. It is therefore straightforward to access the competitiveness of the B2C segment by comparing the B2C trades to the best B2B quote at the same side of the market. We distinguish B2C trades which occur at the ask and compare them to the best B2B ask price prevailing at the same moment in time. Similarly, B2C trades at the bid side of the market are compared to the best available contemporaneous B2B bid price. We refer to this price difference as cross-market spread, defined as

$$\text{Cross-Market Spread (Ask)} = \text{B2C Ask Price} - \text{Best B2B Ask Price}$$

$$\text{Cross-Market Spread (Bid)} = -\text{B2C Bid Price} + \text{Best B2B Bid Price}.$$

How favorable are B2C transaction prices in BondVision relative to the best B2B quote on the same side of the market in the interdealer platform MTS?

Table 3 addresses this question. Reported is the cross-market spread for 5050 ask side trades and 4297 bid side trades for bonds in the 3 maturity groups. The cross-market spreads for each bond maturity are grouped in the 4 quantiles, where Q(1) denotes the 25 percent lowest (best) cross-market spreads and Q(4) represents the 25 percent highest (worst) spreads from the customer perspective. We report the quantile mean as well as the overall mean. The quantile mean is a better measure compared to the quantile limit itself. The latter is afflicted by the tick size clustering and therefore often not very sensitive to differences in the spread distribution.

The first insight from Table 3 is that B2C spreads are surprisingly competitive. The mean cross-market spread is negative for all 3 maturity groups on the ask side and for all but the short run bonds on the bid side of the market. B2C transactions occur on average at or inside the B2B spread. For the 6 long run bonds, for example, the average B2C transaction is 1.51 cents (≈ 1.51 basis points) more favorable than the best B2B quote on the ask side and 0.73 cents (≈ 0.73 basis points) more favorable than the best B2B quote on the bid side.⁸ The right-hand side of panels A and B report distribution of B2B spreads recorded at the time when B2C trades occur. The overall mean B2B half-spread on the ask side is 1.40 cents. The average ask side B2B half-spread is therefore very close to the absolute value of the ask side cross-market spread of -1.45 . B2C ask side trades are therefore centered around the midprice of the interdealer spread, suggesting a near ‘zero’ average spread. This finding is particularly surprising. On the bid side B2C trades are slightly less favorable, but still

⁸One explanation for the negative cross-market spread might be higher volume-based order processing costs charged by MTS for B2B transactions relative to B2C transactions. The higher market power of MTS trading platform in the interdealer market make this assumption plausible. Unfortunately, we were not able to obtain reliable data the fee structure of MTS.

extremely ‘low cost’. B2C trades are centered around a price level half-way between the B2B midprice and the best B2B bid price as the comparison between the average cross-market spread of -0.84 cents and the B2B half-spread of 1.68 reveals.⁹

A second insight concerns the maturity dependence of the cross-market spread. Long run bonds and the very long-run bonds with their high interest rate risk show relatively more favorable cross-market spreads. The overall mean for the cross-market spread decreases along the maturity dimension both on the ask and bid side. A clue as to why this is the case is provided for by the summary statistics on the B2B Spreads. For each B2C transaction, we record the momentary B2B spread and tabulate the quantile means in the same manner as for the B2C spreads. The B2B spreads increase noticeably in maturity in the same magnitude as the cross-market spreads decrease. This suggests that interest rate risk (associated with maturity) widens the B2B spread. Since the B2C spread is measured relative to the B2B spread as cross-market spread, it shows a relative improvement in bond maturity. This also shows that B2C quotes in BondVision are not as sensitive to the interest rate risk compared to the B2B quotes in the MTS interdealer platform.¹⁰

Table 4 explores the volatility dependence of the spread determination. We measure volatility as hourly realized volatility measured over return intervals of 2 minutes. Four different volatility levels are distinguished. ‘Low’ volatility periods are those with hourly realized volatility in the lowest 10 percent quantile. The ‘medium’ volatility captures volatility levels ranging from the 10 percent quantile to the 90 percent quantile. From the 90 percent to the 95 percent quantile we have the ‘high’ volatility range and beyond the 95 percent quantile we refer to ‘very high’ volatility. Table 4 reports quantile means for each volatility level as well as the overall mean. The average cross-market spread is on average negative for each of the 4 volatility levels. It decreases in volatility on the ask side and is almost constant on the bid side of the market. Ask side B2C trades improve (relative to the best B2B quote) in volatility and on the bid side they do not deteriorate as volatility increases. This finding is in contrast with the behavior of the B2B spread itself. B2B spreads show a pronounced increase in volatility both on the ask and the bid side. The increase in the average B2B spread from the lowest to the highest volatility category is 35 percent on the ask side and 12 percent on the bid side. A preliminary conclusion is that B2B spreads have a positive volatility sensitivity, while the

⁹Our findings contrast with Vitale (1998) who reports for the U.K. gilt market that transaction costs in the interdealer market are substantially smaller than those for external customers.

¹⁰It is useful to compare European interdealer spreads with typical spreads on the BrokerTec platform for U.S. Treasuries. Table 2 of Fleming and Mizrach (2008) reports interdealer half spreads which are easy to convert into cents. They are approximately 0.4 cents at the short, 0.75 cents at the long and 1.5 cents at the very long maturity. The corresponding numbers in Table 3 for the European sovereign bond market are approximately 0.4, 1.5 and 5.0. In other words, European spreads are comparable at the short end but much higher for long maturities.

cross-market spread has either none or even a negative one.

Table 5 considers the relation between both the cross-market and B2B spreads and inventory imbalance. We measure inventory imbalance using the (limit order) quantities at the best prices on either side of the B2B market prevailing at each B2C transaction. Imbalances are calculated across all 13 Italian bonds in the sample as the difference between the amount offered at the best ask price and the amount at the best bid price. Imbalance at each B2C bid and each B2C ask side trade are then grouped into four quantiles, which are labeled ‘very negative’, ‘negative’, ‘positive’ or ‘very positive’, respectively. Table 5 reports quantile means of the cross-market spread for each imbalance quantile as well as the overall mean. In general, on the ask-side the cross-market spread is becoming more negative as imbalance becomes more positive. The opposite is true for the bid-side. By contrast, there is no clear relationship between imbalance and the B2B spread measured at the same time as the B2C trades. This indicates that our measure of imbalances primarily characterizes transaction quality in the B2C market.

What explains these findings? Can they be derived from the structure of the European bond market? To explore this issue, we develop a simple model of Europe’s segmented bond market. The model allows for a better understanding of the volatility dependence of the dealer quote behavior in both market segments.

3 A Model of Intermediation in Segmented Markets

Microstructure models of dealer intermediation have incorporated adverse selection and inventory management concerns. We combine inventory management concerns with adverse selection risk in client transactions in a dynamic setting. The adverse selection risk is captured by time varying customer reservation prices which are observed by dealers only with a one period delay. Inventory management concerns are embodied simply as binding constraints on dealer inventory positions. For simplicity, dealer inventories cannot exceed these exogenous thresholds.

Most importantly, our model captures important institutional aspects of the European bond market. First, clients are excluded from participation in the B2B market and have to directly transact with a dealer. This creates a dual market structure with a B2B and B2C segment and differences in their competitive structure. Dealers may acquire a certain degree of market power in their dealer-client relationships. For simplicity, we assume the most extreme form of such market power where dealers do not compete for each other’s ‘clientele’. Second, the B2B segment only serves as a trading venue to intermediate dealer inventory imbalances stemming from transactions in the B2C segment.

Price determination here is competitive and transactions occur at the reservation price of the liquidity supplying dealer. For a highly transparent multi-dealer market this assumption is appropriate relative to a setting with a single market specialist considered by Madhavan and Smith (1993).

The model set-up is simple and nevertheless produces an astonishing richness of results. It allows us to (i) characterize the optimal inventory dependent quote behavior of dealers in the B2C market, (ii) determine the competitive interdealer spread in the B2B market, (iii) compare the cross-market spread and the interdealer spread for different levels of market volatility and (iv) show how aggregate dealer imbalances influence the quote behavior in the B2C segment of the market. The following section spells out the model assumptions in more detail.

3.1 Assumptions

Dealers face a stochastic environment in which potential customers arrive sequentially with uncertain reservation prices.

Assumption 1: Customer Flows

Customer quote requests for buy and sell quotes arrive each period with a constant probability q . Let R^a and R^b denote the customer reservation price such that the customer buys if $R^a > \hat{a}$ and sells if $R^b < \hat{b}$ where the requested ask and bid prices (\hat{a}, \hat{b}) are set one period ahead. Reservation prices have a uniform distribution with density d over the interval $[x_{t+1}, x_{t+1} + \frac{1}{d}]$ and $[x_{t+1} - \frac{1}{d}, x_{t+1}]$ for the ask and the bid, respectively. The mid-price x_{t+1} is a stochastic martingale process known to all dealers only at time $t + 1$. For simplicity we choose $\Delta x_{t+1} = x_{t+1} - x_t \in \{-\epsilon, +\epsilon\}$ with corresponding probabilities $(\frac{1}{2}, \frac{1}{2})$. All transactions concern a quantity of one unit.

Assumption 1 abstracts from all strategic dealer competition, in which the acceptance of a quote would directly depend on competing quotes of other dealers. It implicitly grants dealers a certain degree of monopolistic market power which depends on the distribution of reservation prices governed by the parameter d . A large d increases the monopolistic rents a dealer can earn from her dealer-client relationship. A dealer's monopolistic rents from this customer flow does not depend in any way on the price behavior of the other dealers. A second important aspect concerns the information structure. It is assumed that dealers quote optimal ask and bid prices for period $t + 1$ based on knowledge of the mid-price x_t , but not yet based on the new realization x_{t+1} . Hence dealer-quoted customer prices incorporate demand shocks only with a one period delay. This subjects dealers to an adverse selection

problem which widens spreads. The adverse selection risk increases in the volatility ϵ^2 of the midprice process x_t .

It is useful to denote standardized ask and bid quotes by $a = \hat{a} - x_t$ and $b = \hat{b} - x_t$, respectively.¹¹ Standardized quotes represent the quoted dealer prices relative to the current expected midprice $x_t = \mathcal{E}(x_{t+1})$. We also define cumulative density functions for the acceptance of a dealer quote as

$$\begin{aligned} F^a(R^a \geq \hat{a}) &= F^a(R^a - x_{t+1} \geq \hat{a} - x_{t+1} = a - \Delta x_{t+1}) = 1 - ad + d\Delta x_{t+1} \\ F^b(R^b \leq \hat{b}) &= F^b(R^b - x_{t+1} \leq \hat{b} - x_{t+1} = b - \Delta x_{t+1}) = 1 + bd - d\Delta x_{t+1}, \end{aligned}$$

respectively. A higher dealer ask price a for example decreases the quote acceptance linearly. The term $d\Delta x_{t+1}$ captures changes in the acceptance probability resulting from the exogenous evolution of the reservation price distribution.

For the purpose of inventory management, dealers can resort to an interdealer market with a spread $S = \hat{A} - \hat{B} > 0$.

Assumption 2: Competitive Interdealer Market

Dealers have access to the interdealer market and can buy inventory at an ask price \hat{A} and sell at price \hat{B} . The interdealer prices are cointegrated with the price process x_t with $\hat{A} = x_t + \frac{S}{2}$ and $\hat{B} = x_t - \frac{S}{2}$. We refer to standardized interdealer prices as $A = \hat{A} - x_t = \frac{S}{2}$ and $B = \hat{B} - x_t = -\frac{S}{2}$, respectively and assume $\frac{S}{2} \in [0, \frac{1}{d}]$. The ask and bid (limit order) prices A and B are set competitively (i.e. equal a dealer's reservation price) by a large number of dealers distributed across all inventory levels. Interdealer transactions require order processing costs of τ per transaction for the liquidity providers.¹²

The interdealer market allows a dealer to manage her inventory and respect their inventory constraints. Excessive long or short inventory positions can be reversed or at least be stabilized at prices B and A , respectively. The interdealer spread reflects all public dealer information about the price x_t . An important aspect of the analysis is to develop the (endogenous) equilibrium spread S under a competitive interdealer market structure. A competitive market structure implies that identical dealers with identical inventory levels compete away all rents from liquidity provision in the interdealer market. Hence, perfect interdealer competition makes dealers indifferent between having their limit

¹¹Hereafter, the expression ‘standardized quotes’ means the deviation of the quote from the prevailing B2B mid-price.

¹²MTS charges dealers for executed limit orders a fee which is proportional to trading volume. This brokerage fee may decrease in a dealer's overall MTS trading volume, but details on volume discounts were not disclosed to us. We assume for simplicity a fee structure which is constant for each unit of executed limit order supply.

order executed or not. The latter attribute implies that the interdealer transactions do not modify the value functions of the dealers.¹³

Assumption 3: Dealer Objectives and Inventory Constraints

A dealer sets optimal retail quotes (\hat{a}, \hat{b}) for the ask and bid price in order to maximize the expected payoff under an inventory constraint which limits her inventory level to the three values $I = 1, 0, -1$. She is required to liquidate any inventory above 1 or below -1 immediately in the interdealer market. Let $0 < \beta < 1$ denote the dealer's discount factor.

In order to limit the number of state variables we allow for only 3 inventory levels. This choice greatly facilitates the exposition.¹⁴ Inventory constraints embody the idea that dealers work within managerially pre-set position limits during the course of trading. Considering endogenously determined trading limits might be interesting, but any given limit is unlikely to change over the microstructure horizon we are considering here.

We summarize the sequence of trading in Figure 1. It is assumed that all payoffs come at the end of the period and are therefore discounted. We also note that the optimal B2C quotes generally depend on the inventory level as well as on the known state x_t of the lagged price. The following sections characterize a dealer's value function and her optimal quote behavior.

3.2 A Dealer's Value Function

We denote a dealer's value function for the present value of all future expected payoffs by $V(s, x_t)$. The state variable $s = 1, 0, -1$ represents one of the three possible inventory values. Furthermore, let $p_{s_t s_{t+1}}$ denote the transition probability of state s_t in period t to state s_{t+1} in period $t + 1$. For 3 states, a total of 9 transition probabilities characterize the transition matrix

$$\mathbf{M} = \begin{bmatrix} p_{12} + p_{11} & p_{10} & 0 \\ p_{01} & p_{00} & p_{0-1} \\ 0 & p_{-10} & p_{-1-1} + p_{-1-2} \end{bmatrix}.$$

The matrix element $p_{12} + p_{11}$ in the first row and column arises from two possible events. Starting from a maximum inventory of 1, the dealer remains in that state if she does not conduct any trades

¹³This aspect simplifies the analysis considerably. In a first step we solve for the optimal quote behavior of the dealers under an exogenous B2B spread. A second step consists in deriving the endogenous interdealer spread.

¹⁴It is possible to generalize the model to more inventory states at the cost of a more cumbersome exposition. On the other hand all analytical insights are preserved under the most parsimonious structure of only three inventory states.

in the B2C market and we denote this probability as p_{11} . Alternatively, the dealer might acquire an additional unit if her bid quote is accepted by a customer. In the latter case, the dealer would exceed the maximum inventory level of 1 and has to off-set the excess inventory immediately in the B2B market with a sell transaction. We denote this probability by p_{12} . The symmetric case arises under a negative inventory level of -1 , where we distinguish as p_{-1-2} the probability of a dealer selling an additional unit with the obligation to acquire immediately one unit in the B2B market.

The transition probabilities depend on the standardized state-dependent ask quotes $a(s)$ and bid quotes $b(s)$. We can now characterize the value function for the three inventory states as

$$\mathbf{V}(s, x_t) = \begin{bmatrix} V(1, x_t) \\ V(0, x_t) \\ V(-1, x_t) \end{bmatrix} = \max_{\{\hat{a}(s), \hat{b}(s)\}} \beta \mathcal{E}_t \left[\mathbf{M} \mathbf{V}(s, x_{t+1}) + \tilde{\mathbf{\Lambda}} \right] \quad (1)$$

where \mathcal{E}_t represents the expectation operator, and $\tilde{\mathbf{\Lambda}}$ denotes the period payoff given by

$$\tilde{\mathbf{\Lambda}} = \begin{bmatrix} \tilde{\Lambda}(1) \\ \tilde{\Lambda}(0) \\ \tilde{\Lambda}(-1) \end{bmatrix} = \begin{bmatrix} [\hat{B} - \hat{b}(1)] p_{12} + \hat{a}(1) p_{10} + r x_t \\ -\hat{b}(0) p_{01} + \hat{a}(0) p_{0-1} \\ -\hat{b}(-1) p_{-10} + [\hat{a}(-1) - \hat{A}] p_{-1-2} - r x_t \end{bmatrix}.$$

The payoff in state $s = 1$ includes the profit $\hat{B} - \hat{b}(1)$ in case a dealer's bid quote is executed (which occurs with probability p_{12}) and the expected profit $\hat{a}(1) p_{10}$ if the ask quote is accepted by a customer. The terms $r x_t$ and $-r x_t$ capture the interest revenue and cost in the two states with positive or negative bond inventories, respectively.¹⁵

The optimal quote policy can be characterized in terms of the standardized quotes $(a(s), b(s))$ and hence does not depend on the level of x_t . Quotes need to be optimal relative to any given level of the distribution of customer reservation prices. In other words dealers make their profit based on the spread, and not contingent on any particular price level of the underlying asset. The expected profit from a given spread should be the same independently of whether the bond price is 90 or 110 Euros. As a consequence, for a zero inventory level, the value function has to be independent of the price level, that is $V(0, x_{t+1}) = V(0, x_t) = V(0)$. For a positive or negative inventory level the value function is linear in the process x_t . Here a higher price level for the price process implies that a positive inventory level has a correspondingly higher value function. An analogous remark can be made with respect to a negative inventory. The value difference corresponds to the expected future sales value given by Δx_{t+1}

¹⁵For the interest rate r we assume $1/(1+r) = \beta$. The rate of interest equals the rate of time preference. This assumption assures that the value function takes on its simple linear form expressed in proposition 1.

for a positive inventory and $-\Delta x_{t+1}$ for negative inventory. We conclude that the value functions are fully characterized by two parameter values V and ∇ as summarized in the following proposition:

Proposition 1: Value Function Linearity

The value function of the dealer has the following properties:

$$\begin{aligned} V(1, x_{t+1}) &= V(1, x_t) + \Delta x_{t+1} &= V - \nabla + x_{t+1} \\ V(0, x_{t+1}) &= V(0, x_t) &= V \\ V(-1, x_{t+1}) &= V(-1, x_t) - \Delta x_{t+1} &= V - \nabla - x_{t+1} \end{aligned}, \quad (2)$$

where V and ∇ are two positive parameters.¹⁶

Proof: See Appendix A.

The value function is the discounted expected cash flow from being a dealer, i.e. of intertemporal intermediation in the B2C market and (occasionally) using the B2B market for inventory management. For the states $s = 1$ and $s = -1$ the value function $V(s, x_{t+1})$ accounts for the momentary value of the inventory given by x_{t+1} and $-x_{t+1}$, respectively. We can also show that $V(-1, 0) = V(1, 0) < V(0, 0)$. This is intuitive, as the dealer is in the more favorable position with a zero inventory than with either extreme inventory states. A dealer with no inventory owns the two-way option of being able to absorb both ask and bid transactions in the customer segment without having to resort to the interdealer market. In the extreme inventory states, the dealer owns a one way option. For example, with a positive inventory, a customer sell cannot be internalized and the dealer is forced into the B2B market: this reduces the value function. The parameter ∇ characterizes the concavity of the value function with respect to the inventory level. It embodies a dealer's value loss due to inventory constraints.

3.3 Optimal B2C Quotes

The first order conditions are obtained by differentiating the value function (1) with respect to the bid and ask prices $(\hat{a}(s), \hat{b}(s))$ for each inventory state s . The first order conditions do not depend on the price process x_t . The standardized quotes $(a(s), b(s))$ can be characterized only in terms of the interdealer spread S , the parameter ∇ and the density parameter d for the distribution of reservation prices.

¹⁶A necessary condition for existence is the usual transversality condition which requires that the present value of the future payoff is bounded.

For example, increasing the quoted ask price $a(1)$ in state $s = 1$ marginally by ∂a has two opposite effects. It increases the expected profit on prospective sell transactions which have a likelihood of $qF^a (R^a - x_{t+1} \geq a(1) - \Delta x_{t+1}) = q(1 - a(1)d + d\Delta x_{t+1})$ for the current period. This implies an expected profit increase of $q[1 - a(1)d] \partial a$. But a higher selling price also reduces the number of expected buyers by $(qd) \partial a$ and the value of each transaction is given by $a(1) + \nabla$. The marginal gain and loss are equalized for

$$q[a(1) + \nabla]d = q(1 - a(1)d),$$

which implies for the optimal ask quote

$$a(1) = \frac{1}{2d} - \frac{1}{2}\nabla.$$

Similar expressions are obtained for the two other inventory state and for the optimal bid quotes, which we summarize in proposition 2:

Proposition 2: Optimal B2C Quotes

For every given interdealer spread $0 < S < \frac{2}{d}$ and inventory state s , there exists a unique optimal ask and bid quote $(a(s), b(s))$ given by

$$\begin{bmatrix} a(-1) \\ a(0) \\ a(1) \end{bmatrix} = \begin{bmatrix} \frac{1}{2d} \\ \frac{1}{2d} \\ \frac{1}{2d} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \frac{S}{2} \\ \nabla \\ -\nabla \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} b(-1) \\ b(0) \\ b(1) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2d} \\ -\frac{1}{2d} \\ -\frac{1}{2d} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \nabla \\ -\nabla \\ -\frac{S}{2} \end{bmatrix} \quad (3)$$

which depend linearly on the concavity parameter ∇ and the interdealer spread S . The value function of a dealer follows as the perpetuity value of her future expected payoffs Λ_0 and the expected adverse selection losses Φ . Formally,

$$\mathbf{V}(s, 0) = \begin{bmatrix} V - \nabla \\ V \\ V - \nabla \end{bmatrix} = (\mathbf{I} - \beta \mathbf{M})^{-1} (\Lambda_0 + \Phi). \quad (4)$$

The concavity parameter $\nabla > 0$ is monotonically increasing in S and monotonically decreasing in the volatility ϵ^2 of the mid-price process x_t .

Proof: See Appendix B.

Equation (4) implicitly defines the concavity parameter ∇ as a function of the interdealer half-spread $\frac{S}{2}$. A particular parameter combination $(\frac{S}{2}, \nabla)$ corresponds to optimal B2C quotes. This

equilibrium schedule is graphed in Figure 2 as the B2C equilibrium schedule in a space spanned by $\frac{S}{2}$ and ∇ . The concavity parameter ∇ monotonically increases in the B2B half-spread $\frac{S}{2}$. Intuitively, higher interdealer spreads render inventory imbalances more costly as rebalancing occurs at less favorable transaction prices. An increase in ∇ affects the optimal quotes differently according to a dealer's inventory state. The optimal B2C quotes $a(1)$ and $b(-1)$ become more favorable as dealers seek to substitute B2C trades for more costly B2B trades, while B2C quotes under balanced inventories $a(0)$ and $b(0)$ deteriorate.

The next section develops the equilibrium condition for the interdealer market.

3.4 Competitive B2B Spreads

A competitive market structure for interdealer quotes implies that identical dealers with identical inventory levels compete away all rents in the B2B segment. Interdealer competition makes dealers indifferent between having their limit order executed or not. Hence, interdealer transactions do not modify the value functions of the dealers. The first-order conditions developed in proposition 2 remain therefore valid even if we allow dealers to engage in B2B liquidity supply through an electronic limit order market.

Dealers with extreme inventories have a value function which is lower by $\nabla > 0$. Dealers with a negative inventory position of -1 gain ∇ by increasing their inventory level to zero and dealers with a positive inventory position *also* gain ∇ by decreasing their inventory to zero. Hence, dealers with a short inventory position will provide the most competitive interdealer bid B while dealers with a positive inventory submit the most competitive interdealer ask A . The competitive spread is therefore determined by the two dealers with extreme positions who make a gross gain ∇ by moving to a zero inventory position.

Limit order submission in the interdealer market also amounts to writing a trading option which other dealers can execute. In particular, we assume that a dealer with an inventory position deteriorating from -1 to -2 following a customer buy order immediately needs to rebalance to -1 by resorting to a market buy order in the interdealer market. Under assumption 1, the distribution of the customer reservation prices is assumed to move up or down by ϵ . For example, a rise in the mid-price ($\Delta x_{t+1} = \epsilon > 0$) increases customer demand at the ask. The area of the reservation price distribution which leads to the customer acceptance of a dealer quote at the ask increases by ϵd because the reservation price distribution is uniform. This probability change is multiplied by the probability q of customer arrival to produce an upward demand shift of $\epsilon q d$. Similarly, sales at the bid to a dealer with inventory 1 fall by the same amount. Analogous remarks can be made for the case of a fall in

the mid-price process.

The customer demand increase at the ask price, $a(-1)$, for a dealer with inventory -1 spills over into the B2B market. Similarly, the customer sales decrease at the bid, $b(1)$, faced by dealer with inventory 1 is also passed on to the B2B market. The B2B market order flow is therefore correlated with Δx_{t+1} . Hence, the limit order submitting dealer in the B2B market is exposed to an adverse selection problem. She faces a systematically higher execution probability at the ask price A if the customer moves toward a higher valuation and a lower execution probability for limit orders at the bid price B . The following proposition characterizes the expected adverse selection loss and the competitive B2B half-spread $\frac{S}{2}$.

Proposition 3 : Competitive B2B Quotes

The expected adverse selection loss due to executed limit order at both ask and bid is given by¹⁷

$$L = L^A = L^B = \frac{2\epsilon^2}{\frac{1}{d} - \frac{S}{2}}.$$

Under quote competition in the B2B market, the competitive ask and bid prices are given by

$$\begin{aligned} A &= \max(L - \nabla + \tau, 0) = \frac{S}{2}, \\ B &= \min(-L + \nabla - \tau, 0) = -\frac{S}{2}, \end{aligned} \tag{5}$$

respectively, where τ represents the order processing costs of the liquidity provider and ∇ denotes the concavity parameter of the dealers' value function.

Proof: See Appendix C.

An interesting feature of Proposition 3 is that the expected adverse selection loss of an executed limit order does not depend on the distribution of inventories across the dealers. This seems counter-intuitive at first. A larger number of limit order submitting traders for example reduces the likelihood of execution for any given limit order. However, what matters for the adverse selection loss of executed trades is not the likelihood of execution itself, but the probability of adverse midprice movement conditional on execution. The latter is not contingent on the distribution of dealers across the inventory states. Not surprisingly, the loss function is increasing in the variance ϵ^2 of the market process x_t . It is also increasing in the density d of reservation prices because the more concentrated the distribution,

¹⁷Recall that the properties of the uniform distribution require that the denominator is positive.

the greater is the shift in demand induced by any given price change. Finally, the expected adverse selection loss is increasing in the interdealer spread. Note that dealers adjust their B2C quotes $a(-1)$ and $b(1)$ to a widening B2B spread S . If B2C execution occurs nevertheless, then it is highly correlated with the directional change Δx_t of the reservation price distribution, which implies a high adverse selection risk for the liquidity suppliers in the B2B segment.

The equilibrium condition expressed in the second part of proposition 3 is straightforward. A dealer with a positive inventory submits a sell limit order at the B2B ask with price A . Her expected adverse selection loss conditional on execution is L , but she gains ∇ by moving to a zero inventory if execution occurs. Under the competitive market assumption 2, her expected conditional profit is zero, hence $A + \nabla - L - \tau = 0$, where τ represents the order processing costs. An analogous remark applies at the bid price B . We also note that for the B2B quotes given by equation (5), dealers in inventory states $s = \pm 1$ do not find it optimal to submit market orders as the cost $\frac{S}{2}$ exceeds their benefit ∇ of rebalancing. Only dealers who run against the inventory limits at ± 2 place market orders.

Proposition 3 shows that the B2B spread is given by the difference between the adverse selection loss L and the benefit of moving to a zero inventory. The interdealer quote spread is therefore negatively related to the benefit of moving to a zero inventory position and positively to the adverse selection loss of quote submission. As with the B2C locus, we can graph the B2B locus in the $(\frac{S}{2}, \nabla)$ space. It is the parabola illustrated in Figure 2 with the label B2B. Its intercept and turning point are derived in Appendix D.

For a low B2B spread S , an increase in the B2B spread comes with a decrease in the concavity parameter ∇ . Intuitively, the most competitive B2B quote is provided on the ask side by dealers with positive inventory and on the bid side by dealers with negative inventory. A successful B2B transaction moves the dealer in both cases to the zero inventory state and the associated value gain is given by ∇ . Under competitive B2B bidding, a higher value gain from rebalancing implies a lower B2B spread. Hence the negative link between S and ∇ at low levels of volatility. As the equilibrium spread S becomes large, the expected adverse selection loss L increases non-linearly. For liquidity supplying dealers to still earn a zero expected profit, the benefit of reverting to a zero inventory given by ∇ has to increase as S increases. The steepness of the loss function in S eventually dominates and implies a positive relationship between S and ∇ .

3.5 Existence and Stability of the Equilibrium

The previous sections derive separately the equilibrium relationship for the B2B and B2C markets in the $(\frac{S}{2}, \nabla)$ space. It is shown that the optimal quotes in the B2C market depend on the spread S

in the B2B market. Inversely, the equilibrium spread in the B2B market depends on the concavity parameter ∇ of the value function under optimal B2C quote setting. This market interdependence requires that we solve the model for the joint equilibrium in both markets. The joint equilibrium solution is illustrated in Figure 2 as the intersection of the B2B and B2C graphs. Figure 2 highlights that there could be up to two equilibria. We label the equilibrium, where both $\frac{S}{2}$ and ∇ are high as Z_U in contrast to the equilibrium Z_L with low values of $\frac{S}{2}$ and ∇ . It is straightforward to identify Z_U as the unstable equilibrium. Assume two dealers with opposite inventory positions deviate from equilibrium Z_U to Z_L by quoting the much narrower interdealer spread S_L . Since the effective interdealer spread is determined by the most competitive quote, their quoted spread S_L becomes the new reference point for the customer segment of the market. Hence, all customer quotes in the B2C market adjust also to the new equilibrium Z_L , whereby the previous equilibrium is identified as unstable. Note that the equilibrium Z_L cannot be destabilized by the reverse process of two dealers quoting higher spreads. Their quotes would stand no chance of being executed. Hence these non-competitive quotes are irrelevant and cannot trigger any adjustment in the B2C segment of the market. We can therefore conclude that Z_L is the only stable equilibrium and discard Z_U .

Proposition 4: Equilibrium Existence and Stability

Under assumption (1) to (3) and market volatility ϵ^2 below some threshold $\bar{\epsilon}^2$, there exists a single stable equilibrium pair $(\frac{S}{2}, \nabla)$ for the B2B spread S and the convexity of the dealer value function ∇ , such that (i) dealers make optimal customer quotes as stated in proposition 2 and (ii) these quotes imply a value function with convexity ∇ so that S is the competitive B2B spread as stated in proposition 3.

Proof: See Appendices D.

The uniqueness of the stable equilibrium Z_L allows us to undertake comparative statics with respect to the price volatility ϵ^2 . Note that the price volatility is directly tied to the information asymmetry between customer and dealer and the degree of adverse selection under quote provision. The axis intercepts in Figure 2 shows that a volatility increase (higher ϵ^2) pushes the B2B locus upwards and the B2C locus to the right. The B2B spread unambiguously increases. The same is true for an increase in the order processing costs τ which also shifts the B2B schedule upwards. Again, the interdealer spread S increases as the higher cost of liquidity provision in the B2B market is incorporated into the interdealer spread.

It is also instructive to consider two boundary cases. First, for zero volatility, the B2C schedule passes through the origin, while the intercept for the B2B curve is at the level τ . In the absence of

any adverse selection, the interdealer spread reaches its minimum at a level which is less than the order processing cost because the dealer is still partly compensated by an option value of inventory holding ∇ , which remains positive. For zero order processing costs ($\tau = 0$), the competitive interdealer spread becomes zero. Second, consider a high level of price volatility given by $\epsilon^2 = \frac{1}{4d^2}$. At this level of volatility the B2C equilibrium schedule degenerates to a single point $(\frac{1}{d}, 0)$ without any possible intersection with the B2B locus. We conclude that at very high levels of volatility, the adverse selection effect does not allow for a market equilibrium. The market equilibrium can only exist for a volatility of the process x_t below a critical threshold so that the B2B and B2C schedule still intersect.

The derivation of the joint equilibrium implicitly assumes that there are, at any period, dealers with inventory positions 1 and -1 , who maintain the inside B2B spread S . This assumption is generally fulfilled in a large market with many dealers. However, for dealership markets with only a few dealers this might be more problematic. In this case the positive probability of having to rebalance at a wider interdealer spread has to be incorporated into the model.

4 Empirical Implications

4.1 A Linearized Model Solution

It is straightforward, though tedious, to solve equations (17) and (21) for the B2B and B2C spreads. A more informative representation is obtained by a simple linearization of the model.

Proposition 5: Linear Equilibrium Approximation

A linear approximation to the joint market equilibrium implies inventory-dependent optimal B2C quotes which are linearly dependent on market volatility $Vol = \epsilon^2$ according to

$$\begin{aligned}
 a(-1) &= \gamma_{1c} + \gamma_{1v} \times Vol & b(-1) &= -\gamma_{3c} \\
 a(0) &= \gamma_{2c} & b(0) &= -\gamma_{2c} \\
 a(1) &= \gamma_{3c} & b(1) &= -\gamma_{1c} - \gamma_{1v} \times Vol
 \end{aligned} \tag{6}$$

and a B2B half-spread given by

$$\frac{1}{2}S = \frac{1}{2}(A - B) = \gamma_{4c} + \gamma_{4v} \times Vol, \tag{7}$$

where the parameters fulfill $\gamma_{1c} > \gamma_{2c} > \gamma_{3c} > 0$; $\gamma_{2c} > \gamma_{4c} > 0$ and $\gamma_{4v} > \gamma_{1v} > 0$.

Proof: See Appendix E.

The B2C spread shows a volatility dependence which differs across inventory states. The most unfavorable ask side quote $a(-1)$ increases in volatility and the most unfavorable bid side quote $b(1)$ decreases in volatility. The volatility dependence in these two inventory states reflects the volatility dependence of the B2B spread. In both inventory states it is possible that the dealer has to resort to the B2B market if the respective B2C quotes are executed. In order to avoid trading losses, the B2C quotes deteriorate in volatility. But the volatility dependence of the B2B spread is nevertheless much stronger than for the B2C quotes $a(-1)$ and $b(1)$ as $\gamma_{4v} > \gamma_{1v}$. The four B2C quotes $a(0) > a(1) > b(-1) > b(0)$ are constant in volatility under the linear approximation. Intuitively, the market power of the dealer implies a monopolistic B2C quote with a constant price mark-up determined by the distribution of reservation prices. The mark-up largely absorbs the adverse selection effect under increasing volatility except for the outside quotes $a(-1)$ and $b(1)$ which have to account for the probability of rebalancing in the the B2B market. The competitive nature of the B2B market on the other hand fully reflects the adverse selection effect and therefore features a strong volatility dependence. Figure 3 plots for the ask side (Panel A) and the bid side (Panel B) the exact numerical solutions for B2C quotes and the B2B spread as a function of volatility. The four B2C quotes $a(0) > a(1) > b(-1) > b(0)$ show virtually no volatility dependence and are indistinguishable from a constant. The two outside B2C quotes $a(-1)$ and $b(1)$ deteriorate with higher volatility, but much less so than the best B2B quotes $A = \frac{S}{2}$ and $B = -\frac{S}{2}$. Overall, the model linearization in Proposition 5 provides a rather accurate approximation to the exact quotes for a wide volatility range.

The finding of a strong volatility dependence of the B2B spread and a weak volatility dependence of the B2C spreads implies the following:

Corollary 1: Volatility Dependence of the Cross-Market Spread

Higher volatility improves the quality of the average B2C trade (\bar{a}, \bar{b}) relative to the B2B spreads (A, B) as measured by the average cross-market spreads, $\bar{a} - A$ and $-\bar{b} + B$, respectively. The average cross-market spread decreases in volatility both on the ask and bid sides of the market.

Proof: See Appendix E.

The two graphs on the right hand side of Figure 3 show the average cross-market spread, which is identical for the ask and bid side of the market. The average cross-market spread is decreasing in volatility due to the differential volatility dependence of the B2B and B2C quotes. Interestingly, the

average cross-market spread can become negative beyond a certain volatility level. This corresponds to the findings reported in Table 4, which shows that the average cross-market spread is negative and decreases in volatility. The model nevertheless falls short of fully describing the data on the ask side of the market. While it can qualitatively explain negative cross-market spreads, it cannot explain the magnitude of the negative cross-market spread observed here. The average ask side B2C transaction spread (as measured against the midprice) is approximately zero. The latter finding is not explained by the model.

4.2 Evidence on the Volatility Dependence of Spread

This section applies regression analysis to test for the negative volatility dependence of the cross-market spread predicted in Corollary 1. A linear regression

$$\begin{aligned} \text{Cross-Market Spread (Ask)} &= a - A = \mu_{a0} + \mu_{av} \times Vol + \epsilon \\ \text{Cross-Market Spread (Bid)} &= -b + B = \mu_{b0} + \mu_{bv} \times Vol + \epsilon \end{aligned} ,$$

implies parameter estimates $\mu_{av} = \mu_{bv} < 0$.

A potential problem with this regression is simultaneity bias. For example, relatively high realizations of the best B2B ask quote A changes the cross-market spread on the ask side negatively. But such data points simultaneously increase the volatility measure which is based on variations of the midprice $MidP = \frac{1}{2}(A + B)$. In order to eliminate this simultaneity bias in the regression, an instrumental variable approach is needed. Lagged volatility is fortunately a very good instrument for the contemporaneous volatility measure and it is therefore used in the regression. We also include fixed effects for each bond to control for heterogeneity across bonds.

Table 6 presents the regression results for the cross-market spread. Panel A reports the regression results for the ask side and panel B for the bid side of the market. In each case we run a regression for the full sample of all 13 bonds and the subsample of 6 long-dated bonds. The 6 long-dated bonds form a particularly homogenous subsample in terms of coupon rates, maturity and liquidity characteristics and at the same time represent a large share of the overall bond transactions in Italian long-dated bonds. The regression results are consistent with the findings from Table 4. The cross-market spread on the ask side is almost constant in the volatility and decreasing on the bid side. The decrease on the bid side is statistically significant at the one percent level for both the full sample and the subsample of long maturity bonds. The behavior of the bid side spread is therefore fully consistent with the model prediction. For the ask side we cannot confirm that the predicted cross-market spread decrease in volatility. On the other hand we do not find any positive volatility effect either. Hence, there is no volatility premium on the B2C ask side relative to the best B2B quotes.

The B2B spreads show, as expected, a highly significant positive volatility dependence. The volatility dependence in the full sample is stronger on the bid side than the ask side with coefficients 0.310 and 0.212, respectively. The more positive volatility dependence for the B2B spread on the ask side may explain why we find a more negative volatility dependence for the cross-market spread on the ask side as well. The asymmetry in the spread behavior between the ask and bid side needs to be explained by forces outside the presented model framework. For example, the magnitude of the adverse selection problem faced by dealers may be conditional on up- or down movement of the market price. This may explain why the volatility sensitivity of the B2B market differs between the ask and bid side of the market. But rather than focusing on the bid-ask asymmetries, we next look at the central issue of inventory imbalances and their role in the determination of the B2C quotes.

4.3 Aggregate Inventory Imbalances and B2C Trades

A key insight of the model is that the B2C quotes depend on the inventory state of the dealer. Unfortunately, such inventory data is not directly available. However, inventory imbalances also induce dealers to submit the most competitive B2B quotes. The relative depth of the best B2B quotes therefore indicate the distribution of inventory imbalances within the dealer population. We can therefore infer the aggregate inventory imbalances from the B2B market and verify empirically whether inventory imbalances have the predicted role for the B2C quotes. For example, a large depth in the B2B market at the inside ask quote indicates a willingness of many traders to sell and this should occur under undesirable positive inventory, namely the state $s = 1$ in our model.

To obtain an empirical counterpart to inventory imbalances, consider that n dealers compete in the B2B market. Their distribution over the three inventory states $s = -1, 0, 1$ is denoted by $n(-1)$, $n(0)$ and $n(1)$, respectively. We define the imbalance towards positive inventory as

$$Imb = \frac{n(1) - n(-1)}{n(1) + n(-1)},$$

where $-1 \leq Imb \leq 1$. Since each of the dealers in states $s = -1$ and $s = 1$ submits a unit quantity of liquidity at the best B2B bid and ask price, respectively, we can directly measure the variable Imb without observing dealer specific inventory states.

We can express the (conditional) probability distribution of traders over the three inventory states as a function of the variable Imb . The share of traders with a balanced inventory can be defined as

$$c_t = \frac{n(0)}{n(1) + n(0) + n(-1)}$$

and the expected share as $E(c_t) = \bar{c}$. The number of dealers with unbalanced inventories follows simply

as $n(1) + n(-1) = (1 - c_t)n$. The probability of a particular trader to be in state s is given by

$$p(s) = p(s, Imb, \bar{c}) \begin{cases} E \left[\frac{n(1)}{n} \right] & = \frac{1-\bar{c}}{2}(1 + Imb) & \text{for } s = 1 \\ E \left[\frac{n(0)}{n} \right] & = \bar{c} & \text{for } s = 0 \\ E \left[\frac{n(-1)}{n} \right] & = \frac{1-\bar{c}}{2}(1 - Imb) & \text{for } s = -1 \end{cases} . \quad (8)$$

A high value for imbalances Imb therefore implies a relatively higher expected probability that a representative dealer is in inventory state $s = 1$ and a lower expected probability for him to be in state $s = -1$.

An attractive feature of the aggregate imbalance variable Imb is its observability in the B2B order book data. According to our model, each dealer with a positive inventory submits a bid quote B in the B2B market at the best inside quote. The total liquidity available at the best bid is therefore proportional to the number of dealers with inventory $s = 1$. The same holds for dealers in state $s = -1$, who are the liquidity suppliers at the best B2B ask. We can therefore measure aggregate inventory imbalances as

$$Imb = \frac{Q(\text{Bid}) - Q(\text{Ask})}{Q(\text{Bid}) + Q(\text{Ask})},$$

where $Q(\cdot)$ denotes the limit order book liquidity at the best ask or bid, respectively.

The average B2C quotes (\bar{a}, \bar{b}) depend on the distribution of inventory states $p(s)$. Formally, we have

$$\bar{a} = \sum_{s=-1,0,1} p(s)a(s)g(a(s)) \quad \text{and} \quad \bar{b} = \sum_{s=-1,0,1} p(s)b(s)g(b(s)).$$

where $p(s)$ represents the probability of inventory state s . The functions $g(a(s)) = 1 - a(s)d$ and $g(b(s)) = 1 + b(s)d$ denote the probabilities that customer quotes $a(s)$ and $b(s)$ are accepted. A positive inventory imbalance implies that relatively more dealers are in state $s = 1$ and this implies that more customers receive favorable ask quotes $a(1)$ and unfavorable bid quotes $b(1)$. The expected B2C ask and bid transaction prices (\bar{a}, \bar{b}) should therefore decrease in the inventory imbalance Imb .

Figure 4, panel A plots the average cross-market spread $\bar{a} - A$ on the ask side as a function of the inventory imbalance and the volatility. The corresponding cross-market spread $-\bar{b} + B$ on the bid side is featured in panel B. As before, higher volatility decreases this spread because of the higher volatility sensitivity of the B2B spread S . Moreover, Figure 4 also reveals the dependence of the cross-market spread on the inventory imbalance. A more positive aggregate inventory imbalance, namely more dealers in state $s = 1$ relative to $s = -1$, comes with a lower average ask quote \bar{a} and therefore a lower cross-market spread on the ask side. On the bid side, the cross-market spread increases in the imbalance statistics as depicted in panel B. Intuitively, a positive imbalance comes with a tilt of

the probability distribution of dealer states towards $s = 1$ as described in equation (8). This implies relatively more dealers quote B2C prices $a(1)$ or $b(1)$ relative to $a(-1)$ or $b(-1)$. Hence the average cross-market spread deteriorates on the ask side and improves on the bid side. The dependence of the cross-market spread on both volatility and the inventory imbalance is summarized as follows:

Proposition 6: Transaction Spreads under Dealer Inventory Imbalances

The cross-market spreads on the ask and bid side can be linearly approximated by

$$\begin{aligned} a - A &= \mu_{a0} + \mu_{av} \times Vol + \mu_{aI} \times Imb + \epsilon \\ -b + B &= \mu_{b0} + \mu_{bv} \times Vol + \mu_{bI} \times Imb + \epsilon \end{aligned}$$

where we expect for the coefficients $\mu_{av} = \mu_{bv} < 0$ and $\mu_{aI} = -\mu_{bI} < 0$.

Proof: See Appendix D.

Previous work has found evidence for inventory effects on prices in equity and future markets. Hasbrouck and Sofianos (1993) for example find evidence that inventory shocks influence the quote behavior of NYSE specialists. Manaster and Mann (1996) confirm inventory price effects in future trading and Lyons (1997) for a single FX dealer. The following section takes up this issue for the European bond market.

4.4 Evidence on the Role of Aggregate Imbalances

Extending the previous regression on the nexus between volatility and spreads to inventory imbalances is straightforward. Price outliers in the interdealer market tend to influence both the B2B half-spread and the volatility measurement in the same period. To avoid this simultaneity bias, we use again an instrumental variable approach based on lagged volatility instead of contemporaneous volatility.

Table 7 presents the regression results for the inventory dependence of the cross-market spread. Panel A reports the regression results for the ask side and panel B for the bid side. In each case we run a regression for the full sample of all 13 bonds and the subsample of 6 long-dated bonds. The estimation coefficients have the signs predicted in proposition 6 and are therefore consistent with the numerical results depicted in Figure 4. The point estimates for the volatility dependence of the spread are very similar to those in Table 6. The imbalance measure is almost orthogonal to the volatility measure and its inclusion in the regression is without consequence of the spread-volatility nexus.¹⁸

¹⁸The correlation between imbalances and volatility for the long-dated bonds is miniscule at 0.0076.

The imbalance measure itself is statistically highly significant with t-statistics above 3. For the ask side we find a negative effect on the cross-market spread and for the bid side a positive coefficient as predicted by proposition 6. The intuition is simple. A large number of dealers with positive inventory will tend to increase the liquidity available at the best bid relative to the best ask and therefore generate a positive realization for the imbalance measure. But a positive inventory imbalance by the majority of traders will also imply that the average B2C quote on the ask side is very favorable and on the bid side very unfavorable. As a consequence, the cross-market spread should *ceteris paribus* be low on the ask side and high on the bid side of the market as depicted in Figure 4.

Finally, we highlight that the point estimates for imbalances between 0.289 and 0.418 are also economically significant. To see this assume that inventory imbalances move over half the maximal range from -0.5 to 0.5 . The coefficient estimates then represent the corresponding change in the B2C price quality in cents. Such an inventory related price change is large considering that the B2B half-spreads are on average only 1.40 cents on the ask and 1.68 cents on the bid side whenever B2C trades occur.

5 Conclusions

Microstructure research has typically framed a dealer's intermediation problem within a single market which enable both liquidity provision and inventory rebalancing. The segmented market structure of the European bond market separate both functions. Liquidity provision for customers occurs through request for quote systems like BondVision, while the electronic interdealer platforms like MTS primarily serve dealers' rebalancing needs. Customers generally do not have direct market access to the interdealer platform.

This paper examines the transaction quality in such a segmented market structure. Synchronized price data from both market segments allow us to compare B2C transactions to the prevailing B2B quotes. The price difference between the B2C price and the best B2B quote is referred to as the cross-market spread. Three stylized findings emerge from the data: First, the cross-market spread is on average very low and even negative. Customer transactions are therefore (on average) very favorably priced. Second, the price dispersion of the cross-market spread is nevertheless large. The price difference between the 25 percent best and the 25 percent worst B2C trades on either the bid or the ask side of the market exceeds the average B2B spread. Third, B2B and B2C prices feature different sensitivities with respect to market volatility. As expected, B2B spreads increase in midprice volatility. But the same volatility dependence is not found for the cross-market spread. Cross-market

spreads are constant or even decreasing in volatility and particularly low for long maturity bonds.

The recent literature has argued that price dispersion in dealer-customer transactions may reflect price discrimination between informed and uninformed investors. Dealers may for example earn informational rents on illiquid municipal bonds which are difficult to price for a retail investor. High price transparency of B2B quotes in European sovereign bonds and a sophisticated institutional buy side make such an explanation very implausible for European bond prices. We argue instead that the B2C price dispersion is driven by the dealers inventory management concerns. Under inventory constraints, dealers find it optional to provide B2C price mark-ups or discounts if their dealer-client relationship grants them some degree of market power. Inventory dispersion can thus generate cross sectional B2C price dispersion.

Dealer market power can also explain the volatility puzzle for the cross-market spreads. Quote behavior in the competitive B2B segment is very sensitive to the adverse selection risk which comes with higher volatility. Optimal B2C price quotation by contrast is strongly inventory dependent, but less sensitive to changes in adverse selection risk. Intuitively, monopolistic mark-ups in the customer segment can partly absorb increasing adverse selection losses in customer transactions. Customer trades therefore become relatively more competitive compared to interdealer trades on the same side of the market as volatility increases.

An additional empirical prediction of our model framework is the inventory dependence of the B2C quote behavior. Do dealer inventory effects influence the B2C trade quality? Inventory data is generally not available in multi-dealer markets like the European bond market. But we have access to the limit order book in the interdealer trading platform MTS and can use this information to infer the aggregate state of the dealer inventory. Optimal inventory management through this B2B segment implies that dealers with a positive inventory imbalance tend to submit limit orders at the best bid and dealers with a negative inventory post liquidity at the best ask. The relative depth of the limit order book at the best bid relative to the best ask therefore proxies for the aggregate inventory imbalance among all dealers. We show that the inferred measure of inventory imbalances is indeed a strong predictor of the B2C trade quality. A positive inventory imbalance decreases customer trade costs on the ask side and increases customer trade costs on the bid side. The dealer inventory effect is both statistically and economically significant for the quality of B2C transactions.

References

- [1] Amihud, Y., and H. Mendelson, 1980, Dealership Markets: Market Making with Inventory, *Journal of Financial Economics*, 8, 31-53.
- [2] Barclay, M. J., T. Hendershott, and K. Kotz, 2006, Automation versus Intermediation: Evidence from Treasuries Going Off the Run, *Journal of Finance*, 61:5, 2395-2414.
- [3] Bessembinder, H., W. Maxwell, and K. Venkataraman, 2006, Market Transparency, Liquidity Externalities, and Institutional Trading Costs in Corporate Bonds, *Journal of Financial Economics*, 82:2, 251-288
- [4] Bjønnes, G. H., and D. Rime, 2005, Dealer Behaviour and Trading Systems in Foreign Exchange Markets, *Journal of Financial Economics*, 75, 571-605.
- [5] Brandt, M. W. and K. A. Kavajecz. 2004, Price Discovery In The U.S. Treasury Market: The Impact of Orderflow and Liquidity on the Yield Curve, *Journal of Finance*, 59:6, 2623-2654.
- [6] DeJong, F., Y. Chung, and B. Rindi, 2004, Trading European Sovereign Bonds: The Microstructure of the MTS trading platforms, CEPR Discussion Paper 4285.
- [7] Dunne, P. G., M. Moore, and R. Portes, 2006, European Government Bond Markets: Transparency, Liquidity, Efficiency; City of London, Corporation Monograph commissioned from CEPR, <http://www.cityoflondon.gov.uk/economicresearch>.
- [8] Dunne, P. G., M. Moore, and R. Portes, 2007, Benchmark Status in Fixed-Income Asset Markets, *Journal of Business Finance and Accounting*, forthcoming.
- [9] Edwards, A. K., L. Harris, and M. Piwowar, 2007, Corporate Bond Market Transaction Costs and Transparency, *Journal of Finance*, 62:3, 1421-1451.
- [10] Fleming, M. J., and E. M. Remolona, 1999, Price Formation and Liquidity in the U.S. Treasury Market: The Response to Public Information, *Journal of Finance*, 54:5, 1901-1915.
- [11] Fleming, M. J., and B. Mizrach, 2008, The Microstructure of a U.S. Treasury ECN: The BrokerTec Platform, mimeo, [http://snde.rutgers.edu/wp/\[48\]-WP_brokertec.pdf](http://snde.rutgers.edu/wp/[48]-WP_brokertec.pdf).
- [12] Glosten, L., and P. Milgrom, 1985, Bid, Ask, and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders, *Journal of Financial Economics*, 14:1, 71-100.

- [13] Goldstein, M. A., E. S. Hotchkiss, and E. R. Sirri, 2007, Transparency and Liquidity: A Controlled Experiment on Corporate Bonds, *Review of Financial Studies*, 20:2, 235-273.
- [14] Green, R., B. Hollifield, and N. Schurhoff, 2007, Financial Intermediation and the Costs of Trading in an Opaque Market, *Review of Financial Studies*, 20:2, 275-314.
- [15] Harris, L., and M. Piwowar, 2006, Secondary Trading Costs in the Municipal Bond Market Preview, *Journal of Finance*, 61:3, 1361-1397.
- [16] Hasbrouck, J., and G. Sofianos, 1993, The Trades of Market Makers: An Empirical Analysis of NYSE Specialists, *Journal of Finance*, 48, 1565-1594.
- [17] Kyle, A., 1985, Continuous Auctions and Insider Trading, *Econometrica*, 53:6, 1315-1335.
- [18] Lyons, R. K., 1997, A Simultaneous Trade Model of the Foreign Exchange Hot Potato, *Journal of International Economics*, 42, 275-98.
- [19] Madhavan, A., and S. Smidt, 1993, An Analysis of Changes in Specialist Inventories and Quotations, *Journal of Finance*, 48:5, 1595-1628.
- [20] Manaster, S., and S. Mann, 1996, Life in the Pits: Competitive Market Making and Inventory Control, *Review of Financial Studies*, 9, 953-976.
- [21] O'Hara, M., and G. Oldfield, 1986, The Microeconomics of Market Making, *Journal of Financial and Quantitative Analysis*, 21:4, 361-376.
- [22] Stoll, H., 1978, The Supply of Dealer Services in Securities Markets, *Journal of Finance*, 33, 1133-1151.
- [23] Vitale, P., 1998, Two months in the life of several gilt-edged market makers on the London Stock Exchange. *Journal of International Financial Markets, Institutions and Money*, 8, 413-432.

Appendix A: Value Functions

Proposition 1.

We derive the linear form of the value functions for each of the three inventory states $s = -1, 0, 1$. For this purpose we conjecture that the optimal standardized B2C quotes $(a(s), b(s)) = (\widehat{a}(s) - x_t, \widehat{b}(s) - x_t)$ are independent from the variable x_t . In proposition 2, we show that this is indeed the case under optimal quote setting. Intuitively, dealers earn a cash flow from intertemporal demand intermediation in the B2C market. The expected cash flow created from the customer relationship should therefore not depend on the price level of the asset under consideration. Hence, the value function cannot depend on the process x_t if the dealer starts from a zero inventory level. We therefore impose the condition $V(0, x_t) = V(0) = V$ for all levels of x_t .

For a positive or negative inventory level, however, the value function generally depends on the level of the asset price because the inventory itself is valuable. Next we determine the functional form of $V(1, x_t)$. The case of $V(-1, x_t)$ is analogous. Recall that the stochastic process x_t has binomial innovations $\Delta x_{t+1} \in \{+\epsilon, -\epsilon\}$ of constant and equal probability $\frac{1}{2}$. We further assume that dealers earn (pay) interest on the nominal value $rx_t = \frac{1-\beta}{\beta}x_t$ of their positive (negative) inventory. The transition probabilities follow from Assumption 1 as

$$\begin{aligned}
 p_{12} &= qF^b(R^b - x_{t+1} \leq \widehat{b}(1) - x_{t+1}) &= q(1 + b(1)d - d\Delta x_{t+1}) \\
 p_{11} &= 1 - p_{12} - p_{10} \\
 p_{10} &= qF^a(R^a - x_{t+1} \geq \widehat{a}(1) - x_{t+1}) &= q(1 - a(1)d + d\Delta x_{t+1}) \\
 p_{01} &= qF^b(R^b - x_{t+1} \leq \widehat{b}(0) - x_{t+1}) &= q(1 + b(0)d - d\Delta x_{t+1}) \\
 p_{00} &= 1 - p_{01} - p_{0-1} \\
 p_{0-1} &= qF^a(R^a - x_{t+1} \geq \widehat{a}(0) - x_{t+1}) &= q(1 - a(0)d + d\Delta x_{t+1}) \\
 p_{-10} &= qF^b(R^b - x_{t+1} \leq \widehat{b}(-1) - x_{t+1}) &= q(1 + b(-1)d - d\Delta x_{t+1}) \\
 p_{-1-1} &= 1 - p_{-10} - p_{-1-2} &= \\
 p_{-1-2} &= qF^a(R^a - x_{t+1} \geq \widehat{a}(-1) - x_{t+1}) &= q(1 - a(-1)d + d\Delta x_{t+1})
 \end{aligned} \tag{9}$$

Using the transition probabilities, we express the value functions as

$$\begin{aligned}
 V(1, x_t) &= \frac{1}{2}\beta [V(1, x_t + \epsilon)(1 - p_{10}^+) + [B - b(1)]p_{12}^+ + V(0, x_t + \epsilon)p_{10}^+ + [a(1) + x_t]p_{10}^+] + \\
 &+ \frac{1}{2}\beta [V(1, x_t - \epsilon)(1 - p_{10}^-) + [B - b(1) - c]p_{12}^- + V(0, x_t - \epsilon)p_{10}^- + [a(1) + x_t]p_{10}^-] \\
 &+ \beta rx_t,
 \end{aligned} \tag{10}$$

where $p_{s_1 s_2}^+$ and $p_{s_1 s_2}^-$ denotes the transition probability from inventory state s_1 to s_2 for innovations $\Delta x_{t+1} = +\epsilon$ and $\Delta x_{t+1} = -\epsilon$, respectively. Inspection of equation (10) shows that repeated substi-

tution for the terms $V(1, x_t + \epsilon)$ and $V(1, x_t - \epsilon)$ yields a sequence of discounted terms $\beta^i x_t$ (with $i = 1, 2, 3, \dots$) and a sequence of constants $V(0)$, B , $b(1)$ and $a(1)$ all independent of x_t . A similar consideration follows from the development of

$$\begin{aligned} V(-1, x_t) &= \frac{1}{2}\beta [V(-1, x_t + \epsilon)(1 - p_{-10}^+) + [a(-1) - A]p_{-1-2}^+ + V(0, x_t + \epsilon)p_{-10}^+ + [b(-1) + x_t]p_{-10}^+] + \\ &\quad + \frac{1}{2}\beta [V(1, x_t - \epsilon)(1 - p_{-10}^-) + [a(-1) - A]p_{-1-2}^- + V(0, x_t - \epsilon)p_{-10}^- + [b(-1) + x_t]p_{-10}^-] \\ &\quad - \beta r x_t \end{aligned}$$

Again sequential substitution gives discounted terms only in $\beta^i x_t$ (with $i = 1, 2, 3, \dots$) and a sequence of constants. Under the usual transversality condition that this sequence has an upper bound, there exist some constant k_x for which the value function can be expressed as

$$\begin{aligned} V(1, x_t) &= V(1) + k_x x_t \\ V(-1, x_t) &= V(-1) - k_x x_t \end{aligned} ,$$

for the inventory levels 1 and -1 , respectively. Next we show that $k_x = 1$. Using

$$\begin{aligned} &\frac{1}{2} [V(1, x_t + \epsilon)(1 - p_{10}^+) + V(1, x_t - \epsilon)(1 - p_{10}^-)] \\ &= \frac{1}{2} V(1, x_t + \epsilon)(1 - q(1 + d\epsilon - da(1))) + \frac{1}{2} V(1, x_t - \epsilon)(1 - q(1 - d\epsilon - da(1))) \\ &= V(1, x_t)(1 - q(1 - da(1))) - k_x q d \epsilon^2 = V(1, x_t)(1 - \mathcal{E}_t(p_{10})) - k_x q d \epsilon^2 \end{aligned}$$

and

$$\frac{1}{2} [V(0, x_t + \epsilon)p_{10}^+ + V(0, x_t - \epsilon)p_{10}^-] = V(0, x_t)q(1 - da(1)) = V(0, x_t)p_{10},$$

we rewrite the value function as

$$\begin{aligned} V(1, x_t) &= \beta V(1, x_t)(1 - p_{10}) - \beta k_x q d \epsilon^2 + \beta V(0, x_t)p_{10} + \beta [B - b(1)]p_{12} + \beta [a(1) + x_t]p_{10} + \beta r x_t \\ &= \beta V(1, 0)(1 - p_{10}) - \beta k_x q d \epsilon^2 + \beta V(0, 0)p_{10} + \beta [B - b(1)]p_{12} + \beta a(1)p_{10} + \\ &\quad + \beta k_x x_t(1 - p_{10}) + \beta x_t p_{10} + (1 - \beta)x_t. \end{aligned}$$

A comparison of coefficients with $V(1, x_t) = V(1) + k_x x_t$ implies that $k_x = \beta k_x(1 - p_{10}) + \beta p_{10} + 1 - \beta$ or $k_x = 1$. The value function for the inventory $s = 1$ is therefore given by $V(1, x_t) = V(1) + x_t$. An analogous argument applies to the inventory $s = -1$ where we find also find $k_x = 1$. Defining the concavity parameter $\nabla = V(0) - V(1)$ implies the linear form in proposition 1.

Appendix B: Optimal B2C Quotes

Proposition 2.

The dealer value function (1) can be expanded as

$$\begin{aligned}
V(1, x_t) &= \max_{\{a(s), b(s)\}} \beta \mathcal{E}_t \left[\begin{aligned} &[V(1) + x_{t+1} + B - b(1)] p_{12} + [V(1) + x_{t+1}] p_{11} + \\ &+ [V(0) + a(1) + x_{t+1}] p_{10} + r x_t \end{aligned} \right] \\
V(0, x_t) &= \max_{\{a(s), b(s)\}} \beta \mathcal{E}_t \left[\begin{aligned} &[V(1) + x_{t+1} - b(0) - x_t] p_{01} + V(0) p_{00} + \\ &+ [V(-1) - x_{t+1} + a(0) + x_t] p_{0-1} \end{aligned} \right] \\
V(-1, x_t) &= \max_{\{a(s), b(s)\}} \beta \mathcal{E}_t \left[\begin{aligned} &[V(0) - b(-1) - x_{t+1}] p_{-10} + [V(-1) - x_{t+1}] p_{-1-1} + \\ &+ [V(-1) - x_{t+1} - A + a(-1)] p_{-1-2} - r x_t \end{aligned} \right].
\end{aligned} \tag{11}$$

For each of the three state variables, we find the first order conditions by differentiation with respect to the corresponding quoted B2C prices $a(s)$ and $b(s)$. This implies the 6 first order conditions stated in proposition 2. The second order conditions are trivially fulfilled since the Hessian matrix is $-2d\mathbf{I}_3$ and therefore negative definite.

It is more difficult to derive the equilibrium condition on the concavity parameter ∇ which depends on the B2B spread S . From proposition 1, we know that the value function has a linear representation in the state variable x_t . In order to solve for ∇ , we can write the value function (11) for optimal B2C quotes as

$$\mathbf{V}(s, x_t) = \beta \mathcal{E}_t \left[\mathbf{M}\mathbf{V}(s, x_{t+1}) + \tilde{\mathbf{\Lambda}} \right] = \beta \mathbf{M}\mathbf{V}(s, x_t) + \mathbf{\Lambda}_0 + \mathbf{\Lambda}_x x_t + \mathbf{\Phi} \tag{12}$$

where \mathbf{M} denotes the transition matrix and where we define vectors

$$\mathbf{\Lambda}_0 = \beta \begin{bmatrix} [-\frac{S}{2} - b(1)] p_{12} + a(1) p_{10} \\ -b(0) p_{01} + a(0) p_{0-1} \\ -b(-1) p_{-10} + [a(-1) - \frac{S}{2}] p_{-1-2} \end{bmatrix}, \tag{13}$$

$$\mathbf{\Lambda}_x = \beta \begin{bmatrix} 1 + r \\ p_{0-1} - p_{01} \\ -(1 + r) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix},$$

$$\mathbf{\Phi} = \beta \mathcal{E}_t \begin{bmatrix} \Delta x_{t+1} (p_{12} + p_{11}) \\ \Delta x_{t+1} (p_{01} - p_{0-1}) \\ -\Delta x_{t+1} (p_{-1-1} + p_{-1-2}) \end{bmatrix} = \beta \begin{bmatrix} -qd\mathcal{E}_t (\Delta x_{t+1})^2 \\ -2qd\mathcal{E}_t (\Delta x_{t+1})^2 \\ -qd\mathcal{E}_t (\Delta x_{t+1})^2 \end{bmatrix} = \beta qd\epsilon^2 \begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}.$$

Subtracting the vector $\mathbf{\Lambda}_x x_t$ from both sides in equation (12) we obtain

$$\mathbf{V}(s, 0) = \mathbf{V}(s) = \beta \mathbf{M}\mathbf{V}(s, 0) + \mathbf{\Lambda}_0 + \mathbf{\Phi}.$$

Hence, the concavity parameter $\nabla = V(0) - V(1)$ is implicitly characterized by

$$\mathbf{V}(s) = \begin{bmatrix} V(1) \\ V(0) \\ V(-1) \end{bmatrix} = \begin{bmatrix} V - \nabla \\ V \\ V - \nabla \end{bmatrix} = (\mathbf{I} - \beta\mathbf{M})^{-1} (\Lambda_0 + \Phi). \quad (14)$$

The vector Λ_0 denotes the expected payoffs in each state. It is independent of both the current price process x_t and its innovation Δx_{t+1} . The vector Φ captures the state specific adverse selection risk with respect to shocks to the price process x_t . The matrix \mathbf{M} of transition probabilities can be written as

$$\mathbf{M} = \mathcal{E}_t \begin{bmatrix} p_{12} + p_{11} & p_{10} & 0 \\ p_{01} & p_{00} & p_{0-1} \\ 0 & p_{-10} & p_{-1-1} + p_{-1-2} \end{bmatrix} = \quad (15)$$

$$= \begin{bmatrix} 1 - [q\{1 - a(1)d\}] & q\{1 - a(1)d\} & 0 \\ q\{1 + b(0)d\} & 1 - q\{1 + b(0)d\} - q\{1 - a(0)d\} & q\{1 - a(0)d\} \\ 0 & q\{1 + b(-1)d\} & q\{1 + b(-1)d\} \end{bmatrix}. \quad (16)$$

Substituting the relevant elements of (9) into (13) and using (15), we can rewrite

$$(\mathbf{I} - \beta\mathbf{M}) \mathbf{V}(s) - \Lambda_0 - \Phi = \mathbf{0}$$

or

$$\begin{bmatrix} 8\beta q + \beta d^2 q (4\nabla^2 + S^2 - 16\epsilon^2) + 4d \{2\nabla (2 + \beta(q - 2)) - \beta q S + 4V(0)(\beta - 1)\} \\ V(0) - \frac{\beta q \{4d^2 \epsilon^2 - (d\nabla - 1)^2\}}{2d(\beta - 1)} \\ 8\beta q + \beta d^2 q (4\nabla^2 + S^2 - 16\epsilon^2) + 4d \{2\nabla (2 + \beta(q - 2)) - \beta q S + 4V(0)(\beta - 1)\} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

The second equation can be solved for $V(0)$ in terms of ∇ . The first and third equations are identical and we substitute $V(0)$ into either to obtain

$$f_{b2c}(\nabla, S, \epsilon^2, q, d) = \frac{1}{4} (\beta d q \nabla^2) + \nabla \left(-1 + \beta - \frac{3\beta q}{2} \right) - \frac{\beta q \{S(dS - 4) + 16d\epsilon^2\}}{16} = 0. \quad (17)$$

This B2C schedule characterizes the inventory concavity parameter ∇ of a dealer's value function under optimal B2C quotes and for any B2B spread S . It is depicted in Figure 2.

Appendix C: Competitive Pricing in the B2B Market

Proposition 3.

To determine the expected loss of liquidity provision in the interdealer market, it is useful to denote by $(n(1), n(0), n(-1)) > 0$ the number of traders with inventories 1, 0, and -1 , respectively. We assume furthermore that the probability q of customer arrival in the B2C market is sufficiently small so that $\frac{1}{2}q < \frac{n(1)}{n(-1)} < \frac{2}{q}$ holds. Liquidity at the best B2B ask price is only demanded by dealers who experience an negative inventory shock from -1 to -2 and are therefore forced to rebalance. The respective probability p_{-1-2} (see equations (9)) is given by $q(1 - a(-1)d + d\epsilon)$ when $\Delta x_{t+1} = \epsilon$ (with probability $\frac{1}{2}$) and $q(1 - a(-1)d - d\epsilon)$ when $\Delta x_{t+1} = -\epsilon$ (with probability $\frac{1}{2}$). The liquidity supplying dealer (at the ask) experiences an expected loss if the liquidity demand is more likely to occur for $\Delta x_{t+1} = \epsilon$ than $\Delta x_{t+1} = -\epsilon$. If market orders (due to rebalancing needs) in the B2B market were unrelated to the dynamics of Δx_{t+1} , then the expected (adverse selection) loss L^A of liquidity provision at the ask would follow as

$$L^A = \frac{1}{2}\epsilon + \frac{1}{2}(-\epsilon) = 0.$$

But since execution probabilities for limit order supplies depend on Δx_{t+1} , we have instead

$$L^A = \text{prob}(\Delta x_{t+1} = \epsilon \mid \text{Execution}) \epsilon + \text{prob}(\Delta x_{t+1} = -\epsilon \mid \text{Execution}) (-\epsilon), \quad (18)$$

where $\text{prob}(\Delta x_{t+1} = \epsilon \mid \text{Execution}) > \frac{1}{2}$ denotes the probability of $\Delta x_{t+1} = \epsilon$ conditional on execution of the liquidity supply at the ask. Using Bayes rule implies

$$\text{prob}(\Delta x_{t+1} = \epsilon \mid \text{Execution}) = \frac{\text{prob}(\Delta x_{t+1} = \epsilon \cap \text{Execution})}{\text{prob}(\Delta x_{t+1} = \epsilon \cap \text{Execution}) + \text{prob}(\Delta x_{t+1} = -\epsilon \cap \text{Execution})} \quad (19)$$

$$\text{prob}(\Delta x_{t+1} = -\epsilon \mid \text{Execution}) = \frac{\text{prob}(\Delta x_{t+1} = -\epsilon \cap \text{Execution})}{\text{prob}(\Delta x_{t+1} = \epsilon \cap \text{Execution}) + \text{prob}(\Delta x_{t+1} = -\epsilon \cap \text{Execution})} \quad (20)$$

We calculate the expected number of (unit) market order as $n(-1)q(1 - a(-1)d \pm d\epsilon)$ (for $\Delta x_{t+1} = \pm\epsilon$, respectively) and the number of (unit) liquidity supplies at the best ask as $n(1)$. The execution probability for each liquidity supplying dealer then follows as

$$\begin{aligned} \text{prob}(\Delta x_{t+1} = \epsilon \cap \text{Execution}) &= \frac{1}{2} \frac{n(-1)q(1 - a(-1)d + d\epsilon)}{n(1)} \\ \text{prob}(\Delta x_{t+1} = -\epsilon \cap \text{Execution}) &= \frac{1}{2} \frac{n(-1)q(1 - a(-1)d - d\epsilon)}{n(1)}. \end{aligned}$$

Both expressions are bounded between 0 and 1 for $\frac{1}{2}q < \frac{n(1)}{n(-1)}$. Substitution into equations (19) and (20) implies

$$\begin{aligned} \text{prob}(\Delta x_{t+1} = \epsilon \mid \text{Execution}) &= \frac{(1 - a(-1)d + d\epsilon)}{2(1 - a(-1)d)} = \frac{1}{2} + \frac{d\epsilon}{2(1 - a(-1)d)} \\ \text{prob}(\Delta x_{t+1} = -\epsilon \mid \text{Execution}) &= \frac{(1 - a(-1)d - d\epsilon)}{2(1 - a(-1)d)} = \frac{1}{2} - \frac{d\epsilon}{2(1 - a(-1)d)}. \end{aligned}$$

The expected loss of B2B liquidity supply at the best ask stated in (18) follows as

$$L^A = \frac{d\epsilon^2}{[1 - a(-1)d]} = \frac{\epsilon^2}{\frac{1}{d} - a(-1)} = \frac{\epsilon^2}{\frac{1}{d} - \frac{S}{4} - \frac{1}{2d}} = \frac{2\epsilon^2}{\frac{1}{d} - \frac{S}{2}},$$

and an analogous expression holds for $L^B = L^A = L$.

The equilibrium condition equalizes the adverse selection costs L^A with benefits of a balanced inventory ∇ , the transaction revenue $\frac{S}{2}$ and order processing costs τ . If all rents from liquidity disappear under perfect supply, we obtain as the B2B equilibrium condition

$$f_{b2b}(\nabla, S, \epsilon^2, q, d) = \tau - \frac{S}{2} - \nabla + \frac{4d\epsilon^2}{2 - Sd} = 0. \quad (21)$$

and (for $S \geq 0$)

$$\begin{aligned} A &= \max(L - \nabla + \tau, 0) = \frac{S}{2} \\ B &= \min(-L + \nabla - \tau, 0) = -\frac{S}{2}. \end{aligned}$$

Appendix D: Existence and Uniqueness of the Equilibrium

Proposition 4:

First, we show that the two equilibrium schedules (17) and (21) have exactly two intersections in the $(\frac{S}{2}, \nabla)$ space as long as the volatility ϵ^2 of the midprice process x_t is below some threshold $\bar{\epsilon}^2$. This situation is graphed in Figure 2. Second, we argue that only one of the two equilibria is stable. Third, for high levels of volatility with $\epsilon^2 > \bar{\epsilon}^2$ no equilibrium exists in which both the B2B and B2C market function simultaneously.

To characterize the shape of the B2C equilibrium schedule, we calculate the partial derivatives of the implicit function f_{b2c} giving

$$\begin{bmatrix} \frac{\partial f_{b2c}}{\partial S} \\ \frac{\partial f_{b2c}}{\partial \nabla} \\ \frac{\partial f_{b2c}}{\partial \epsilon^2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{8}\beta q (dS - 2) > 0 \\ -1 + \frac{1}{2}\beta [2(1 - q) + qd(\nabla - \frac{1}{d})] < 0 \\ -\beta dq < 0 \end{bmatrix}. \quad (22)$$

We have $\frac{\partial f_{b2c}}{\partial S} > 0$ because the uniform distribution was restricted to have $\frac{S}{2} < \frac{1}{d}$. Moreover, $\frac{\partial f_{b2c}}{\partial \nabla} < 0$, because $q < 1$ and $\nabla < \frac{1}{d}$. To verify the condition $\nabla < \frac{1}{d}$, take into consideration that the ask quote $a(1) = \frac{1}{2}(\frac{1}{d} - \nabla) > 0$ in equation (3) needs to be positive. The B2C schedule has the derivatives

$$\frac{\partial \nabla_{b2c}}{\partial S} = -\frac{\frac{\partial f_{b2c}}{\partial S}}{\frac{\partial f_{b2c}}{\partial \nabla}} = \frac{\frac{1}{8}\beta q (dS - 2)}{-1 + \frac{1}{2}\beta [2(1 - q) + qd(\nabla - \frac{1}{d})]} > 0 \quad \text{and} \quad \frac{\partial^2 \nabla_{b2c}}{\partial S^2} < 0.$$

In the $(\frac{S}{2}, \nabla)$ space the B2C schedule is therefore increasing in S with a decreasing slope.

Next, we examine the B2B schedule (21). Its intercept with the vertical axis is found by evaluating equation (21) at $S = 0$, which gives $2d\epsilon^2 + \tau$. The B2B schedule has derivatives

$$\frac{\partial \nabla_{b2b}}{\partial S} = -\frac{1}{2} + \frac{4d^2\epsilon^2}{(2 - Sd)^2} \quad \text{and} \quad \frac{\partial^2 \nabla_{b2b}}{\partial S^2} < 0. \quad (23)$$

At $S = 0$, we find $\frac{\partial \nabla_{b2b}}{\partial S} = -\frac{1}{2} + 2d^2\epsilon^2 < 0$, because the maximum value of ϵ^2 is $\frac{1}{4d^2}$. Equation (21) is quadratic. Its minimum is obtained for $\frac{S}{2} = \frac{1}{d} - \sqrt{2\epsilon^2}$. For $\frac{1}{d} - \sqrt{2\epsilon^2} < \frac{S}{2} < \frac{1}{d}$, the slope is positive. Importantly, $\frac{\partial^2 \nabla_{b2b}}{\partial S^2} > 0$ for the B2B schedule and $\frac{\partial^2 \nabla_{b2c}}{\partial S^2} < 0$ for the B2C schedule implies that both schedules intersect exactly twice as long as the volatility ϵ^2 is not too large. Of the two equilibria Z_L and Z_H shown in Figure 2, only Z_L with lower values of S and ∇ is stable. Deviation of a liquidity supplier in the B2B market to a lower spread S immediately attracts all the market orders from other dealers. The less favorable B2B quotes become irrelevant. The reverse argument does not hold, which demonstrates the stability of equilibrium Z_L . Finally, as ϵ^2 becomes large, the B2B and B2C schedule no longer intersect and no market equilibrium exists. The volatility level ϵ^2 at which both schedule touch in one tangency point characterizes the threshold value $\bar{\epsilon}^2$ for breakdown of the joint equilibrium in both markets.

Appendix E: Approximation to Solution

Proposition 5:

The market equilibrium is characterized by the B2C schedule (17) and B2B schedule (21), respectively. This equilibrium is linearize around the value $\epsilon^2 = 0$. Let $(\bar{S}, \bar{\nabla})$ denote the equilibrium values which correspond to $\epsilon^2 = 0$, which fulfill $\bar{\nabla} = \tau - \frac{\bar{S}}{2}$. The first order approximation of the B2B equation (21) follows as

$$\nabla = \tau - \frac{S}{2} + \alpha_{1v}\epsilon^2, \quad (24)$$

with a parameter $\alpha_{1v} = 4/(\frac{2}{d} - \bar{S}) > 0$. The corresponding approximation of the B2C equation (17) follows as

$$0 = \alpha_{2c} + \alpha_{2s}\frac{S}{2} + \alpha_{2\nabla}\nabla + \alpha_{2v}\epsilon^2, \quad (25)$$

with parameters

$$\begin{aligned} \alpha_{2c} &= \frac{1}{4}\beta qd \left[\left(\frac{\bar{S}}{2}\right)^2 - (\bar{\nabla})^2 \right] < 0, & \alpha_{2s} &= -\frac{1}{4}\beta qd \left[\bar{S} - \frac{2}{d} \right] > 0, \\ \alpha_{2\nabla} &= \beta - 1 + \frac{1}{2}\beta q (d\bar{\nabla} - 3) < 0, & \alpha_{2v} &= -\beta qd < 0. \end{aligned}$$

Substitution of equation (24) into equation (25) implies

$$\frac{S}{2} = -\frac{\alpha_{2c} + \alpha_{2\nabla}\alpha_{1c}}{\alpha_{2s} + \alpha_{2\nabla}\alpha_{1s}} - \frac{\alpha_{2v} + \alpha_{2\nabla}\alpha_{1v}}{\alpha_{2s} + \alpha_{2\nabla}\alpha_{1s}}\epsilon^2 = \gamma_{4c} + \gamma_{4v}\epsilon^2, \quad (26)$$

where we find for the coefficients

$$\gamma_{4c} = -\frac{\alpha_{2c} + \alpha_{2\nabla}\alpha_{1c}}{\alpha_{2s} + \alpha_{2\nabla}\alpha_{1s}} = \tau - \frac{\alpha_{2c} + \alpha_{2s}\tau}{\alpha_{2s} - \alpha_{2\nabla}} < \tau, \quad \gamma_{4v} = -\frac{\alpha_{2vol} + \alpha_{2\nabla}\alpha_{1v}}{\alpha_{2s} + \alpha_{2\nabla}\alpha_{1s}} = \alpha_{1v}.$$

The inventory concavity parameter follows (after substitution of equation (26) into equation (24)) as

$$\nabla = \tau - \frac{S}{2} + \alpha_{1v}\epsilon^2 = \tau - \gamma_{4c}, \quad (27)$$

and substitution into the first order conditions implies

$$\begin{bmatrix} a(-1) \\ a(0) \\ a(1) \end{bmatrix} = \begin{bmatrix} \frac{1}{2d} \\ \frac{1}{2d} \\ \frac{1}{2d} \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \frac{S}{2} \\ \nabla \\ -\nabla \end{bmatrix} = \begin{bmatrix} \gamma_{1c} \\ \gamma_{2c} \\ \gamma_{3c} \end{bmatrix} + \begin{bmatrix} \gamma_{1v} \\ 0 \\ 0 \end{bmatrix} \epsilon^2,$$

where $\gamma_{1c} = \frac{1}{2d} + \frac{1}{2}\gamma_{4c}$, $\gamma_{2c} = \frac{1}{2d} + \frac{1}{2}(\tau - \gamma_{4c})$, $\gamma_{3c} = \frac{1}{2d} - \frac{1}{2}(\tau - \gamma_{4c})$ and $\gamma_{1v} = \frac{1}{2}\gamma_{4v} < \gamma_{4v}$. It follows directly that $\gamma_{1c} > \gamma_{2c} > \gamma_{3c}$ and $\gamma_{2c} > \gamma_{4c}$. Analogous relationships apply at the bid side. The accuracy of the linear approximation can be inferred from Figure 3, which shows the exact solution as solid line and the linear approximation to as a dashed line.

Corollary 1:

Let $p(s) = E(\frac{n(s)}{n})$ denote the probability distribution of traders over the three inventory states s and assume that it does not depend on the volatility ϵ^2 . We define the expected B2C spreads

$$\bar{a} = \sum_{s=-1,0,1} p(s)a(s)g(a(s)) \text{ and } \bar{b} = \sum_{s=-1,0,1} p(s)b(s)g(b(s)).$$

where $g(a(s))$ and $g(b(s))$ denotes the probability that the respective customer quote is accepted. Furthermore, $g(a(s)) = 1 - a(s)d$ with $0 < a(s) < \frac{1}{d}$. The expected B2C ask price is given by

$$\bar{a} = \sum_{s=-1,0,1} p(s)a(s)g(a(s)) = \sum_{s=-1,0,1} p(s)a(s)[1 - a(s)d] = \sum_{s=-1,0,1} p(s)[a(s) - a(s)^2d].$$

The derivative with respect to volatility ϵ^2 follows as

$$\frac{\partial \bar{a}}{\partial \epsilon^2} = \sum_{s=-1,0,1} p(s)[1 - 2a(s)d] \frac{\partial a(s)}{\partial \epsilon^2} < \frac{\partial a(-1)}{\partial \epsilon^2} = \gamma_{1v} < \gamma_{4v} = \frac{\partial S}{\partial \epsilon^2} = \frac{\partial A}{\partial \epsilon^2}.$$

A similar argument applies to the bid side. Hence,

$$\frac{\partial}{\partial \epsilon^2} [\bar{a} - A] < 0 \quad \text{and} \quad \frac{\partial}{\partial \epsilon^2} [-\bar{b} + B] > 0.$$

Proposition 6.

For the volatility dependence see proposition 5. Combining $a(-1) > a(1)$, $b(-1) > b(1)$ and $\frac{\partial p(-1)}{\partial Imb} < 0$, $\frac{\partial p(1)}{\partial Imb} > 0$ from equation (8) implies $\frac{\partial(\bar{a}-A)}{\partial Imb} = \frac{\partial \bar{a}}{\partial Imb} < 0$ and $\frac{\partial(-\bar{b}+B)}{\partial Imb} = -\frac{\partial \bar{b}}{\partial Imb} > 0$.

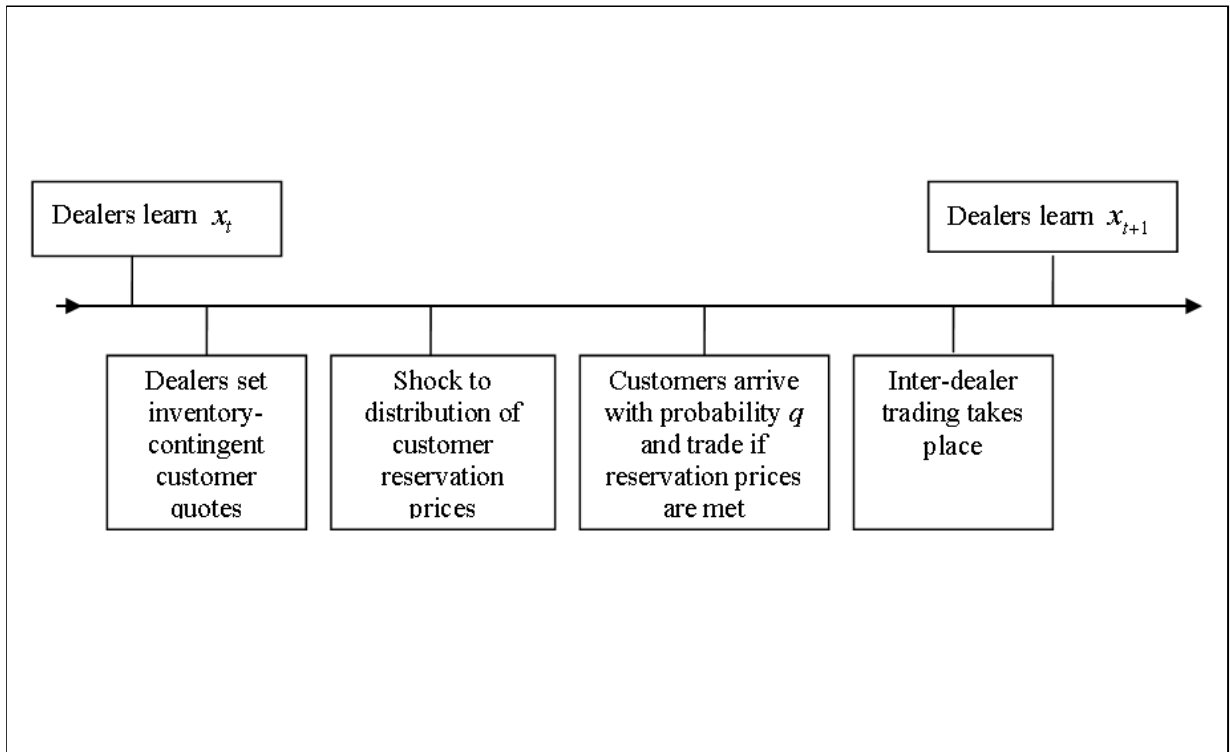


Figure 1: Time line for the trading process

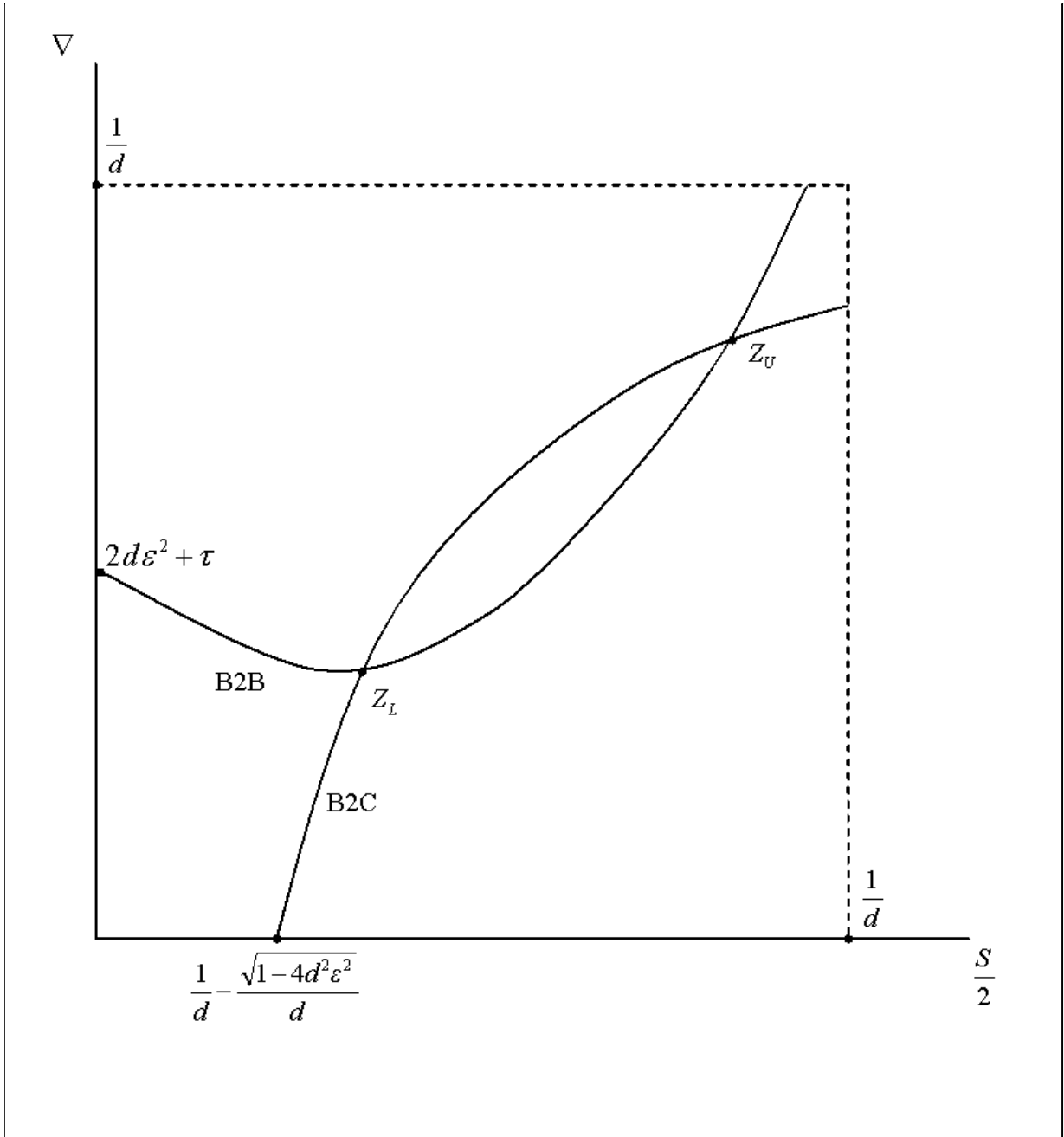


Figure 2: The B2C schedule characterizes the inventory concavity parameter ∇ for optimal B2C quotes under any B2B spread S . The B2B schedule defines the competitive B2B spread S for dealers which have ∇ as their inventory concavity parameter. The two intersextions fulfill the equilibrium conditions in both the B2B and B2C market. Of the two equilibria, only one, Z_L , is stable.

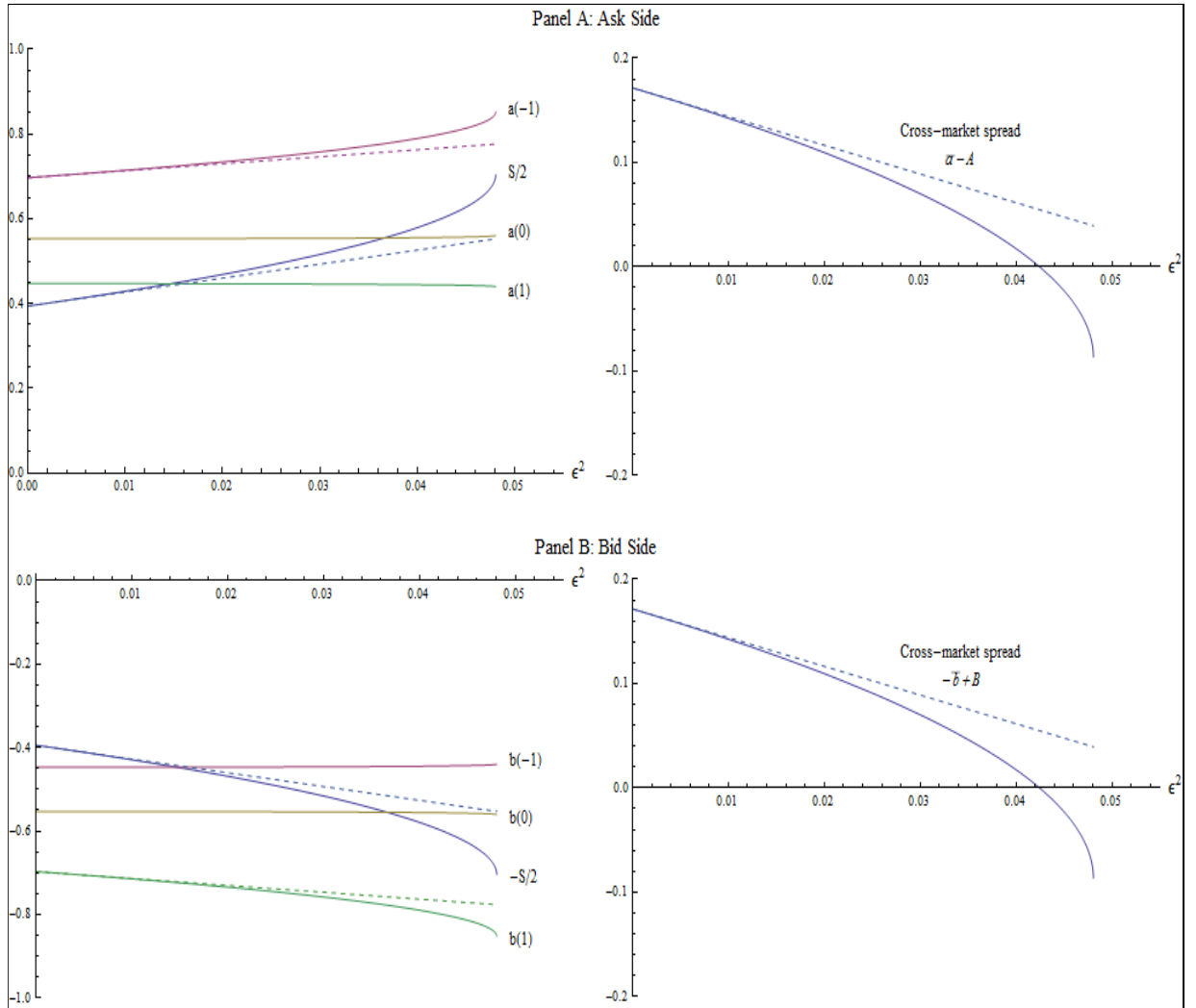


Figure 3: For the ask side (panel A) and the bid side (panel B) we plot on the left the B2B half-spread $\frac{S}{2}$ and the three B2C spreads (for the three different inventory states) as a function of volatility ϵ^2 . The picture on the right shows in each case the average cross market spread as a function of volatility. The order processing cost parameter is chosen as $\tau = 0.5$; the probability of customer arrival is $q = 0.5$; the discount rate $\beta = 0.99$ and the density of the customer price reservation distribution d is set at 1. The dashed lines represent the linear approximation as stated in Proposition 5.

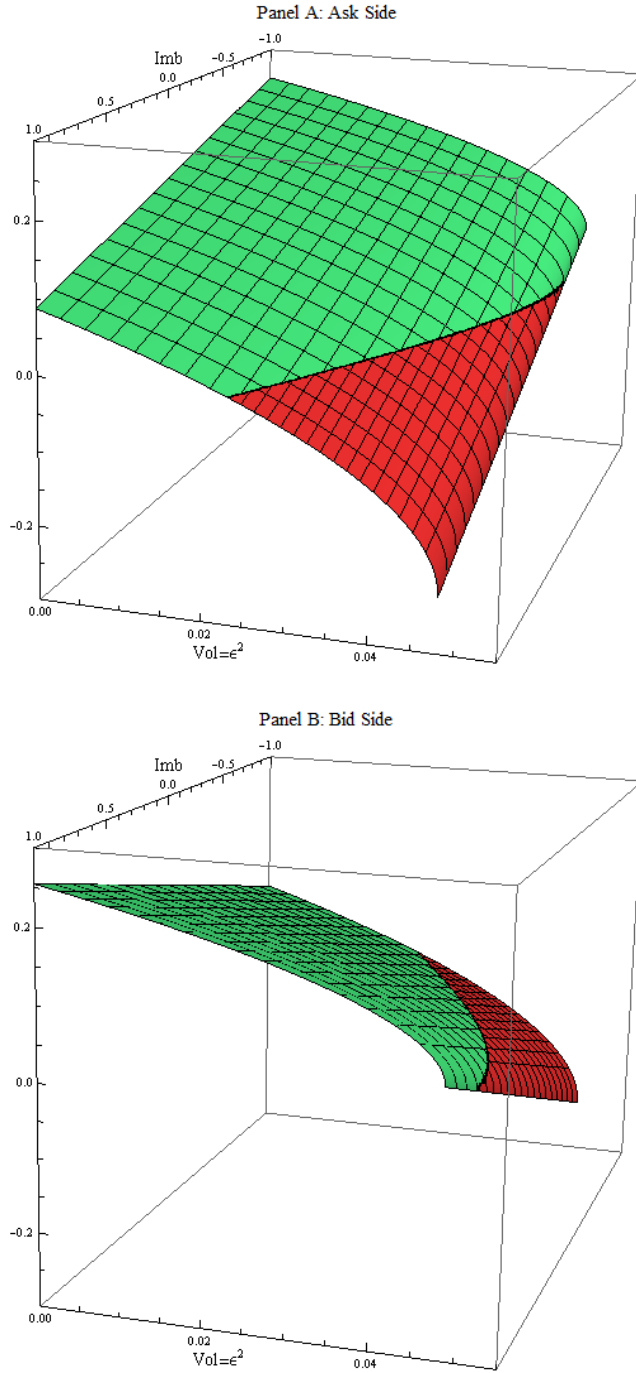


Figure 4: For the ask side (panel A) and the bid side (panel B) we plot vertically the average cross-market spread as a function of volatility (ϵ^2) and the aggregate inventory imbalance (Imb). The red area marks the region for which the average B2C spread is more favorable than the B2B spread. The order processing cost parameter is chosen as $\tau = 0.5$; the probability of customer arrival is $q = 0.5$; the discount rate $\beta = 0.99$; the density of the customer price reservation distribution d is set at 1.

Table 1: European Bond Market by Country and MTS Sample Size

The size of the European government bond market in terms of bond value outstanding is described by country for 2005. The MTS data sample of the B2B market extends over the last 3 quarters of 2005. The coverage ratio reports the ratio of MTS trading volume divided by the value outstanding in each market. For the Italian market the coverage ratio corresponds to a 100 percent market share since MTS benefits from a legal monopoly in B2B trading. Assuming that the volume to value ratio for Italy applies to all countries, we estimate the percentage market share of MTS in each country (with 100 percent as the upper boundary). Transaction volumes are stated in billions of Euros.

Country	European Bond Market	MTS Sample of B2B Market			Coverage Ratios	
	(2005)	(2005q2 to 2005q4)			Volume/Value	MTS Market Share
	Value Outstanding	Bonds	Volume	Trades		
Austria	155.1	14	29.30	3,304	0.189	19.7
Belgium	277.9	54	141.46	14,776	0.509	52.9
France	877.0	136	177.76	20,727	0.203	21.1
Germany	900.1	86	145.66	19,436	0.162	16.9
Greece	163.4	23	12.64	12,641	0.077	8.0
Ireland	31.3	5	7.28	915	0.233	24.2
Italy	1,213.0	134	1,261.91	239,525	1.040	100.0
Netherlands	220.5	43	79.13	6,514	0.359	37.3
Portugal	94.0	27	122.62	13,931	1.304	100.0
Spain	319.2	54	98.28	10,229	0.308	32.0
All	4,251.5	576	2,076.04	341,998	0.488	50.8

Table 2: Italian Bond Market Size by Bond Maturity and MTS Sample Size

The size of the Italian government bond market is described by maturity based on MTS data. The MTS sample extends over the last 3 quarters of 2005 and covers the interdealer market (B2B) and the customer dealer market (B2C). Transaction volumes are stated in billions of Euros. The number of bonds included is constrained to be the number of bonds in the B2C market.

Bond Maturity	MTS Sample of B2B Market (2005q2 to 2005q4)						MTS Sample of B2C Market (2005q2 to 2005q4)							
	All			Selected Bonds			All				Selected Bonds			
	Bonds	Volume	Trades	Bonds	Volume	Trades	Bonds	Volume	Trades	Quote Requests	Bonds	Volume	Trades	Quote Requests
Short-Medium	46	563.42	115,060	6	139.52	32,207	46	73.64	15,792	24,007	6	20.18	3,541	5,385
Long	15	231.73	36,057	6	188.26	29,816	15	15.03	5,603	10,151	6	12.37	4,979	9,422
Very Long	11	50.61	13,296	1	10.72	2,956	11	7.41	4,992	9,528	1	1.42	1,086	2,177
All	72	845.76	164,413	13	338.5	64,979	72	96.08	26,387	43,686	13	33.97	9,606	16,984

Table 3: Cross-Market Spreads and B2B Spreads by Bond Maturity

The average of the cross-market spread within each quantile, and overall, is shown for each of the three main maturity categories of European sovereign bonds. Panel A reports average spreads for transactions at the ask quotes while Panel B reports spreads for bid transactions. The cross-market spread is defined as the difference between the B2C transaction price (a or b for B2C ask or bid, respectively) and the prevailing best B2B price (A or B for B2B ask or bid, respectively). Alongside the cross-market spread we also report the averages of the B2B spreads for the corresponding maturity categories measured (relative to the midprice $MidP$ between the best B2B ask and bid) at the same moment in time when the B2C transactions occurs. The sample is based on 5050 ask transactions and 4297 bid transactions in the B2C market. Measures of the cross-market spread and the B2B spread are given in cents. At par, these amount to basis points.

Panel A: Ask Side Spreads											
Cross-Market Ask Spread $a - A$						B2B Ask Spreads $A - MidP$					
Quantile Means	Quality	Bond Maturity			All	Quantile Means	Quality	Bond Maturity			All
		Short-Medium	Long	Very Long				Short-Medium	Long	Very Long	
Mean of $Q(1)$	Best	-2.70	-2.52	-4.96	-3.41	Mean of $Q(1)$	Best	0.30	0.50	0.50	0.31
Mean of $Q(2)$		-1.07	-1.34	-1.17	-1.30	Mean of $Q(2)$		0.50	0.96	1.00	0.70
Mean of $Q(3)$	Worst	-1.00	-1.00	-1.00	-1.00	Mean of $Q(3)$	Worst	1.01	1.28	1.33	1.26
Mean of $Q(4)$		0.01	-0.34	0.67	-0.11	Mean of $Q(4)$		2.50	2.17	5.13	3.34
Overall Mean		-0.61	-1.51	-4.59	-1.45	Overall Mean		0.41	1.46	5.10	1.40

Panel B: Bid Side Spreads											
Cross-Market Bid Spreads $-(b - B)$						B2B Bid Spreads $-(B - MidP)$					
Quantile Means	Quality	Bond Maturity			All	Quantile Means	Quality	Bond Maturity			All
		Short-Medium	Long	Very Long				Short-Medium	Long	Very Long	
Mean of $Q(1)$	Best	-1.80	-2.01	-4.15	-2.96	Mean of $Q(1)$	Best	0.34	0.50	0.50	0.35
Mean of $Q(2)$		-0.57	-0.81	-1.00	-0.78	Mean of $Q(2)$		0.65	0.97	1.00	0.88
Mean of $Q(3)$	Worst	0.00	0.00	0.00	0.00	Mean of $Q(3)$	Worst	1.50	1.51	1.50	1.51
Mean of $Q(4)$		0.44	0.28	1.78	0.38	Mean of $Q(4)$		2.50	2.66	5.22	3.98
Overall Mean		0.12	-0.73	-3.58	-0.84	Overall Mean		0.41	1.56	5.17	1.68

Table 4: Cross-Market Spreads and B2B Spreads by Volatility

The average of the cross-market spread within each quantile, and overall, is shown for the ‘Low’, ‘Medium’, ‘High’ and ‘Very-High’ volatility level. Panel A reports average spreads for transactions at the ask quotes while Panel B reports those at the bid. The cross-market spread is defined as the difference between the B2C transaction price (a or b for B2C ask or bid, respectively) and the prevailing best B2B price (A or B for B2B ask or bid, respectively). Volatility is measured as realized volatility. ‘Low’ volatility is defined as volatility lying below the 10th percentile of observed realized volatilities, ‘Medium’ is realized volatility between the 10th and 90th percentiles, ‘High’ is between the 90th and the 95th percentiles, and ‘Very High’ volatility corresponds to levels above the 95th percentile. Alongside the cross-market spread we also report the averages of the B2B spreads (relative to the midprice $MidP$ between the best B2B ask and bid) for the corresponding groups measured at the same moment in time when the B2C transactions occurred. The sample is based on 5050 ask transactions and 4297 bid transactions in the B2C market. Measures of the cross-market spread and the B2B spread are given in cents. At par, these amount to basis points.

Panel A: Ask Side Spreads													
		Cross-Market Ask Spread $a - A$					B2B Ask Spreads $A - MidP$						
		Volatility Level					Volatility Level						
Quantile Means	Quality	Low	Medium	High	Very High	All	Quantile Means		Small	Medium	High	Very High	All
Mean of $Q(1)$	Best	-3.24	-3.36	-3.79	-4.18	-3.41	Mean of $Q(1)$	Best	0.26	0.31	0.36	0.37	0.31
Mean of $Q(2)$		-1.15	-1.28	-1.53	-1.75	-1.30	Mean of $Q(2)$		0.60	0.70	0.78	0.85	0.70
Mean of $Q(3)$	Worst	-1.00	-1.00	-1.00	-1.00	-1.00	Mean of $Q(3)$	Worst	1.19	1.24	1.51	1.56	1.26
Mean of $Q(4)$		-0.11	-0.12	0.00	0.02	-0.11	Mean of $Q(4)$		3.22	3.24	4.00	4.39	3.34
Overall Mean		-1.37	-1.44	-1.58	-1.74	-1.45	Overall Mean		1.32	1.37	1.66	1.78	1.40

Panel B: Bid Side Spreads													
		Cross-Market Bid Spreads $-(b - B)$					B2B Bid Spreads $-(B - MidP)$						
		Volatility Level					Volatility Level						
Quantile Means		Low	Medium	High	Very High	All	Quantile Means		Low	Medium	High	Very High	All
Mean of $Q(1)$	Best	-3.09	-2.93	-2.84	-3.27	-2.96	Mean of $Q(1)$	Best	0.35	0.35	0.36	0.38	0.35
Mean of $Q(2)$		-0.81	-0.76	-1.00	-0.91	-0.78	Mean of $Q(2)$		0.86	0.88	0.96	1.03	0.88
Mean of $Q(3)$	Worst	0.00	0.00	-0.03	0.00	0.00	Mean of $Q(3)$	Worst	1.56	1.48	1.61	1.83	1.51
Mean of $Q(4)$		0.37	0.37	0.40	0.56	0.38	Mean of $Q(4)$		4.29	3.89	4.33	4.67	3.98
Overall Mean		-0.88	-0.83	-0.87	-0.91	-0.84	Overall Mean		1.76	1.65	1.81	1.97	1.68

Table 5: Cross-Market Spreads by Imbalance

The average of the cross-market spread within each quantile, and overall, is shown by quantiles of the imbalance. Imbalance is measured as the difference between the B2B liquidity at the best ask and the best bid across all 13 sample bonds at the moment when a B2C transaction takes place. We form four quantiles for the imbalance measure denoted as ‘Very Negative’, ‘Negative’, ‘Positive’ and ‘Very Positive’. Panel A reports average spreads for transactions at the ask quotes while Panel B reports those at the bid. The cross-market spread is defined as the difference between the B2C transaction price (a or b for B2C ask or bid, respectively) and the prevailing best B2B price (A or B for B2B ask or bid, respectively). Imbalance is measured as the difference between best limit order quantities relative to their sum across all 13 bonds. Alongside the cross-market spread we also report the averages of the B2B spreads (relative to the midprice $MidP$ between the best B2B ask and bid) for the corresponding groups measured at the same moment in time when the B2C transactions occurred. The sample is based on 5050 ask transactions and 4297 bid transactions in the B2C market. Measures of the cross-market spread and the B2B spread are given in cents. At par, these amount to basis points.

Panel A: Ask Side Spreads													
Cross-Market Ask Spread							B2B Ask Spreads						
$a - A$							$A - MidP$						
		Imbalance Level							Imbalance Level				
Quantile Means	Quality	Very Negative	Negative	Positive	Very Positive	All	Quantile Means		Very Negative	Negative	Positive	Very Positive	All
Mean of $Q(1)$	Best	-3.23	-3.39	-3.41	-3.58	-3.41	Mean of $Q(1)$	Best	0.29	0.30	0.32	0.32	0.31
Mean of $Q(2)$		-1.22	-1.36	-1.29	-1.34	-1.30	Mean of $Q(2)$		0.70	0.68	0.69	0.72	0.70
Mean of $Q(3)$		-1.00	-1.00	-1.00	-1.00	-1.00	Mean of $Q(3)$		1.25	1.27	1.28	1.23	1.26
Mean of $Q(4)$	Worst	-0.06	-0.19	-0.13	-0.17	-0.11	Mean of $Q(4)$	Worst	3.18	3.33	3.48	3.36	3.34
Overall Mean		-1.35	-1.49	-1.46	-1.52	-1.45	Overall Mean		1.35	1.39	1.44	1.40	1.40

Panel B: Bid Side Spreads													
Cross-Market Bid Spreads							B2B Bid Spreads						
$-(b - B)$							$-(B - MidP)$						
		Imbalance Level							Imbalance Level				
Quantile Means		Very Negative	Negative	Positive	Very Positive	All	Quantile Means		Very Negative	Negative	Positive	Very Positive	All
Mean of $Q(1)$	Best	-2.97	-3.29	-3.03	-2.54	-2.96	Mean of $Q(1)$	Best	0.36	0.36	0.34	0.34	0.35
Mean of $Q(2)$		-0.86	-0.94	-0.80	-0.53	-0.78	Mean of $Q(2)$		0.89	0.90	0.87	0.87	0.88
Mean of $Q(3)$		0.00	0.00	0.00	0.00	0.00	Mean of $Q(3)$		1.46	1.62	1.58	1.39	1.51
Mean of $Q(4)$	Worst	0.36	0.35	0.37	0.45	0.38	Mean of $Q(4)$	Worst	3.78	4.49	4.07	3.60	3.98
Overall Mean		-0.87	-0.97	-0.87	-0.66	-0.84	Overall Mean		1.62	1.84	1.71	1.55	1.68

Table 6: Cross Market Spread and B2B Spread Estimation

Reported are instrumental variable estimates of the relation between the cross-market spread and volatility and between the B2B spread and volatility. Results are provided for the full-sample of bonds and for the sub-sample containing the 6 long bonds. In all cases we include bond-specific fixed-effects to control for spread differences across bonds. The ask side results are presented in Panel A and the bid side results are presented in Panel B. Volatility is measured by the log realized volatility of the mid-price returns over 1 minute intervals computed for every full hour. The IV regression uses a constant and volatility lagged by one hour as instruments. Standard errors and t-statistics are adjusted for heteroscedasticity. Spreads are the dependent variable and are expressed in cents. At par, these amount to basis points.

Panel A: Ask Side Spreads				
Dep. Variable:	Cross-Market Spread		B2B Spread	
	$a - A$		$A - MidP$	
	Full Sample	Long Bonds	Full Sample	Longs Bonds
Constant	-0.577	-1.220	-0.347	0.004
StdErr	(0.233)	(0.375)	(0.148)	(0.229)
T-Stat	[-2.478]	[-3.250]	[-2.353]	[0.017]
Volatility	-0.003	-0.016	0.212	0.295
StdErr	(0.057)	(0.090)	(0.035)	(0.055)
T-Stat	[-0.062]	[-0.174]	[6.022]	[5.367]
Obs.	4975	2498	4975	2498
Fixed Bond Effects	Yes	Yes	Yes	Yes
Panel B: Bid Side Spreads				
Dep. Variable:	Cross-Market Spread		B2B Spread	
	$-(b - B)$		$-(B - MidP)$	
	Full Sample	Long Bonds	Full Sample	Long Bonds
Constant	0.949	0.299	-0.694	0.099
StdErr	(0.267)	(0.390)	(0.177)	(0.232)
T-Stat	[3.554]	[0.765]	[-3.919]	[0.425]
Volatility	-0.249	-0.218	0.310	0.278
StdErr	(0.065)	(0.090)	(0.043)	(0.055)
T-Stat	[-3.825]	[-2.418]	[7.204]	[5.069]
Obs.	4247	2385	4247	2385
Fixed Bond Effects	Yes	Yes	Yes	Yes

Table 7: Cross Market Spread and Inventory Imbalances

Reported are instrumental variable estimates of the relation between the cross-market spread, volatility and inventory imbalances. Results are provided for the full-sample of bonds and for the sub-sample containing the 6 long bonds. In all cases we include bond-specific fixed-effects to control for spread differences across bonds. The ask side results are presented in Panel A and the bid side results are presented in Panel B. Volatility is measured by the log realized volatility of the mid-price returns over 1 minute intervals computed for every full hour. The construction of the imbalance variable is described in the main text. The IV regression uses a constant and volatility lagged by one hour as instruments. Standard errors and t-statistics are adjusted for heteroscedasticity. Spreads are the dependent variable and are expressed in cents. At par, these amount to basis points..

Panel A: Ask Side Spreads			Panel B: Bid Side Spreads		
Dep. Variable:	Cross-Market Spread $a - A$		Dep. Variable:	Cross-Market Spread $-(b - B)$	
	Full Sample	Long Bonds		Full Sample	Longs Bonds
Constant	-0.604	-1.228	Constant	0.915	0.266
StdErr	(0.240)	(0.375)	StdErr	(0.266)	(0.389)
T-Stat	[-2.519]	[-3.270]	T-Stat	[3.436]	[0.682]
Volatility	-0.004	-0.013	Volatility	-0.241	-0.210
StdErr	(0.057)	(0.090)	StdErr	(0.065)	(0.090)
T-Stat	[-0.079]	[-0.146]	T-Stat	[-3.711]	[-2.334]
Imbalance	-0.289	-0.379	Imbalance	0.418	0.300
StdErr	(0.095)	(0.083)	StdErr	(0.108)	(0.091)
T-Stat	[-3.048]	[-4.552]	T-Stat	[3.862]	[3.307]
Obs.	4960	2490	Obs.	4232	2377
Fixed Bond Effects	Yes	Yes	Fixed Bond Effects	Yes	Yes