Phil. 138 b: Philosophy of Mathematics
Prof. Yourgrau
Fall, 2017

Course Description:

What exactly is mathematics? In particular, is it a science? Historically, it has been seen as "the queen of the sciences." (Carl Friedrich Gauss so referred to it.) Whereas Gottlob Frege, the father of modern logic and the philosophy of language, based his philosophy of mathematics on the assumption that it is a science, Ludwig Wittgenstein denied it was. Some take it to be an art; others, a game. If it is a game, it is rather strange that it is so useful, say, in building airplanes and bombs. And if it is either an art or a game, it is a puzzle why it seems to be so concerned with truth -- the domain of the sciences. As we will see, much of Frege's philosophy of math -- which will be the centerpiece of this course -- is a direct consequence of his taking it to be a science.

If mathematics is a science, what distinguishes it from other sciences: its objects or its method (or both)? What appears, at first glance, most characteristic of its method is its use of proof. But what kind of proof? Deductive proof, surely, (where the consequences necessarily follow, given the truth of the premises) not inductive proof, (which characterizes the empirical sciences, which merely renders the conclusions of arguments more likely or more probable). But is proof all there is to math? It might at first seem so, but on reflection, it can't be all, since proofs, after all, rest on assumptions or axioms, which aren't proved, and on principles of inference (i.e. principles of deductive logic), which also aren't proved. The investigation of those elements of mathematics is too often neglected in mathematics itself. If mathematics is science, like physics, surely it aims at the truth, whereas a proof of something demonstrates only that if follows from our assumptions.

There is also the general question of how empirical beings like us can have any grasp of things like numbers and geometrical figures, since those do not appear to be empirical objects (like atoms and genes). Thus the very possibility of knowledge in the field Gauss called “the Queen of the Sciences” comes into question.

These are some of the questions Gottlob Frege investigates in his groundbreaking study, The Foundations of Arithmetic, which will be the focus of this class. We will also study contemporary attempts to answer these questions by some of the leading contemporary philosophers of mathematics.

Texts:

1) G. Frege, The Foundations of Arithmetic

2) P. Benacerraf and H. Putnam, eds., Philosophy Mathematics: Selected Readings
3) Various essays

**Writing Assignments**: several short papers (3 – 4 ppg), and one final, longer paper (4 – 5 ppg).

**Note**:

a) No ipads, iphones, or any such computer related device to be used in class, except, strictly, for taking notes. The only "i" I want to see in class is you.

b) Attendance in class is mandatory. Attendance will be taken. After two unexcused absences, there will be a grade penalty for each additional unexcused absence.

**Learning Goals**: Students will learn how to approach fundamental issues in mathematics from a perspective that may well be new to them, namely, philosophical issues such as whether mathematics, like other sciences, has a subject matter peculiar to it – as stars are to astronomers, and species are to biologists – or whether it is a different kind of science. And if it is a different kind of science, what kind of science is it. And further, if it isn’t a science at all, what exactly is it? They will not be learning new theorems of mathematics, as in a mathematics class, but learning, rather, to think clearly about mathematics, as such, which, strangely, one can have advanced far into mathematics itself without having done. They will learn to develop intellectual muscles that, previously, they may not have known existed.

**Disabilities**: If you are a student with a documented disability on record at Brandeis and need to have special accomodations for this class, please see me immediately. Also please note that any case of dishonesty (cheating on a test, using materials, including from the internet, and failing to cite its origin) is a serious academic infraction and is subject to disciplinary action.

**Academic Integrity**: You are expected to be honest in all of your academic work. Please consult the university webpage for all policies and procedures related to academic integrity. Allegations of alleged dishonesty will be forwarded to the Director of Academic Integrity. Sanctions for academic dishonesty can include failing grades and/or suspension fom the university.