Dealer Trading at the Fix

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Abstract

This paper analyzes dealer trading at "fixes," which are benchmark financial prices set at specific times of day. Extreme returns and quick retracements are common around fixes and often prompt suspicions of collusion and market manipulation, but the connections between price dynamics and dealer behavior are poorly understood. I examine a model of trading at the fix in which dealers can engage in three prohibited behaviors: front-running, sharing information a-bout customer orders, and colluding. The model shows that dealers will engage a strategy akin to front-running regardless of whether they compete or collude, causing quick retracements after the fix. Collusion shuts down free-riding among dealers while information sharing intensifies it. Therefore collusion intensifies, and information-sharing reduces, pre-fix volatility, post-fix retracements, and the convexity of the pre-fix price path.

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This paper develops a model of dealer trading around "fixes," which are specially-calculated benchmark prices for financial assets. The paper is motivated by the dramatic price dynamics associated with fixes, which are illustrated in Figure 1 for the London 4 pm fix in foreign exchange. Prices sweep to extreme values at an accelerating pace and then quickly retrace some of the move after the fix price is calculated. Observers often suspect that these price dramas manifest illegal trading behavior. Referring to the afternoon gold fix, for example, one authority states: "the unusually large (and typically downward) price movements around the PM fixing, often followed by upward movements showing price recovery … are consistent with the possibility of collusion and manipulation of the PM Fixing" (Abrantes-Metz, 2014).

Others see no reason to be concerned about these price patterns, arguing they are statistically unreliable or that they reflect legal behavior unrelated to manipulation (e.g., Fertig, 2014). Nonetheless, authorities around the world have chosen to investigate fix trading in markets including foreign exchange (CFTC 2015a, 2015b; U.S. Department of Justice, 2015, FCA 2014c-2014g), gold bullion (Harvey, 2014; FCA 2014b), other precious metals including silver (McLaughlin and Schoenberg, 2015), interest rate derivatives (Leising and van Voris, 2014), and Treasury securities (Stempel, 2014).

This paper analyzes a model in which dealers can engage in prohibited trading behaviors including front-running, sharing information about customer orders, and outright collusion. This permits a careful examination of the connections from dealer behavior to price dynamics, which are not yet well understood. According to Evans (2015), price dynamics represent "a challenge to theories of trading behavior around the Fix" (p. 4). Melvin and Prins assert that these dynamics are "not well accounted for in existing microstructure models market" (2011, p. 1).

A front-running dealer opens a position for his own account before trading on behalf of the customer. This creates harmful distortions for customers and is generally prohibited. The model shows, however, that dealers will choose front-running, or closely related strategies, regardless of whether they trade independently, share information, or collude. Front-running causes partial price retracements after the fix, consistent with the patterns observed in foreign exchange and gold markets.

Front-running is advantageous due to a special feature of fix trades: the customers' price is set *after* the trade quantity has been agreed. At the London 4 pm fix in foreign exchange, for example, dealers must receive all customer fix trading instructions by 3:45 pm. The dealer thus has 15 minutes in which to influence the customer's price (Melvin and Prins, 2011). Some of this influence happens naturally when he accumulates the inventory required to service the customer order: the purchases (sales) required to serve a customer buy (sell) order will naturally raise (lower) the price and increase the dealers own profits. Front-running intensifies that profitable price move. For normal trades, by contrast, prices and quantities are determined simultaneously so the dealer has no opportunity to influence the customer's price. Indeed, with normal trades dealers rationally minimize, rather than maximize, their market impact (Bertsimas and Lo, 1998).

Information sharing and collusion influence price dynamics through a form of free riding that, to my knowledge, has not been discussed in the literature. Fix orders are positively correlated in foreign exchange, as shown in Melvin and Prins (2015). This leads dealers to anticipate a monotonic price trend throughout the pre-fix period, caused in part by the trading of other dealers. A free-riding dealer attempts to exploit the trend-creating trading of other dealers by accelerating his inventory accumulation before the fix. This is profitable in anticipation because any inventory purchased early in the pre-fix trading interval can be expected to appreciate as the price continues to trend. The acceleration of inventory accumulation changes the shape of the pre-fix price path, speeding the trend immediately after fix trading begins and slowing the trend as the fix calculation moment approaches.

The dealers' incentive to free ride is maximized when dealers share Information about customer fix orders. This information is essentially a signal of the upcoming price trend and every dealer will rationally exploit it. A dealer with zero fix orders, who would abstain from trading altogether in the absence of information-sharing, will open a speculative position immediately if he knows the fix orders received by his competitors. Information sharing reduces average dealer profitability because many dealers will be liquidating speculative positions and countering the dominant trend just when the majority-direction dealers need the initial trend to continue. With lower profits, dealers have less incentive to front-run, so prices become less volatile, retracements become less pronounced, and the pre-fix price path accelerates less – or decelerates more – as the fix approaches than it would if dealers maintained the confidentiality of their customer orders.

Collusion shuts down free riding. Its effects on dealer behavior and price dynamics are thus opposite to the effects of information sharing. Average dealer profitability is higher, as is aggregate front-running. Prices at the fix are more volatile, post-fix retracements are more pronounced, and the pre-fix price path accelerates more as the fix moment approaches.

What can be inferred from dramatic price dynamics at the fix? According to the model, high volatility will occur around a fix whether or not dealers engage in prohibited behaviors. Price retracements imply front-running, which also intensifies volatility. The effects of information sharing and collusion on price dynamics are relatively subtle. They dampen or intensify already-high volatility and already-evident price retracements. Information sharing reduces, and collusion increases, the convexity of the price path, meaning its tendency to accelerate as the fix calculation moment approaches. Empirically these effects can only be established by comparison with some benchmark for independent trading at the fix.

Fix prices serve many of the same purposes as closing prices, and concerns about market dynamics around closing prices are nothing new. The one theoretical treatment of dealer behavior of which I am aware, Hillion and Souminen (2004), focuses on order choice and bid-spreads at the close. The model developed here follows Kyle (1985) in assuming batch trading and thus abstracts from both order choice and spreads. Pagano et al. (2013) show that a call market at the close can reduce volatility, though Ellul et al. (2005) suggest closing calls may have high failure rates for small and medium-sized firms. Cushing and Madhavan (2000) analyze price dynamics at the close for Russell 1000 stocks in the late 1990s, prior to the NASDAQ's implementation of a closing call in 2004. Closing-price dynamics at the time included exceptionally high volatility and price retracements, consistent with patterns observed around fixing prices. They provide evidence that post-close retracements reflect price pressures (Huang and Stoll, 1997; Hendershott and Menkveld, 2014), referring to the tendency of dealers to temporarily lower (raise) prices when they hold excess (insufficient) inventory. The only academic research related to fixing prices, per se, is Melvin and Prins (2013), but their focus is not the fix itself. Instead they focus on fix trading as a period in which equity hedging transactions of institutional asset managers are highly concentrated, and they show that such risk-motivated trades influence exchange rates.

Formal investigations of fixes have provided constant fodder for the financial media in recent years. In 2014 the U.K.'s Financial Conduct Authority (FCA) began attending the London gold fix, a benchmark set since 1919 by twice-daily meetings of a few prominent bankers. Later in 2014 Barclays was fined $44 million for allowing a dealer to manipulate the fix (Slater and Jones, 2014). Also in 2014 authorities announced an investigation of the ISDA fix, which sets rates for interest-rate derivatives (Van Voris and Leising, 2014). Formal investigations of trading around foreign exchange (forex) fixes began in 2013 after media reports that forex dealers were colluding over electronic chat rooms with colorful names like "the cartel" and "the mafia" (Bloomberg, 2013). Since then the major currency dealing banks have paid over $11 billion in fines in the U.S. (CFTC 2015a, 2015b; U.S. Department of Justice, 2015), the U.K. (FCA 2014a), and Switzerland (Bray, Anderson, and Protess, 2014). In May of 2015 major forex dealing banks pled guilty in the U.S. to market manipulation (Department of Justice, 2015)[1] and paid over $2 billion to settle a U.S. class-action suit (Raymond, 2015).

More important than media buzz have been the ensuing changes in fix-price calculation methodologies. In 2015 the time interval over which the London 4 pm forex fix is calculated was extended from one to five minutes. Meanwhile the traditional gold fix was replaced with a more transparent auction on an electronic platform provided by the CME and the 117-year-old silver fix was similarly transformed.

[1] Given the guilty plea I do not describe the behavior in forex as "alleged."

The rest of this paper has five sections. Section I outlines the model. Section II analyzes profit-maximizing trading and exchange-rate dynamics around the fix when dealers trade independently. Section III examines the equilibrium when risk-neutral dealers share information about customer fix orders or set trading strategies collusively. Section IV analyzes fix trading when dealers are risk averse. Section V concludes.

Before moving on, a quick caveat. This paper provides a positive analysis of what dealers might choose to do around fixes, not a normative analysis. The paper does not advocate prohibited behaviors and the author deplores them when they occur in reality.

I. The Model

The structure of the model outlined below is motivated by the W.M. Reuters London 4 pm fix. This fix was established in 1994 to serve as the functional equivalent of a closing price for the foreign exchange market, given that they never formally close (Bloomberg, 2013). Many of that fix's key features are shared by other fixes.

Agents: Customer fix orders are managed by $N+1=D<\infty$ identical dealers. These agents interact in the interdealer market with a fringe of atomistic dealers. Foreign exchange trading is highly concentrated, consistent with the model, even though there are hundreds of foreign exchange dealers worldwide: the market share of the top four banks exceeds 50% (*Euromoney*, 2013). Fix trading is yet more concentrated because small and regional banks generally pass fix orders on to the dominant few. The gold market appears to have been similarly concentrated, given that just five banks participated in the fix calls before they ceased in 2015.

Customer Fix Orders: Before trading begins, each dealer receives a random set of customer fix orders. We focus on a representative dealer, specifically the *N*+1st dealer, indicated by subscript d. Dealer d's net fix order, F_d, includes a component shared by all other dealers, Φ, and a dealer-specific deviation, η_d : $F_d = \Phi + \eta_d$. Both Φ and η_d are i.i.d. and mutually uncorrelated with mean zero and variances σ_Φ^2 and σ_η^2, respectively. The correlation in fix orders is $\rho = \sigma_\phi^2 / (\sigma_\phi^2 + \sigma_\eta^2)$>0. Without loss of generality the discussion assumes that dealer d's customers are buyers, $F_d > 0$, so dealer d is a buyer in the interdealer market.

The model takes customer fix orders as exogenous but their origin in reality is well understood. Cochrane (2015) points out that international equity funds worth $9 trillion are benchmarked to the MSCI indexes and another $2 trillion are benchmarked to the Citi World Government Bond Index, all of which are marked to market with the WM/Reuters Closing Spot Rates. These institutions have a high incentive to avoid tracking risk, which they can achieve by trading exactly at the fix price. Melvin and Prins (2015) substantiate the influence of portfolio hedging with evidence that month-end forex fix flows are related to recent equity returns.

Time: The pre-fix trading interval has two trading periods, periods 1 and 2, during which representative dealer d trades quantities D_{1d} and D_{2d} in the interdealer market at prices P_1 and P_2, respectively. Orders to trade at the London 4 pm fix must be received by 3:45 pm, which suggests that periods 1 and 2 could, in reality, be roughly 7.5 minutes long. It is also possible, however, that dealers wait to trade until just before the fix price is actually calculated, and the functional equivalent of a period is a minute or less.

Inventory management: The model follows the literature in assuming that each dealer's inventory is at its target level when fix orders arrive and is restored to that level by the end of fix trading, though that level of desired inventory is left unspecified. Representative dealer d

may take a proprietary position, where $X_d \equiv D_{1d} + D_{2d} - F_d$, which he will liquidate immediately after the fix in the third and last period of fix trading at interdealer price P_3.

The fix: The fix price is set equal to the period-2 interdealer price, $P_F = P_2$, and dealer d trades F_d with his customers at that price. Our analysis should be relevant to fixing prices calculated with a variety of methodologies. Indeed, foreign exchange dealers appear to have behaved similarly at both the London 4 pm fix and the ECB fix, which occurs at 2:15 European Central time, though the methodologies for determining these fixing prices are distinct. Until December, 2014, the W.M. Reuters 4 pm fix used (roughly) the median traded interdealer price over the 60-second interval centered on the hour. The ECB fix is set according to a central bank "concertation procedure," the details of which are not published.[2] The results may also transcend asset class given the observation by the director of the Swiss financial authority, FINMA, Mark Branson, that "[t]he behaviour patterns in precious metals were somewhat similar to the behaviour patterns in foreign exchange" (Harvey, 2014).

Dealer Objectives: Dealers are initially assumed to be risk-neutral profit maximizers; Section IV investigates the implications of dealer risk aversion. The analysis does not incorporate potential costs to dealers or their employers of violating laws, regulations, or bank policies.

Dealer d's revenues comprise $P_F F_d$ from selling to customers at the fix plus $P_3 X_{1d}$ from liquidating any proprietary position after the fix. His costs come from purchasing inventory in periods 1 and 2: $P_1 D_{1d} + P_2 D_{2d}$. Interest expense is irrelevant because fix trading occurs

[2] The methodology is described with consistent wording, and thus consistent ambiguity, on every central bank's website: "The reference rates are based on the relevant price prevailing in the market at this point in time. … Since the exchange rates of the aforementioned currencies against the euro are averages of buying and selling rates, they do not necessarily reflect rates at which actual market transactions have occurred. The exchange rates against the euro published by the ECB are released for reference purposes only; therefore the ECB does not necessarily trade at these rates.
http://dsbb.imf.org/Pages/SDDS/BaseSMReport.aspx?ctycode=EMU&catcode=EXR00&ctyType=SDDS

intraday. We abstract from the cost of bank equity capital following the literature. Dealer d's profits, π_d, are:

$$\pi_d = P_F F_d + P_3 X_{1d} - P_1 D_{1d} - P_2 D_{2d} \quad . \tag{1}$$

This analysis of dealer behavior relies heavily on the following decomposition of profits:

$$\pi_d = D_{1d}\left(P_2 - P_1\right) + X_d\left(P_3 - P_2\right). \tag{2}$$

The first term on the right represents the interaction between period-1 inventory trades and the period-2 return. Dealer d's period-1 inventory purchase will be profitable in expectation when he sells it to his fix customers at $P_F = P_2$ so long the interdealer price moves in a profitable direction during period 2. The second term on the right captures gains or losses incurred upon liquidating his proprietary position after the fix. In the model's equilibria the first term is always positive in expectation and the second term is always negative in expectation. That is, in equilibrium the interaction between period-1 inventory and period-2 returns is the sole reason that fix trading is profitable.

Price Generating Process: When executing fix trades, fix dealers trade against each other and against the atomistic fringe. Their trades have a linear contemporaneous price impact proportional to $\theta > 0$. Returns are also driven by trading shocks unrelated to the fix, ε_t, such as the arrival of public information. These shocks are i.i.d. with zero mean and variance σ_ε^2:

$$P_t - P_{t\text{-}1} = \theta\left(D_{td} + \sum_{N \neq d} D_{tn} + \varepsilon_t\right), \qquad t = \{1,2,3\}. \tag{3}$$

Equation (3) implies that, outside of the fix trading interval, the price follows a random walk, $P_t - P_{t-1} = \theta\varepsilon_t$, with one-period return variance $\theta^2\sigma_\varepsilon^2$. The proportionality coefficient captures the permanent effect of order flow on price.

Closely related papers consistently assume a linear and permanent impact of order flow on price (e.g., Bertsimas and Lo, 1998; Cushing and Madhavan, 2000). Nonetheless, the assumption that order flow has a permanent price impact deserves some justification since it may seem to violate efficient-markets theory. This assumption is amply supported by rigorous empirical research that began to emerge over three decades ago and now covers all the major asset classes.[3] It is also supported by long-established microstructure theory. Indeed, Equation (3) is often derived as an equilibrium relation in models of rational market making that include asymmetric information (Kyle, 1985; Glosten and Milgrom, 1985) or finite elasticity of demand (Evans and Lyons, 2002). Under asymmetric information uninformed agents rationally extract information from observed order flow. Uninformed agents will infer from a price rise that someone else may be buying based on private information and adjust their own expectations accordingly. When they are correct the price moves permanently. Finite elasticity of demand refers to the idea that a given amount of end-user purchases (sales) may require a rise (decline) in the price to elicit the required selling (purchasing) by other end-users. This effect arises in any speculative market with risk-averse speculators (Evans and Lyons, 2002) or hedgers (meaning agents who trade exclusively to reduce risk). In foreign exchange this effect also arises from firms engaged in international trade, who naturally respond to the changes in relative prices induced by changes in exchange rates (Osler, 2006).[4]

[3] For equities see, e.g., Shleifer (1986), Hasbrouck (1991), Huang and Stoll (1994), Chordia, Roll, and Subrahmanyam (2002); for exchange rates see Evans and Lyons (2002), Berger et al. (2006), and Rime, Sarno, and Sojli (2007); for bonds see Simon (1991, 1994), Fleming (2003), Brandt and Kavajecz (2005), and Pasquariello and Vega (2007).

[4] The model abstracts from questions of order choice – meaning the choice between making and taking liquidity – because influence of order flow on financial prices arises whether informed agents make or take liquidity. If a dealer takes liquidity the price necessarily moves by the bid-ask spread. If he makes liquidity by placing a limit

II. Independent Trading

This section examines trading and price dynamics when fix dealers trade independently, consistent with the law. That is, they do not share confidential information about customer orders and they do not collude on trading strategies. Representative dealer d faces a fairly standard intertemporal optimization problem in which he must analyze his later trading decisions before making his initial trading decision. Dealer d's final trading decision occurs in period 2, not period 3, because period-3 trading is dictated by the requirement that he restores inventory to its initial level.

A. The Period-2 Trading Decision

The analysis is streamlined with a change of variables. Let α_{1d} represent the share of dealer d's net fix order that he trades in period 1, $\alpha_{1d} \equiv D_{1d} / F_d$, and $\overline{\alpha}_{1d}$ represent the average of the corresponding fraction for all other dealers: $\overline{\alpha}_1 \equiv \sum_N D_{1n} \Big/ \sum_N F_n$. In period 2 dealer d takes α_{1d} and $\overline{\alpha}_1$ as given and chooses X_d , his own proprietary trading:

$$\underset{X_d}{Max}\ E_{2d}\{\pi_d\} = \alpha_1 F_d \theta \left\langle [(1-\alpha_1)F_d + (1-\overline{\alpha}_1)E_{2d}\{\sum_{N\neq d} F_n\}] \right\rangle + (\alpha_1 F_d - X_d)\theta(X_d + E_{2d}\{\sum_{N\neq d} X_n\}). \quad (4)$$

The first-order condition shows that X_d depends positively on the dealer's own first-period purchase, D_{1d}, and negatively on the other dealers' expected proprietary trading:

$$X_d = \frac{1}{2}\left[\alpha_1 F_d - E_{2d}\{\sum_{n\neq d} X_n\}\right]. \quad (5)$$

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order the price will move in the same direction in expectation because liquidity on the bid (ask) side reduces the likelihood of a price decline (rise).

From here dealer d forms his rational expectation of the other dealers' proprietary trades, $E_{2d}\{\sum_{n\neq d} X_n\}$. Details of the solution are reported in the Appendix. The results show that dealer d's proprietary trading is proportional to his period-1 trade with proportionality coefficient q_d:

$$X_d = \frac{\alpha_{1d} F_d}{2+\rho N} \equiv q_d \alpha_{1d} F_d \, , \qquad q_d \equiv \frac{1}{2+\rho N} \quad . \tag{6}$$

The ratio of dealer d's proprietary position to his own fix order will be denoted x_d:

$x_d \equiv X_d / F_d = q_d \alpha_{1d}$.

B. The Period-1 Trading Decision

Having identified the functional form for his period-2 decision, dealer d solves the following optimization problem to choose his period-1 trades:

$$\underset{\alpha_{1d}}{Max}\; E_{1d}\{\pi_d\} = \alpha_1 F_d \theta \left[F_d(1-\alpha_1) + (1-\overline{\alpha}_{1n})\rho N F_d\right] + \alpha_{1d} F_d (1-q)\theta \left[q\alpha_{1d} F_d + E_{1d}\{\sum_N X_n\}\right] \tag{7}$$

The profit-maximizing value of α_1 depends on the other dealers' expected behavior as captured by $\overline{\alpha}_1$ and $\overline{q} \equiv E_{2d}\{\sum_N X_n \big/ \sum_N F_n\}$:

$$\alpha_{1d} = \frac{(1+\rho N) - \overline{\alpha}_1 \rho N[1-\overline{q}(1-\overline{q})]}{2[1-q(1-q)]} \quad . \tag{8}$$

In market equilibrium, dealer symmetry implies $\alpha_{1d}{=}\overline{\alpha}_1$ and $q_d = \overline{q}$, which closes the model.

Lemma 1 characterizes equilibrium trading shares using the correlation between an individual dealer's fix order, such as F_d, and aggregate fix orders of the fix dealing community, $\sum_{N+1} F_n$: $\sigma_F^2 \equiv (N+1)\sigma_\phi^2 + \sigma_\eta^2$.

Lemma 1: In competitive equilibrium ($N \geq 1$) with positively correlated fix orders ($\rho > 0$), a dealer's trades are proportional to his own fix orders in every period:

a. $\frac{2}{3} < \alpha_1 = \frac{D_{1d}}{F_1} = \frac{(2+\rho N)(1+\rho N)}{(2+\rho N)^2 - (1+\rho N)} < 1$, (14a)

b. $0 < \alpha_2 = \frac{D_{2d}}{F_d} = \frac{(2+\rho N)}{(2+\rho N)^2 - (1+\rho N)} < \frac{2}{3}$, (14b)

c. $0 < x \equiv \frac{X_d}{F_d} = \frac{\alpha_1}{2+\rho N} = \frac{(1+\rho N)}{(2+\rho N)^2 - (1+\rho N)} < \frac{1}{3}$. (14c)

d. Each fix dealer's trades decline over time in absolute magnitude: $\alpha_1 > \alpha_2 > x$.

e. Expected profits are positive: $E_0\left\{\overline{\pi}^{Indep}\right\} = \theta\sigma_F^2 x$ **.**

Profit-maximizing fix trading has three critical properties: it is distributed across both pre-fix periods ("distributed trading"), it is concentrated in period 1 ("free riding"), and it exceeds customer fix orders ("proprietary trading"). Each of these is best understood by comparing the market under competitive dealing (N≥0) and zero competition (N=0).

Distributed trading before the fix: In equilibrium all dealers with non-zero fix orders trade in both pre-fix periods. In the extreme case of zero competition it is essential for dealer d to trade in both periods. Without his own period-1 purchase, any price rise between periods 1 and 2 brings zero profits in expectation because he has no inventory to appreciate. Without his own period-2 purchase, however, he can expect no price rise between periods 1 and 2 and thus no profits from any period-1 inventory purchase. The profit-maximizing strategy with N = 0 is to trade equal amounts in both periods: $\alpha_1^{N=0} = \alpha_2^{N=0}$ =2/3.

Free-riding: When dealers face competition, or $N \geq 1$, they continue to trade in both periods. However, they now have an incentive to free ride by shifting some trading from period 2 to period 1. This is profitable because every dealer expects the other dealers to trade in his same direction in period 2, given the positive correlation among fix orders: $E_{1d}\{\sum_N F_n\} = \rho N F_d$. Each dealer therefore expects an appreciation in his period-1 inventory even if he himself does not trade in period 2. Of course, if the other dealers apply the same logic they will all skip trading in period 2, the period-2 return will be zero and fix profits will be zero. In market equilibrium each dealer shifts some but not all of his period-2 trading to period 1 relative to the equilibrium with N = 0. This increases the share of total pre-fix trading that is executed in period 1, Π_d:

$$\frac{1}{2} \leq \Pi_d^{Indep} \equiv \frac{D_{1d}}{D_{1d} + X_d} = \frac{\alpha_1}{1+x} = \frac{1+\rho N}{2+\rho N} < 1. \tag{15}$$

Algebraic manipulation shows that free-riding is always increasing the number of other dealers. This relation is intuitively logical, since the costs of free-riding will be distributed equally across all dealers and as N rises each individual dealer can expect to bear less of those costs when making his period-1 trading decision.

Proprietary trading: Profit-maximizing dealers accumulate more inventory than required to fulfil their customer fix orders: $x_d \equiv X_d / F_d > 0$. In the absence of competition (N = 0) this proprietary trading is exactly 1/3 of customer fix orders:

$$(\alpha_{1d}^{N=0} + \alpha_{2d}^{N=0} - 1)F_d \equiv x^{N=0} F_d = \tfrac{1}{3} F_d. \tag{16}$$

For $N \geq 1$ the dealers continue to take proprietary positions in parallel with their customer orders but these positions shrink as a share of customer fix orders: $x^{N \geq 1} < \tfrac{1}{3}$. Proprietary trading maximizes profits due to the unique incentives faced by dealers at the fix. In contrast to normal trades, where prices and quantities are agreed simultaneously, the fix price is set after dealer and customer have set the quantity to trade. The dealer can therefore enhance his revenues from fix trades, $F_d P_F$, by maximizing the price impact of his own inventory accumulation prior to the fix. By trading for his own account in addition to trading for his customers, the dealer can move the price farther, enhancing his profits.

The profit-maximizing amount of proprietary trading is finite because these trades incur a loss when they are liquidated in period 3. In the simple non-competitive case (N=0) the gains from proprietary trading, which are captured by the first term in Equation (2), are proportional to the dealer's period-1 inventory position, D_{1d} , and to the proprietary trading itself, X_d, which drives the period-2 return. For given D_{1d} , therefore, the gains are positive at $X_d = 0$ and grow linearly with X_d. The losses, which are captured by the second term in Equation (2), are proportional to the square of proprietary trading, ${X_d}^2$, because proprietary trading drives the magnitude of the period-3 price change as well as the proprietary position. The losses begin at zero for $X_d = 0$ and grow exponentially. At $X_d = 0$, therefore, the gains to proprietary trading are positive and the losses are zero, so it is optimal to take at least a small proprietary position; as X_d grows the gains grow more slowly than the losses so profit-maximizing proprietary trading is finite.

In standard (non-fix) trading environments the dealers' incentive is generally to minimize price impact (Bertsimas and Lo, 1998). Because the price of a normal trade is agreed at the same time as the quantity traded, the dealer's subsequent inventory management trades cannot influence the price. Additional proprietary trading in such situations raises the dealer's costs and, on average, it generates a loss equal to the proprietary position times the price impact per unit.

A bit of algebraic manipulation shows that proprietary trading is inversely related to the number of other dealers. This is because rising competition brings an increase in the aggregate proprietary position liquidated in period 3, other things equal. This implies a larger retracement between periods 2 and 3, which reduces an individual dealer's profits from proprietary trading.

The prediction that dealers take proprietary positions around the fix is consistent with dealer comments: "Three [forex dealers] said that when they received a large [fix] order they would adjust their own positions knowing that their client's trade could move the market" (Bloomberg, 2013). Likewise the Swiss financial authority, FINMA, reported repeated instances of front-running at the silver fix (Harvey, 2014). In this model under independent trading, front-running and the form of proprietary trading that maximizes profits are functionally equivalent because the batch trading format does not indicate a precise trade sequence.

Front-running is banned in most markets because it is harmful to customers: it raises (lowers) the prices at which they purchase (sell) assets. It is not illegal in currency markets, however, in part because currencies are neither securities nor financial instruments and therefore they do not fall under Europe's Markets in Financial Instruments Directive or the U.S.'s Dodd-Frank Act. More fundamentally, foreign exchange dealing has no worldwide equivalent of the SEC or FSA to set and enforce regulations. The market is thus internationally footloose: when one country attempts to regulate trading the business just moves elsewhere.

Front-running is certainly considered unethical in foreign exchange markets, even though it is not technically illegal. The Non-Investment Products Code, signed by all the major foreign exchange banks in 2011 under the auspices of the Bank of England, states:

> The handling of customer orders requires standards that strive for best execution for the customer in accordance with such orders subject to market conditions. In particular, caution should be taken so that customers' interests are not exploited when financial intermediaries trade for their own accounts. … Manipulative practices by banks with each other or with clients constitute unacceptable trading behaviour. (Bank of England, 2011)

Proposition 1 summarizes the key features of equilibrium fix trading when dealers trade independently:

Proposition 1: Under competitive fix trading ($N \geq 1$) with positively correlated fix orders ($\rho > 0$), equilibrium dealer trading exhibits three key features:

- ***a. Distributed trading: Pre-fix inventory accumulation takes place in both periods 1 and 2***
- ***b. Free riding: Dealers shift pre-fix inventory accumulation from period 2 to period 1 relative to the equilibrium without competition ($N=0$).***
- ***c. Parallel proprietary trading: Every fix dealer accumulates more inventory before the fix than required to service his customer fix orders.***

C. Fix Price Dynamics When Dealers Trade Independently

This model predicts high pre-fix volatility and post-fix trend reversals, both of which have long been apparent around the London 4 pm forex fix (see Figure 1). According to Evans (2015), "across all time periods and currency pairs changes in rates before and after the Fix are regularly of a size rarely seen in normal trading activity" and "pre- and post-Fix rate changes also display a strong degree of negative autocorrelation that is not found elsewhere during normal forex trading" (p. 44).

Volatility before the fix, Σ, is measured here as the variance of returns from P_0 to $P_F = P_2$:

$$\Sigma^{Indep} \equiv E\left\{(P_F - P_0)^2\right\} = \theta^2 E\left\{\left(\sum_{N+1} F_n(1+x) + \varepsilon_1 + \varepsilon_2\right)^2\right\}.$$
$$= \theta^2 2\sigma_\varepsilon^2 + \theta^2(N+1)\sigma_F^2 + \theta^2(N+1)\sigma_F^2 x(2+x). \tag{17}$$

Volatility at the fix naturally exceeds volatility during normal times due to the high concentration of customer orders executed within a short time frame. Two-period volatility at normal times would be $2\theta^2\sigma_\varepsilon^2$, the first term on the right in Equation (17). The direct effect of customer fix orders on volatility is $\theta^2(N+1)\sigma_F^2$, the second term on the right. Without parallel proprietary trading, therefore, fix volatility would be $\theta^2 2\sigma_\varepsilon^2 + \theta^2(N+1)\sigma_F^2$. Proprietary trading intensifies volatility at the fix, contributing the amount $\theta^2(N+1)\sigma_F^2 x(2+x) > 0$.

Volatility at the fix could naturally exceed volatility at other times if the price impact coefficient, θ, is higher at the fix than otherwise. Cushing and Madhavan (2000) show that order flow has a stronger impact on equity prices just prior to the close than at other times. If fix order flow is informative this is economically logical, given Kyle's (1985) demonstration that the sensitivity of price to order flow increases with the extent to which trades are informed.

Post-fix retracements: Negative autocorrelation of returns around the fix, denoted Λ**,** is measured as the coefficient from a regression of total post-fix returns, $P_3 - P_F$, on pre-fix returns, $P_F - P_0$:

$$\Lambda^{Indep} = \frac{-\theta^2 M\sigma_F^2(1+x)x}{2\sigma_\varepsilon^2 + \theta^2 M\sigma_F^2 + \theta^2 M\sigma_F^2 x(2+x)} < 0. \tag{18}$$

Equation (18) shows that post-fix retracements are due entirely to proprietary trading. Without such trading x = 0 and $\Lambda^{Indep} = 0$.

Cushing and Madhavan (2000) suggest a different source of price retracements at the close for U.S. equities: price pressures (Huang and Stoll, 1997; Hendershott and Menkveld, 2014). Price pressures emerge when dealers with excess (insufficient) inventory lower (raise) the price, hoping to attract trades that rectify the imbalance. Though price pressures are not explicitly incorporated in this model, they seem inevitable given the magnitude of order imbalances at fixes. The price pressure and proprietary-trading hypotheses for post-fix retracements are not mutually exclusive, and the comments of dealers and regulators confirm that front-running has also been an issue.

Convexity: The model has implications for the curvature of the pre-fix price path. If $N = 0$ and dealers are risk-neutral the price path is linear: returns neither accelerate nor decelerate during the pre-fix trading interval. If $N \geq 1$, free-riding causes returns to be smaller (in absolute magnitude) in period 2 than in period 1, or equivalently the trend decelerates. This property will be referred to as a decline in the "convexity" of the price path, though technically this application of the term may is accurate for positive F_d.

Proposition 2 summarizes the fix price dynamics predicted by the model when fix dealers trade independently:

Proposition 2: Under competitive fix trading ($N \geq 1$) with positively correlated orders,

a. ***Volatility:*** **Return volatility before the fix will be higher than normal**
b. ***Post-fix retracements:*** **The pre-fix trend will be partially retraced after the fix**
c. ***Convexity:*** **Free-riding reduces the convexity of the price path relative to the equilibrium with zero competition.**

These findings show that high volatility at the fix need not reflect either information sharing or collusion. Nonetheless, when dealers trade independently volatility is intensified and quick retracements will become a regular feature of fix dynamics due to parallel proprietary trading, a behavior akin to the prohibited practice of front-running.

One might wonder whether high pre-fix volatility and predictable retracements are consistent with efficient markets or whether they might ultimately be competed away by the dealing community. Indeed, strong predictable price dynamics often disappear, in the long run, for just this reason (Lo, 2004). Trading and price patterns cannot be competed away, however, when they represent strategic complements, meaning the behavior of different agents reinforces each other (Bulow et al., 1985). At the fix, index funds become strategic complements when they place fix orders. When some firms place fix orders to avoid tracking error, returns at the fix become more extreme and other firms face a stronger incentive to place fix orders. The participation choices of small and mid-sized dealers could reinforce this effect. They have apparently chosen to step back from trading at the fix, given the high fix volatility, opting to pass their orders along to the biggest banks. By stepping back they reduce liquidity at the fix, increase the price impact of fix orders, and ultimately contribute to yet higher volatility. All in all, there is no implication that the market dynamics identified here should eventually disappear. Dealers appear to be strategic complements when they trade at the fix because their individual trading strategies intensify pre-fix volatility, which in turn encourages more fix trading and deters liquidity provision and arbitrage.

III. Information Sharing and Collusion

Recent regulatory activity and judicial actions confirm that fix prices have been "manipulated" by dealers in multiple markets (FCA 2014c-2014g; Harvey, 2014). This section examines the behavior of dealers who go beyond front-running in transgressing regulatory, and legal limits. It first considers dealers who share information about customer orders and then considers dealers who collude outright in selecting trading strategies.

A. Information Sharing

Sharing information about customer orders with another dealer is considered unethical, at a minimum, because it puts the customer at risk of manipulation. Dealers know this because bank compliance departments regularly communicate that message. Nonetheless, conversation transcripts available to the public show that forex dealers shared information about customer orders in private chat rooms (FCA, 2014).

To model such behavior I assume that each fix dealer gives accurate information to the other fix dealers about his customer net fix order as soon as all orders have arrived (e.g., at 3:45 pm for the London fix). Each dealer still undertakes a two-stage analysis to determine his own trades. He first identifies how their period-2 trades will depend on his period-1 trade and then chooses his period-1 trade. In market equilibrium representative dealer d's trading is still influenced by his own customer order, F_d. It is also influenced, however, by the average customer order, $\overline{F} = (F_d + \sum_{N \neq d} F_n)/(N+1)$. Continuing to denote with α the influence of the dealer's own fix order, I denote with δ the influence of average fix orders, $\overline{F}$:

Lemma 2: In competitive equilibrium (N≥1) with information sharing, dealer d trades exclusively for his own account in periods 1 and 3 and accumulates inventory for customers only in period 2.

a. $$D_{1d} \equiv \alpha_1 F_d + \delta_1 \overline{F} = \left(\frac{(1+N)(2+N)}{(2+N)^2 - (1+N)} \right) \overline{F} : \ \alpha_1 = 0, \ \delta_1 > 0 \tag{19a}$$

b. $$D_{2d} \equiv \alpha_2 F_d + \delta_2 \overline{F} = F_d - \left(\frac{(1+N)^2}{(2+N)^2 - (1+N)} \right) \overline{F} : \ \alpha_2 = 1, \ \delta_s < 0 \tag{19b}$$

c. $$X_d \equiv -D_{3d} = \alpha_3 F_d + \delta_3 \overline{F} = \left(\frac{1+N}{(2+N)^2 - (1+N)} \right) \overline{F} : \ \alpha_3 = 0, \ \delta_3 > 0 \tag{19c}$$

The proprietary trading under information sharing is fully consistent with front-running, though they are front-running the average order, rather than their own customer's order. In period 1 the dealers take the average fix order as a signal of the direction of pre-fix returns and take a proprietary position based on that signal. In fact, every dealer takes exactly the same proprietary position in period 1. A dealer with no fix orders of his own, or with net fix orders in the opposite direction to the majority, nonetheless takes the same inventory position in period 1 as the other dealers. By contrast, recall that under independent trading every dealer trades a fraction of his own fix order in every period. If he has no fix order he does not trade; if his customers are buying when all the other customers are selling, he buys.

Free riding continues to be an optimal strategy; indeed, it becomes more intense when dealers have full information about market-wide customer orders than when dealers trade independently. The consequences of free riding begin to be apparent in Period 2, when dealers whose fix orders are against the majority begin liquidating the proprietary position opened in period 1. These trades undermine the efforts of dealers with majority-direction fix orders to extend the initial trend, an extension they need to make money (Equation (2)).

Other properties of trading under information sharing are described in Proposition 3:

Proposition 3: In competitive equilibrium ($N \geq 1$) with information sharing, dealer fix trading is distributed across both pre-fix periods, dealers free-ride, and dealers undertake proprietary trades, as observed under independent trading. Under information-sharing, however,

a. Free-riding is more intense

$$\frac{1}{2} < \Pi^{Indep} = \frac{1+\rho N}{2+\rho N} < \Pi^{InfoShare} = \frac{1+N}{2+N} < 1 \tag{20a}$$

b. The additional free-riding undermines dealer profits

$$0 < E_0\{\overline{\pi}^{Indep}\} = \theta\sigma_F^2 x \leq E_0\{\overline{\pi}^{Share}\} = \theta\sigma_F^2\left(\frac{1+N}{(2+N)^2-(1+N)}\right)^2 , \tag{20b}$$

c. ***On average, dealers take smaller proprietary positions.*** Letting $\overline{\chi} \equiv \sum_{N+1} X_n \Big/ \sum_{N+1} F_n$ represent total proprietary trading across all dealers as a share of total fix orders:

$$0 < \overline{\chi}^{Share} = \left(\frac{1+N}{(2+N)^2-(1+N)}\right) \leq x^{Indep} \leq \frac{1}{3} \ . \tag{20c}$$

Proposition 4 summarizes how information sharing affects fix price dynamics:

Proposition 4: In competitive equilibrium ($N \geq 1$) with information sharing, fix volatility is higher than under normal trading conditions and front-running generates trend reversals immediately after the fix. Because there is more free riding and less proprietary trading than under independent trading,

a. Volatility is less pronounced than under independent trading

$$\Sigma^{Share} \equiv E\{(P_F - P_0)^2\} = \theta^2 2\sigma_\varepsilon^2 + \theta^2 M\sigma_\phi^2(1+\overline{\chi})^2 < \Sigma^{Indep} \ \textbf{.} \tag{21a}$$

b. Post-fix trend reversals are less pronounced than under independent trading

$$\Lambda^{Indep} < \Lambda^{Share} = -\frac{M\sigma_F^2(1+\overline{\chi})\overline{\chi}}{2\sigma_\varepsilon^2 + M\sigma_F^2(1+\overline{\chi})^2} < 0 \ \textbf{.} \tag{21b}$$

c. ***Convexity: The average pre-fix price path becomes less convex than under independent trading*** (see Equation (20a)**.**

B. Collusion

Collusion violates antitrust laws as well as bank policies and standard regulatory limits. As such, the penalties for collusion, which can include jail time, are significantly more severe than the penalties for information sharing. One might therefore wonder why dealers who already share information might also collude. This section shows that collusion has a big advantage over information sharing: it's more profitable.

Collusive strategies could be arranged in a variety of different ways. At the London 4 pm fix, dealers in the "cartel" often assigned a single dealer to control all customer fix trades for the group (FCA, 2014c – 2014e). But the same collusive strategy could be executed with multiple dealers sharing the trading responsibilities, so long as they trust each other. For convenience I refer to a single controlling dealer and denote pre-fix inventory accumulation under collusion by D_1, D_2. Proprietary trading under collusion will be denoted $X \equiv \chi^{Collude} \sum_{N+1} F_n$.

The profit-maximizing collusive strategy depends entirely on total customer fix orders, $F_{Tot} \equiv \left(F_d + \sum_{N \neq d} F_n \right)$. Otherwise the strategy is identical to the strategy of a single dealer (N= 0) under independent trading:

Lemma 3: A dealing cartel will trade 2/3 of total fix orders in period 1 and again in period 2:

$$\frac{D_1}{\sum_{N+1} F_n} = \frac{D_2}{\sum_{N+1} F_n} = \frac{2}{3} \quad . \tag{22a}$$

The cartel's proprietary position will be 1/3 of total fix orders:

$$\frac{X}{\sum_{N+1} F_n} = \frac{1}{3} \quad . \tag{22b}$$

Collusion dominates information sharing because it shuts down free riding. As was true for the zero competition equilibrium under independent trading, pre-fix inventory accumulation will be evenly distributed between periods 1 and 2. By eliminating free riding collusion brings higher per-dealer profits and encourages parallel proprietary trading.

Proposition 5: When dealers collude, dealer fix trading is distributed across both pre-fix periods and dealers trade for their own account, as observed under independent trading and information sharing. Under collusion, however,

- ***a. Dealers cannot free-ride***
- ***b. Expected profits are maximized***

$$E_0\left\{\bar{\pi}^{Collude}\right\} = \theta\sigma_F^2 \big/ 3 > E_0\left\{\bar{\pi}^{Indep}\right\} = \theta\sigma_F^2 x^{Indep} > E_0\left\{\bar{\pi}^{Share}\right\} = \theta\sigma_F^2 \bar{\chi}^{Share}. \tag{23}$$

- ***c. Proprietary trading is maximized:*** $\chi^{Collude} = 1\big/3 > x^{Indep} > \bar{\chi}^{Share}$**.**

These effects on dealer trading strategies have clear implications for price dynamics:

Proposition 6: In equilibrium under collusion

- ***a. Pre-fix volatility is maximized***

$$\Sigma^{Collude} = \theta^2 2\sigma_\varepsilon^2 + \theta^2 M\sigma_\phi^2 (1+\chi^{Collude})^2 > \Sigma^{Indep} > \Sigma^{Share}. \tag{24a}$$

- ***b. Post-fix reversals are most pronounced***

$$\Lambda^{Collude} = -\frac{M\sigma_F^2 \chi^{Share}(1+\chi^{Share})}{2\sigma_\varepsilon^2 + M\sigma_F^2(1+\chi^{Share})^2} < \Lambda^{Indep} < \Lambda^{Share} < 0 \tag{24b}$$

- ***c. Convexity is maximized relative to the other trading environments***

$$\Pi^{Collude} = \frac{1}{2} < \Pi^{Indep} = \frac{1+\rho N}{2+\rho N} < \Pi^{InfoShare} = \frac{1+N}{2+N} < 1. \tag{24c}$$

The effects of collusion and information sharing are thus subtle and in opposite directions. Information sharing reduces per-dealer profits; collusion increases profits. Information-sharing discourages proprietary trading; collusion encourages it. Information sharing is associated with lower pre-fix volatility and less pronounced post-fix retracements; collusion is associated with more pre-fix volatility and more pronounced post-fix retracements. Information sharing brings stronger free riding and a pre-fix price path that is less convex; collusion shuts down free riding and brings a more convex price path.

IV. Risk-Aversion and Convexity

Sections II and III show that with risk-neutral dealers this model captures the high pre-fix volatility and post-fix retracements associated with the London 4 pm fix (see Figure 1). There remains, however, a notable feature of those price dynamics that the model with risk-neutral dealers does not capture: outright convexity of the average pre-fix price path. Indeed, the model under risk neutrality consistently implies a non-convex price path unless dealers collude, in which case it implies a linear price path. But the pre-fix trend tends to accelerate in foreign exchange markets. This section shows that when dealers are risk averse the model is consistent with a convex – or equivalently an accelerating – price path.

Following common practice, I assume that dealers have mean-variance utility with risk aversion $\gamma/2$:

$$Max\ E\{\pi_d\} - \frac{\gamma}{2} Var(\pi_d)\,. \tag{25}$$

The analysis once again considers independent dealing, information sharing, and collusion.

A. Independent Trading

Representative dealer d once again analyzes period 2 before period 1. His expected profits, conditional on period-2 information, were already presented in Equation (4), which is repeated here for convenience:

$$E_{2d}\{\pi_d\} = \alpha_1 F_d \theta \left\langle [(1-\alpha_1)F_d + (1-\overline{\alpha}_1)E_{2d}\{\sum_N F_n\}] \right\rangle + (\alpha_1 F_d - X_d)\theta(X_d + E_{2d}\{\sum_N X_n\}) \qquad (26)$$

The conditional variance of dealer d's profits will be determined by the non-fix trade shocks, ε_2 and ε_3, by his forecast errors with respect to the other dealers' fix orders, $\vartheta_d \equiv \sum_N F_n - E_{2d}\{\sum_N F_n\}$, and by his forecast errors with respect to the other dealers' proprietary trading $\mu_d \equiv \sum_N X_n - E_{2d}\{\sum_N X_n\}$. Unexpected profits are:

$$\pi_d - E_{2d}\{\pi_d\} = D_{1d}\theta(1-\overline{\alpha})(\vartheta + \varepsilon_2 + \varepsilon_3) + (D_{1d} - X_d)\theta(\mu - \varepsilon_3) \quad . \qquad (27)$$

The variance of profits, conditional on period-2 information, is thus:

$$\begin{aligned} Var_2(\pi_d) = \theta^2 \left\langle \alpha_{1d}^2 F_d^2 \left[2\sigma_\varepsilon^2 + (1-\overline{\alpha})^2 \sigma_\vartheta^2\right] + (\alpha_{1d}F_d - X_d)^2(\sigma_\varepsilon^2 + \sigma_\mu^2) \right\rangle \\ + \theta^2 \left\langle 2\alpha_{1d}F_d(\alpha_{1d}F_d - X_d)\left[(1-\overline{\alpha})Cov(\mu,\vartheta) - \sigma_\varepsilon^2\right] \right\rangle. \end{aligned} \qquad (28)$$

This depends on the variance of non-fix trading shocks, σ_ε^2 as well as on the variance and covariance of dealer d's prediction errors, σ_μ^2 and σ_ϑ^2, and $Cov(\mu,\vartheta)$, respectively. The prediction-error properties are partially endogenous because they depend on the dealer's proprietary trading. Later it will be shown that these errors, as well as their variances and covariance, depend on the three underlying sources of randomness: ϕ, η, and ε.

To identify utility-maximizing period-2 trading, dealer d applies Equations (26) and (28) to his overall optimization problem, Equation (25). The first-order condition for X_d implies:

$$X_d = \frac{D_1(1+R_d) - E_{2d}\{\sum_{N\neq d} X_n\}}{2+R_x}. \qquad (29)$$

This solution differs from the risk-neutral solution, Equation (5), due to the introduction of two risk terms, $R_x \equiv \gamma\theta(\sigma_\varepsilon^2 + \sigma_\mu^2)$ and $R_d \equiv \gamma\theta[2\sigma_\varepsilon^2 + \sigma_\mu^2 + (1-\overline{\alpha})Cov(\mu, \vartheta)]$. These should be considered endogenous because they depend on α_{1d}. Further progress in the analysis of dealer behavior requires identifying how dealer d estimates the proprietary trading of other dealers. This analysis, which is presented in the Appendix, reveals that dealer d's proprietary trading is again linear in his period-1 trading and the proportionality coefficient, q_d, now depends non-linearly on risk:

$$X_d = \frac{1+R_d}{2+R_x+\rho N}\alpha_{1d}F_d \equiv q_d\alpha_{1d}F_d \quad , \quad q_d = \frac{1+R_d}{2+R_x+\rho N}. \tag{30}$$

With Equation (30) the endogenous risk variables can be expressed in terms of underlying sources of risk: $\sigma_\mu^2 = q^2\alpha_1^2\sigma_\vartheta^2$, $Cov(\mu,\vartheta) = q\alpha_1\sigma_\vartheta^2$, and $\sigma_\vartheta^2 = \frac{N^2\sigma_\phi^2\sigma_\eta^2}{\sigma_\phi^2+\sigma_\eta^2} + N\sigma_\eta^2$.

Having identified the function that determines utility-maximizing period-2 trading, dealer d next identifies the variance of profits from the perspective of period 1:

$$Var_1(\pi_d) = \sigma_\varepsilon^2\left[2 - q(1-q)\right] + \sigma_\vartheta^2[(1-\overline{\alpha})^2 + (1-q)^2q^2\overline{\alpha}^2 + 2(1-\overline{\alpha})(1-q)q\overline{\alpha}]. \tag{31}$$

To identify his period-1 trading, dealer d solves:

$$\begin{aligned} \underset{\alpha_{1d}}{Max} \;\; & \alpha_{1d}F_d\theta[F_d(1-\alpha_{1d}) + \sum_{N\neq d}F_n(1-\overline{\alpha}_{1n})] + \alpha_{1d}F_d(1-q)\theta\left[q_d\alpha_{1d}F_d + \sum_{N\neq d}X_n\}\right] \\ & - \frac{\gamma}{2}(\alpha_{1d}F_d)^2\theta^2 Var_1(\pi_d). \end{aligned} \tag{32}$$

Lemma 4: In competitive market equilibrium (N≥1), risk-averse dealers operating independently will trade the following shares of their initial customer fix order:

a. $\frac{2}{3} < \alpha_1 \equiv \frac{D_{1d}}{F_d} = \frac{(1+\rho N)}{(2+\rho N)[1-q(1-q)]+\gamma\theta V_1(\pi_d)} \leq 1$, (33a)

b. $0 < \alpha_2 = 1-\alpha_1(1-q_d) < 1$, (33b)

c. $x \equiv \frac{X_d}{F_d} = \frac{1+R_d}{2+R_x+\rho N}\alpha_1$. (33c)

This solution is highly non-linear. Period-1 trading as a share of fix orders, α_{1d}, depends on the risk terms R_x and R_d as well as q_d; these, in turn, depend non-linearly on each other as well as α_1. The solution cannot be expressed in closed form and comparative statics are inconclusive, presumably due to the existence of the multiple equilibria associated with higher-order polynomials. Multiple equilibria are not surprising in financial markets (Biais and Green, 2007) and thus we proceed. Simulations for a wide variety of parameters confirm that there exists a solution akin to the solutions identified in closed form for the risk-neutral case. Figure 2 provides examples for a wide variety of parameter combinations.

These solutions display a clear and consistent shift of trading away from the beginning and towards the end of the pre-fix interval, a shift that makes the price path either less concave or outright convex. By definition the path will be outright convex, and returns will accelerate on average between periods 1 and 2, if $\Pi_d^{Indep} < 1/2$. This occurs if risk aversion is high, if the underlying risk variables (σ_η^2, σ_ϕ^2, and σ_ε^2) are high, or if there are many competing fix dealers. The number of dealers influences this property of equilibrium not through the intensity of competition but through the variance of total fix order flow. This variance – and dealer uncertainty – increase with N, which reduces the dealers' willingness to trade in period 1.

B. Information Sharing

When dealers share information about customer fix orders they no longer face any risk associated with predicting the orders of other dealers and the only source of risk is non-fix trading noise, ε. The variance of profits when the dealer makes his period-2 decision is $\theta^2\sigma_\varepsilon^2(X_d^2 + D_{1d}^2)$. Conditional on the first-period trading of the other dealers, the utility-maximizing choice of X_d becomes:

$$X_d = \frac{\sum_{N+1} D_{1n}}{(1+N)(2+N+\gamma\theta\sigma_\varepsilon^2)} \tag{34}$$

When dealer d's makes his period-1 decision, the relevant profit variance is $\theta^2\sigma_\varepsilon^2[D_{1d}^2 + X_d^2]$. The solution does not involve recursive non-linearities and utility-maximizing trading shares can be expressed in closed form.

Lemma 5: In competitive information-sharing equilibrium with risk-averse dealers, a dealer's own fix orders once again drive his trading only in period 2. In periods 1 and 3 every dealer trades the same amount, which depends on the market-wide average fix order:

a. $$D_{1d} \equiv \alpha_1 F_d + \delta_1 \overline{F} = \left(\frac{(1+N)(2+N+\theta\gamma\sigma_\varepsilon^2)}{(2+N+\theta\gamma\sigma_\varepsilon^2)^2 - (1+N)}\right)\overline{F};\ \ \alpha_1 = 0,\ \ \delta_1 > 0, \tag{35a}$$

b. $$D_{2d} \equiv \alpha_2 F_d + \delta_2 \overline{F} = F_d - \left(\frac{(1+N)(1+N+\theta\gamma\sigma_\varepsilon^2)}{(2+N+\theta\gamma\sigma_\varepsilon^2)^2 - (1+N)}\right)\overline{F};\ \alpha_2 = 1,\ \delta_2 < 0 \tag{35b}$$

c. $$X_d \equiv -D_{3d} = \alpha_3 F_d + \delta_3 \overline{F} = \left(\frac{(1+N)}{(2+N+\theta\gamma\sigma_\varepsilon^2)^2 - (1+N)}\right)\overline{F}:\ \alpha_3 = 0, \delta_3 > 0 \tag{35c}$$

$$1 > \delta_1 > |\delta_2| > \delta_3 > 0 \tag{35d}$$

Under information sharing, risk-averse dealers unambiguously trade less in period 1 and more in period 2 than risk-neutral dealers: $\partial D_{1d}/\partial\gamma$ <0, $\partial D_{2d}/\partial\gamma$ >0. The price path is therefore less concave than it would be under risk neutrality. The path will be outright convex, consistent with Figure 1, if $N(3+2N) < 2\gamma\theta\sigma_{\varepsilon}^{2}$. Intuitively, the path will accelerate outright if dealers are highly risk averse, if risk is high, or if competition is low. The extent of competition matters because with less competition there is less free riding.

C. Collusion

When dealers collude the only source of risk remains non-fix trading noise, ε, and the equilibrium trading strategy can once again be expressed in closed form. The variance of profits, from the perspective of period 1, is: $\theta^{2}\sigma_{\varepsilon}^{2}(X^{2}+D_{1}^{2})$.

Lemma 6: Under risk aversion, collusive dealers trade less in period 1 than period 2:

a. $$D_1 = \left(\frac{2+\theta\gamma\sigma_{\varepsilon}^{2}}{(2+\theta\gamma\sigma_{\varepsilon}^{2})^{2}-1}\right)\sum_{N+1} F_n \,, \tag{36a}$$

b. $$D_2 = \frac{(2+\theta\gamma\sigma_{\varepsilon}^{2})(1+\theta\gamma\sigma_{\varepsilon}^{2})}{(2+\theta\gamma\sigma_{\varepsilon}^{2})^{2}-1}\sum_{N+1} F_n \,, \tag{36b}$$

c. $$X = \left(\frac{1}{(2+\theta\gamma\sigma_{\varepsilon}^{2})^{2}-1}\right)\sum_{N+1} F_n \,. \tag{36c}$$

When risk-averse dealers collude the pre-fix price path is unambiguously convex. Proposition 7 summarizes the implications of risk aversion for the pre-fix price path:

Proposition 7: When dealers are risk-averse, trading shifts towards the end of the pre-fix period regardless of whether dealers trade independently, share information, or collude. The price path will be convex under collusive dealing and may be convex otherwise.

V. Conclusion

This paper examines dealer behavior at fixes. Fix prices serve as benchmarks for portfolio valuation in markets including foreign exchange, precious metals, interest rate derivatives, and government bonds. In foreign exchange alone index funds valued at $11 trillion are benchmarked to a single fix price. Funds that wish to avoid tracking risk need to trade at fix prices and must inform their dealers in advance if they wish to trade at the fix on a given day. Because the fix price is set after these instructions are received, dealers can potentially profit from moving the price.

Price behavior around fixing prices is often dramatic, sweeping quickly to extreme levels and then retracing some of those gains after the fix price is set. Many observers suspect manipulative or collusive trading practices. Academics have been circumspect, but they nonetheless note that such behavior does not seem to fit the efficient markets paradigm.

In recent years regulators and judicial authorities around the world have investigated fixes in many markets, and evidence of collusive and manipulative practices has indeed emerged. But it is not clear how such behavior influences price dynamics. The paper examines a simple model of dealer trading at the fix in which dealers can engage in prohibited behaviors including front-running, sharing information about customer orders, and outright collusion.

The model shows that high volatility is inevitable around fixes, given the magnitude of customer orders that typically must to be executed within a short time frame. The model traces quick trend reversals, however, to proprietary trading akin to front-running: that is, dealers trade for their own account in parallel with their customer orders. This proprietary trading also intensifies volatility before the fix.

The model shows that information sharing and collusion affect price dynamics through a form of free riding that involves trading more at the beginning of the fix trading interval and less towards the end. The larger early inventory position can be expected to appreciate as a result of continued fix trading by the other dealers. Under information sharing free riding intensifies relative to independent trading. Average profitability declines and parallel proprietary trading is discouraged. With lower proprietary trading there is lower volatility, post-fix retracements become less pronounced, and the pre-fix price path is less convex. Collusion has the opposite effects because it shuts down free riding. This raises average dealer profits and encourages proprietary trading. Stronger proprietary trading brings more pronounced volatility and post-fix retracements. In the absence of free riding the path of prices prior to the fix is more convex. Collusion and information sharing could therefore potentially be detected through a change in the convexity of the pre-fix price path.

The analysis may prove useful in evaluating alternative approaches to setting benchmark prices. In forex, for example, the time interval over which the London 4pm fix is calculated was extended in 2015 from 60 seconds to five minutes. The model presented here suggests that this shift will not change the nature of the dealers' incentives, though it could affect their strength by increasing risk. Though dealing banks seem to have cracked down severely on collusion, market observers seems to find fix price dynamics to be little changed (Saks-McLeod, 2015).

The model raises questions about the efficacy of other suggested approaches to setting benchmarks which include a clearing auction and peer-to-peer matching. According to the model, price dynamics at the fix are driven by the net value of customer fix orders. This net quantity has zero price elasticity, in effect: customers have instructed their banks to trade that amount regardless of the market price. The ideal fix system will identify sufficient opposite-side liquidity without incentivizing dealers to engage in prohibited practices. The NYSE has attempted to elicit such liquidity by publishing market-in-close imbalances prior to the close (Cushing and Madhavan, 2000). While this could help reduce overall volatility, it does not discourage front-running: to the contrary, the model shows that when dealers know the net order imbalance they still trade for their own account.

The analysis presented in this paper could be extended in a number of potentially useful directions. Dealers are reported to have reasons for influencing fix prices that are not yet encompassed in the model. For example, when dealers trade in other assets, such as options, they might benefit from influencing the fix so as to ensure that a given option matures in-the-money (see, e.g., Slater and Jones, 2014). Future research could expand the model to allow dealers to trade in other assets. An expanded model could also usefully endogenize the extent to which customers place fix orders.

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Appendix: Rational Expectations Equilibrium Under Independent Trading

The appendix identifies how the dealers trading independently form expectations of each others' proprietary trading.

A.1. Risk Neutrality

Dealer d's period-2 expectation of the other dealers' proprietary trading, $E_{2d}\{\sum_{n\neq d} X_n\}$, necessarily depends on his period-2 information set, $\Omega_{2d} \equiv \{F_d, D_{1d}, P_1 - P_0\}$. To identify the functional form of this expectation, assume that it is linear and apply the method of undermined coefficients:

$$E_{2d}\{\sum_{n\neq d} X_n\} = A(P_1 - P_0) + BD_{1d} + CF_d. \tag{A.1}$$

The coefficients A, B, and C will be identified from rationality constraints. The first is that dealers should expect their own proprietary trading, as a share of their net fix order, to be neither more nor less than the unconditional expected value of that share:

$$E_{1d}\{X_d\}/F_d = E_0\left\{\sum_M X_n \Big/ \sum_M F_n\right\}. \tag{A.2}$$

This implies the following equality which can only be satisfied if A = 0:

$$\frac{E_{1d}\{X_d\}}{F_d} = \frac{\alpha_1}{2}\left[1 - B - \frac{A}{\theta}(1+\rho N)\right] = \frac{\alpha_1}{2}\left[1 - B - \frac{A}{\theta}(1+N)\right] = E_0\left\{\frac{\sum_M X_n}{\sum_M F_n}\right\}. \tag{A.3}$$

A second rationality constraint is that dealers should not make predictable forecast errors, or:

$E_{1d}\{\Psi_d\} \equiv E_{1d}\left\{\sum_{n\neq d} X_n - E_{2m}\left\{\sum_{n\neq d} X_n\right\}\right\} = 0.$ With A = 0, this implies:

$$E_{1d}\{\Psi_d\} = E_{1d}\left\{\left(\frac{1-B}{2}\right)\sum_N D_{1n} - BD_{1d} - \frac{C}{2}(\sum_N F_n + 2F_d)\right\} = 0. \tag{A.4}$$

This can be solved for B and C by considering (a) the model's symmetry, which implies that $\alpha_{1n} = \alpha_{1m}$ for all n, and (b) the structure of fix orders, which implies $E_{1d}\{F_n\} = \rho F_c$ where $\rho \equiv \sigma_\phi^2 / (\sigma_\phi^2 + \sigma_\eta^2)$. Equation (A.4) becomes $E_{1d}\{\Psi_d\} = 0 = \alpha_1[B(2+\rho N) - \rho N] - C(2+\rho N)$ or

$$C = \alpha_1\left(\frac{\rho N}{(2+\rho N)} - B\right). \tag{A.5}$$

Applying this to Equation (A.1) reveals that $E_{2m}\left\{\sum_{n\neq d} X_n\right\}$ depends only on D_{1d}:

$$E_{2m}\left\{\sum_{n\neq d} X_n\right\} = \frac{\rho N}{2+\rho N} D_{1d} \quad . \tag{A.6}$$

Thus $B = \rho N/(2+\rho N)$ and C =0. In combination with Equation (5), this implies:

$$X_d = \frac{1}{2+\rho N} D_{1d} \equiv q D_{1d}. \tag{A.7}$$

A.2 Risk aversion

We again assume agnostically that $E_{2d}\{\sum_{n\neq d} X_n\}$ is linear in the dealer's information: $E_{2d}\{\sum_{n\neq d} X_n\} = A(P_1 - P_0) + BD_{1d}$ (this excludes F_d based on the results of Section II).[5] We once again infer that A = 0 from the rational expectation constraint that a dealer expects his overtrading, as a share of his fix orders, to equal the unconditional average share of overtrading. B can once again be identified from the rational expectation constraint that the dealer's period-2 expectation error should have expected value of zero conditional on period-1 information: $B = \rho N(1+R_d)/(2+R_x+\rho N)$.

[5] The irrelevance of F_d is confirmed in unreported analysis.

Figure 1: Exchange Rate Dynamics Around the London 4 pm Fix

Mean price path from 60 minutes before to 60 minutes after the London 4 pm fix using tick-by-tick data for EURUSD, USDCHF, USDJPY, and GBPUSD over 2003-2014. The level at 3:45 pm is taken to be 0 on the vertical axis; price changes are measured in basis points. Prices that rise and prices that decline over 3:45-4:00 are taken separately. Blue lines represent the average for intramonth days; Black lines represent the average for end-month days; red lines represent the largest (in absolute magnitude) 25th-percentile return on end-month days.

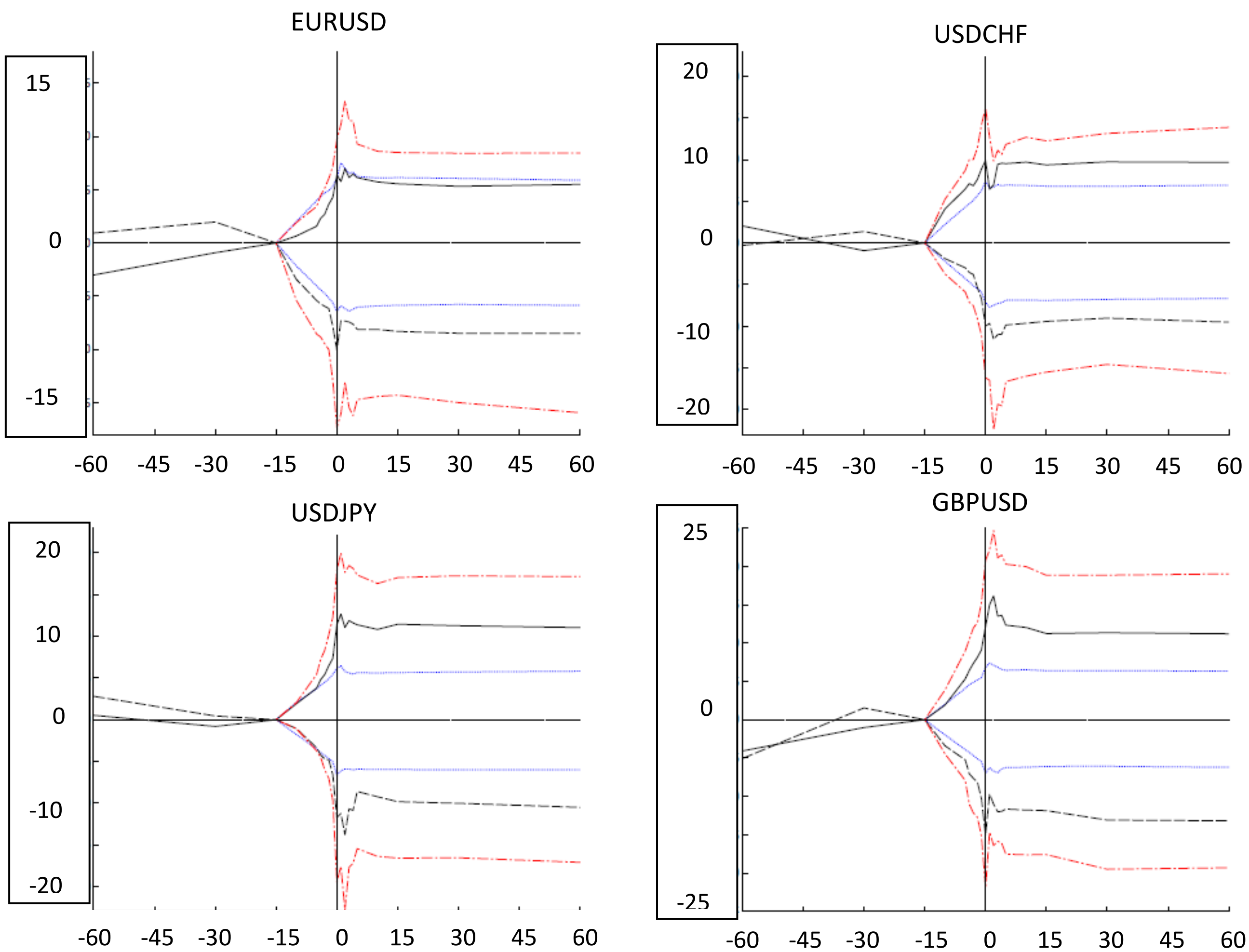

Figure 2: Simulated equilibria when risk- neutral and risk-averse dealers trade independently. For high-risk simulations $\sigma_\eta^2 = 1$; for low-risk simulations $\sigma_\eta^2 = 0.1$; For high-correlation simulations $\sigma_\phi^2 = \sigma_\eta^2$. For low-correlation simulations, $\sigma_\phi^2 = \sigma_\eta^2 / 2$. The risk of non-fix dealing is constant across simulations at $\sigma_\varepsilon^2 = 1$.