Forecasting stock returns: A predictor-constrained approach

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Abstract

We develop a novel method to impose constraints on univariate predictive regressions of stock returns. Unlike the previous approaches in the literature, we implement our constraints directly on the predictor, setting it to zero whenever its value falls below the variable's past 12-month high. Empirically, we find that relative to standard unconstrained predictive regressions, our approach leads to significantly larger forecasting gains, both in statistical and economic terms. We also show how a simple equal-weighted combination of the constrained forecasts leads to further improvements in forecast accuracy, with predictions that are more precise than those obtained either using the Campbell and Thompson (2008) or Pettenuzzo, Timmermann, and Valkanov (2014) methods. Subsample analysis and a large battery of robustness checks confirm that these findings are robust to the presence of model instabilities and structural breaks.

Keywords: Equity premium; Predictive regressions; Predictor constraints; 12-month high; Model combinations.

JEL Classifications: C11; C22; G11; G12

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1 Introduction

Stock return forecasts affect many areas in finance ranging from asset pricing to portfolio allocation and risk management. Not surprisingly, over the years, the quest for accurate and reliable return forecasts has attracted significant interest by both practitioners and academics. A large number of earlier studies have shown convincing evidence of in-sample predictability (Campbell and Shiller, 1988; Fama and French, 1988, 1989; Ferson and Harvey, 1993; Keim and Stambaugh, 1986; Lettau and Ludvigson, 2001; Pontiff and Schall, 1998). However, as noted by Welch and Goyal (2008), it remains difficult to find models that can improve on even the most naive benchmarks out-of-sample.¹

Recently, it has been shown that constraining in some ways the parameters or the forecasts implied by otherwise standard predictive regressions can lead to sharper predictions of aggregate returns. For example, Pástor and Stambaugh (2009, 2012) propose a Bayesian approach with priors that are set in a way to ensure that shocks to unexpected and expected returns are negatively correlated. Campbell and Thompson (2008) (CT hereafter) show that either constraining the return forecasts to be non-negative or forcing the regression coefficients to have the theoretically expected signs leads to clear gains in out-of-sample predictability. Along the same lines, Pettenuzzo, Timmermann, and Valkanov (2014) (PTV hereafter) propose a Bayesian approach to impose non-negative equity premia and bounds on the conditional Sharpe ratio of univariate predictive regressions, and find that their approach leads to very accurate forecasts.²

We build on the above-mentioned work and propose a novel approach to imposing constraints on univariate predictive regressions of stock returns. Unlike the previous

¹In this paper, we focus on the standard and widely used predictors suggested by Welch and Goyal (2008) because their data span a time period long enough for us to reliably test our method. There is also a branch of studies that has uncovered new predictive variables for aggregate stock returns, including short interest (Rapach, Ringgenberg, and Zhou, 2016), variance risk premia (Bollerslev, Tauchen, and Zhou, 2009), oil price changes (Driesprong, Jacobsen, and Maat, 2008), technical indicators (Neely, Rapach, Tu, and Zhou, 2014), news implied volatility (Manela and Moreira, 2017), economic policy uncertainty (Brogaard and Detzel, 2015), aligned investor sentiment (Huang, Jiang, Tu, and Zhou, 2014), and skewness in expected macro fundamentals (Colacito, Ghysels, Meng, and Siwasarit, 2016). These predictors tend to span a much shorter time period, and for this reason we do not focus on them in our study. In addition, we restrict our attention to U.S. stock returns and leave the question of international stock return predictability (see, e.g., Jordan, Vivian, and Wohar, 2014; Lawrenz and Zorn, 2017) to future research.

²Chib and Zeng (2016) extend the PTV approach to a multivariate regression using a large set of return predictors, and find similar gains in forecast accuracy.
methods in the literature, we implement our restrictions directly on the predictor, setting it to zero whenever its value falls below the predictor’s rolling 12-month high. Our main motivation for this approach comes from the idea that investors are likely to pay extra attention to stock return fundamentals and macroeconomic indicators when these undergo extreme changes, and react to the arrival of such information. In this, we draw heavily from the recent work on limited attention in behavioral finance, in which a growing body of evidence has uncovered the fact that investors’ attention changes over time in a way that affects stock returns (Da, Engelberg, and Gao, 2011; Barber and Odean, 2008; Barber, Odean, and Zhu, 2009; DellaVigna and Pollet, 2009). For example, Huberman and Regev (2001) show that stock prices respond to new information only when investors pay attention to it. Along the same lines, Barber and Odean (2008) report that investors have limited attention, and only pay attention to specific categories of stocks and/or extreme returns. Similarly, George and Hwang (2004) suggest that traders use the stock’s 52-week high as an anchor in evaluating whether a recent increase in stock price implies new information. Li and Yu (2012) propose using the proximity of a stock price to its 52-week high as a measure of investor under-reaction, and find that their indicator has predictive power for stock returns.3

A second motivation for our approach comes directly from the notion that asset prices change in response to unexpected, rather than expected, fundamental information (see, e.g., Ball and Brown, 1968; Aharony and Swary, 1980). For example, Dew-Becker, Giglio, Le, and Rodriguez (2017) find that only the unexpected and transitory stock return variance is priced, and investors are not willing to pay to hedge risks due to expected economic uncertainty. In the context of stock return predictability, it is well known that most of the popular predictors (e.g., the dividend/price ratio or interest rates) display strong persistent behavior (see, e.g. Stambaugh, 1999), which implies that a large fraction of the future variance of these predictors is dominated by their persistent time series behaviors, rather than their unexpected shocks. It follows that identifying times when unexpected information dominates the changes in the predictors can offer a potential new way to improve the forecasting performance of predictive

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3In the context of oil prices, Hamilton (1996, 2011) finds that the effects of oil price changes on U.S. GDP are significant only when the current oil price exceeds the nearby past summit.
regressions.

To evaluate our approach empirically, we use a total of 11 well-known monthly predictors of stock returns, covering a time period spanning January 1929 to December 2015. We use the first 20 years of data as a warm-up period to train the various models, and generate recursive forecasts of Standard and Poor (S&P) 500 excess returns starting in January 1949 and continuing through December 2015. Overall, we find that imposing our novel constraint on the predictors improves the statistical accuracy of the univariate predictive models, as measured by the Out-of-Sample $R^2$ ($R^2_{OoS}$), for seven of the 11 predictors, with particularly large gains occurring for the Treasury bill rate ($R^2_{OoS}$ goes from -1.142% to 0.371%), long-term government bond yield ($R^2_{OoS}$ goes from -1.470% to 1.047%), and stock variance ($R^2_{OoS}$ goes from -0.005% to 2.117%). In all three cases, the return forecasts from the constrained regression models are significantly more accurate than the benchmark no-predictability forecasts, as indicated by the Clark and West (2007) test. We also investigate the economic value of using the returns forecasts generated using our constrained regressions to make portfolio decisions. We consider an investor with constant relative risk aversion (CRRA) utility who allocates his/her wealth between the S&P 500 and the short-term Treasury bill, and evaluate the portfolio performance using certainty equivalent returns (CERs). Our results show that imposing our constraint on the predictors leads to large CER gains for all 11 predictors.

The previous results refer to univariate regression models with a single predictor variable. Although our constrained approach is quite successful in the univariate setting, it does not offer a way to deal with the issue of model uncertainty, which is quite pervasive in the context of stock return predictability. In addition, it is very well known that predictive regressions of the sort considered here are plagued by model instability, and that the predictive accuracy of these regressions fluctuates over time. For example, (Pettenuzzo, Timmermann, and Valkanov, 2014, Table 2) show that their univariate constrained models outperform the naive no-predictability benchmark in a statistically significant way for 8 of

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4Our main results rely on a look-back period of 12 months when constraining the predictors. As a robustness check, we experiment with alternative look-back periods of 6, 18, and 24 months. For comparability across the specifications, we omit the first 24 months of data (January 1927–December 1928), and as a result, the sample period starts in January 1929 for all our analyses.
their 16 predictors in the full sample, but the evidence in favor of predictability weakens significantly in the second half of their sample. To tackle these issues, we extend our analysis to a multivariate setting by considering equal-weighted combinations of univariate constrained predictive regressions. We separately generate equal-weighted combinations of either unconstrained (EW-U) or constrained (EW-C) models, and find that the combination of constrained regressions yields an $R^2_{OoS}$ of 0.957%, three times as large as the $R^2_{OoS}$ obtained using an equal-weighted combination of unconstrained forecasts. Most notably, the gains in forecasting performance of the EW-C combination are highly robust to the subsample considered. After splitting the entire sample into two halves, we find that in the first subsample (1949-1981) the EW-C and EW-U model combinations perform very similarly, with $R^2_{OoS}$ of 0.984% and 0.939% respectively. However, in the second subsample the EW-C model combination performs significantly better than its unconstrained counterpart, with an $R^2_{OoS}$ of 0.936%. In contrast, the $R^2_{OoS}$ of the EW-U model combination in the second half of the sample reaches a mere -0.127%. In terms of economic predictability, over the entire sample the CER gains of the equal-weighted combination methods improve from -6.4 basis points to 83.1 basis points after implementing our constraints. With respect to subsample performance, we find that in the first subsample, the EW-C leads to portfolios with a CER that is 48.2 bps higher than its unconstrained counterpart. As with the statistical predictability results, the superior performance of the EW-C is more impressive in the second subsample, with a CER gain of 92.1 basis points, whereas EW-U leads to a negative CER gain of -37.4 basis points. We also compare the forecasting performance of our EW-C approach against the constrained approaches of Campbell and Thompson (2008) and Pettenuzzo, Timmermann, and Valkanov (2014). Our results indicate that our EW-C model combination performs better than the equal-weighted combinations of either CT or PTV constrained univariate regressions, with gains that are particularly large in the second subsample.

The remainder of this paper is organized as follows. Section 2 introduces the econometric methodology we rely on to estimate and forecast the equity premium subject to our constraint

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5 As pointed out by Baumeister and Kilian (2015), forecast combination methods also provide insurance against possible model misspecification.

6 In particular, for the PTV approach we focus on their Sharpe-ratio constrained model.
on the predictor variable. Section 3 describes the data, and Section 4 presents results on both the statistical and economic predictability of our approach. Section 5 reports the results of a number of extensions and robustness checks, and Section 6 provides some concluding remarks.

2 Methodology

In this section, we introduce our predictor-constrained approach and show how to estimate and forecast the equity premium subject to this constraint.

2.1 Predictive regression

As is customary in the literature on stock return predictability, we model $r_{\tau+1}$, the stock returns in excess of the risk free rate at time $\tau + 1$, as a linear function of a lagged predictor variable $x_\tau$:

$$r_{\tau+1} = \alpha + \beta x_\tau + \epsilon_{\tau+1}, \quad \tau = 1, \ldots, t - 1$$

$$\epsilon_{\tau+1} \sim N\left(0, \sigma^2_\epsilon\right) \tag{1}$$

We consider a sample of $T$ observations and use the first $M$ observations in the sample as a training period. Next, we forecast the excess stock returns over the remaining $P$ observations, where $P = T - M$. Out-of-sample forecasts are easily obtained as

$$\hat{r}_{t+1|t} = \hat{\alpha}_t + \hat{\beta}_t x_t \quad \tag{2}$$

where $\hat{\alpha}_t$ and $\hat{\beta}_t$ are the ordinary least squares (OLS) estimates of $\alpha$ and $\beta$ from (1), obtained by regressing $\{r_{\tau+1}\}_{\tau=1}^{t-1}$ on a constant and $\{x_{\tau}\}_{\tau=1}^{t-1}$. Repeating this procedure for $t = M, \ldots, T - 1$ yields a time series of $P$ one-step-ahead (unconstrained) forecasts, which we denote with $\{\hat{r}_{t+1|t}\}_{t=M}^{T-1}$.

2.2 Constrained predictive regression

Following Hamilton (1996, 2011), we introduce a simple truncation of the predictor variable $x$ and define the following non-linear transformation of the original variable as follows:

$$x^*_\tau(n) = \begin{cases} x_\tau & \text{if } x_\tau > \max(x_{\tau-1}, x_{\tau-2}, \cdots, x_{\tau-n}) \\ 0 & \text{otherwise} \end{cases} \quad \tag{3}$$

where $\tau = n + 1, \ldots, T$ and $n$ is the “look-back” period. The main idea behind this simple truncation approach is to allow for a non-linear response of stock returns to changes in the
predictor variable, with the response being potentially more (or less) pronounced in those instances when the predictor exceeds its past \( n \)-period high. Next, a constrained out-of-sample forecast can be easily generated as follows,

\[
\hat{r}_{t+1|t}(n) = \hat{\alpha}_t(n) + \hat{\beta}_t(n)x_t(n)
\]

where \( \hat{\alpha}_t(n) \) and \( \hat{\beta}_t(n) \) are the ordinary least squares (OLS) estimates of \( \alpha \) and \( \beta \) obtained by regressing \( \{r_{\tau+1}\}_{\tau=n+1}^{T-1} \) on a constant and \( \{x_{\tau}(n)\}_{\tau=n+1}^{T-1} \). Repeating this procedure for \( t = M, \ldots, T - 1 \) yields a time series of \( P \) one-step-ahead (constrained) forecasts, which we denote with \( \{\hat{r}_{t+1|t}(n)\}_{t=M}^{T-1} \).

It is important to note that the newly defined predictor variable \( x_t(n) \) and, as a by-product, the constrained forecast \( \hat{r}_{t+1|t}(n) \) depend on the number of periods used to compute the recent past high. In particular, a larger \( n \) leads to more zeros in the time series of the constrained predictor, \( \{x_{\tau}(n)\}_{\tau=n+1}^{T-1} \). This is because as \( n \) increases, the value of the unconstrained predictor \( x_{\tau} \) (\( \tau = n + 1, \ldots, T - 1 \)) is compared to a longer sequence of past observations. Recent studies on limited investor attention and psychological anchors show that investors are often relying on a one-year time span when computing past highs (or lows) and assessing increments in stock values (George and Hwang, 2004; Li and Yu, 2012). Accordingly, our main results are based on a one-year look-back period (i.e., \( n = 12 \) with monthly data). As a robustness check, we also experimented with alternative look-back periods, and include the results of this sensitivity analysis in Section 5.

### 2.3 Forecast combinations

So far, we have followed much of the finance literature on return predictability and focused on (unconstrained and constrained) univariate prediction models. However, the prevailing view (Campbell and Thompson, 2008; Rapach, Strauss, and Zhou, 2010; Pettenuzzo, Timmermann, and Valkanov, 2014) is that univariate predictive regressions tend to generate less accurate and less stable forecasts than even the simplest benchmark models. In this section, we extend the previous analysis to a multivariate setting, in which instead of conditioning on a single predictor variable we exploit the information contained in \( N \) predictors. In particular, we combine the predictive information contained in the different
variables by using forecast combination methods (see for example Rapach et al., 2010; Dangl and Halling, 2012) that pool together the individual forecasts from the $N$ univariate models. Our unconstrained forecast combination is given by:

$$
\hat{r}_{t+1|t}^{\text{comb}} = \sum_{i=1}^{N} w_{i,t} \times \hat{r}_{i,t+1|t},
$$

where $\hat{r}_{i,t+1|t}$ denotes the unconstrained return forecast in (1) generated using predictor $i$ at time $t$, $t = M, ..., T - 1$, and $N$ is the total number of predictors. Our constrained forecast combination is given by:

$$
\hat{r}_{t+1|t}^{\star,\text{comb}}(n) = \sum_{i=1}^{N} w_{i,t}^{*}(n) \times \hat{r}_{i,t+1|t}^{*}(n),
$$

where $\hat{r}_{i,t+1|t}^{*}(n)$ denotes the constrained return forecast in (4) generated using predictor $i$ at time $t$, $t = M, ..., T - 1$, and $n$ is the length of the look-back period.

The choice of weights in pooling together the individual univariate forecasts (i.e, $w_{i,t}$ and $w_{i,t}^{*}(n)$) is of key importance. A popular approach is to use the equal-weighted average of forecasts (i.e., $w_{i,t} = w_{i,t}^{*}(n) = 1/N$). Recent empirical studies (Rapach, Strauss, and Zhou, 2010; Claeskens, Magnus, Vasnev, and Wang, 2016) have shown that in the context of return predictability, the equal-weighted combination method is a very competitive model, and often outperforms more sophisticated combination schemes. Accordingly, we rely on this simple combination method in our analysis. As a robustness check, in Section 5 we consider the sensitivity of our results to the use of more sophisticated model combination schemes.

3 Data

Our empirical analysis uses data on stock returns along with a set of 11 predictor variables originally analyzed by Welch and Goyal (2008) and subsequently extended to 2015 by the same authors.7 Stock returns are computed from the S&P 500 index and include dividends. A short T-bill rate is subtracted from stock returns to capture excess returns. The data on

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7A number of recent papers investigating predictability in stock returns and stock return volatility also use the same data. See Dangl and Halling, 2012; Neely, Rapach, Tu, and Zhou, 2014; Rapach, Strauss, and Zhou, 2010 for examples of the former, and Christiansen, Schmeling, and Schrimpf, 2012 as an example of the latter.
these series are readily available on Amit Goyal’s website.\textsuperscript{8} Below we provide a list of the predictors we relied on in our analysis, along with a short description:\textsuperscript{9}

- Dividend-price ratio (DP): the log of dividends on the S&P 500 index minus the log of stock prices;
- Dividend yield (DY): the log of dividends on the S&P 500 index minus the log of lagged stock prices;
- Earning-price ratio (EP): the log of earnings on the S&P 500 index minus the log of stock prices;
- Stock variance (SVAR): the sum of squared daily returns on the S&P 500 index;
- Book-to-market ratio (BM): the ratio between book value at the end of the previous year and the end-of-month market value of the Dow-Jones Industrial Average (DJIA) index;
- Net equity expansion (NITS): the ratio between the twelve-month moving sum of net equity issues by NYSE-listed stocks and the total end-of-year market capitalization of NYSE stocks;
- Treasury bill rate (TBL): the 3-month US Treasury bill rate from the secondary market rate;
- Long-term yield (LTY): the long-term government bond yield;
- Default yield spread (DFY): the difference between Moody’s corporate BAA- and AAA-rated bond yields;
- Default return spread (DFR): the difference between the returns on long-term corporate and long-term government bonds;

\textsuperscript{8}We thank Amit Goyal for making his data available at http://www.hec.unil.ch/agoyal/.
\textsuperscript{9}Compared to the original work of Welch and Goyal (2008), we exclude the following predictors from our analysis: (i) Dividend-Payout ratio, because it is obtained as the difference between DP and EP; (ii) Term Spread, because it is obtained as the difference between LTY and TBL; and (iii) Long-Term bond return, because it is not highly persistent.
- Inflation (INFL): the change in the Consumer Price Index (CPI) for all urban consumers.\footnote{To account for the delayed release of the CPI index, we lag the inflation one extra month.}

Our sample spans from January 1927 through December 2015, for a total of 1,068 monthly observations. The summary statistics for both predictors and stock returns are provided in Table 1. As can be easily seen by inspecting the Ljung-Box $Q$ statistics $Q(1)$ and $Q(10)$ at the bottom of the table, all the predictors we focus on are highly persistent.

Table 1 about here

4 Equity premia forecasts

We now turn our attention to the out-of-sample performance of the constrained and unconstrained predictive regressions described in Section 2. As previously mentioned, we use a look-back period ($n$) of 12 months to implement the constraints on the predictor variables in (3). We also experiment with a look-back period of 6, 18, and 24 months, and separately reports the results of this sensitivity analysis in section Section 5. We initially estimate our unconstrained and constrained regression models over the period February 1929–December 1948, and use the estimated coefficients to forecast excess returns for January 1949.\footnote{To keep our main results consistent with the robustness analysis presented later in Section 5, we exclude the first 24 months of data from the estimation sample, and begin estimating all our models in 1929.} We next include January 1949 in the estimation sample, giving a sample of February 1929–January 1949, and use the corresponding estimates to predict excess returns for February 1949. We proceed in this recursive fashion until we reach the last observation in the sample, producing a time series of unconstrained and constrained one-step-ahead forecasts spanning the period from January 1949 to December 2015.

Figure 1 plots the time evolution of the unconstrained and constrained equity premia forecasts over the full forecast evaluation period, 1949–2015. We also include in the figure the forecasts obtained using a no-predictability benchmark, $\bar{r}_{t+1|t} = (1/ (t - n)) \sum_{\tau=n+1}^{t} r_{\tau}$. In the interest of space we focus our discussion on three of the predictor variables, namely DP, TBL, and LTY. In all three cases, the forecasts display a marked switching pattern, alternating between a stable level slightly higher than the benchmark forecasts and a level close to the
average unconstrained forecasts. In particular, we observe that when the predictive variables exceed their past 12-month high, our constrained forecasts switch towards the unconstrained forecasts, while in all other instances they hover around the benchmark forecasts. Similarly, Figure 2 displays the time series behavior of the unconstrained and constrained forecast combinations introduced in (5) and (6), along with the benchmark forecasts. Not surprisingly, the EW-C forecasts exhibit a similar switching pattern, alternating between the forecasts obtained using the benchmark and the unconstrained equal-weighted combination model.

**Figure 1 to Figure 2 about here**

### 4.1 Statistical Predictability

We now turn to evaluating the predictive accuracy of the constrained and unconstrained predictive regressions. As is customary in the literature, the predictive performance of each model is evaluated against the no-predictability forecasts, \( \{ \tilde{r}_{t+1|t} \}_{t=M}^{T-1} \). We rely on the out-of-sample \( R^2 (R^2_{OoS}) \), which is defined as the percent reduction in the mean squared predictive error (MSPE) of the model of interest (\( MSPE_{model} \)), relative to the MSPE of the benchmark model (\( MSPE_{bench} \)), i.e.

\[
R^2_{OoS} = 1 - \frac{MSPE_{model}}{MSPE_{bench}}
\]  

(7)

where \( MSPE_{model} = \frac{1}{T-M} \sum_{\tau=M+1}^{T}(r_{\tau} - forc_{\tau|\tau-1})^2 \) with \( forc_{\tau|\tau-1} \) denotes the equity premia forecast produced at time \( \tau - 1 \) by either the unconstrained or constrained univariate model (or model combination), and \( MSPE_{bench} = \frac{1}{T-M} \sum_{\tau=M+1}^{T}(r_{\tau} - \tilde{r}_{\tau|\tau-1})^2 \).

A positive \( R^2_{OoS} \) implies that the model of interest generates more accurate forecasts than the benchmark model. We also evaluate the statistical significance of the changes in \( R^2_{OoS} \) using the Clark and West (2007) (CW hereafter) test statistic, which allows us to test the null hypothesis that the benchmark forecast MSPE is less than or equal to the competing forecast MSPE against the one-sided (upper-tail) alternative hypothesis that the benchmark forecast MSPE is greater than the competing forecast MSPE. This statistic is a correction of the Diebold and Mariano (1995) statistic and is demonstrated to be more suitable for nested models.\(^{12}\)

\(^{12}\)As discussed in Diebold (2015), the p-values resulting from such test statistics should be interpreted as a measure of the relative accuracy of the sequence of forecasts.
4.1.1 Results

The second and third columns of Table 2 report the $R^2_{OoS}$ of the constrained and unconstrained univariate predictive regressions, as well as their equal-weighted combinations. Starting with the forecasting performance of the univariate models, we notice that constraining the predictor variables as in (3) leads to $R^2_{OoS}$ improvements in seven out of 11 cases. In particular, in three of the seven cases (TBL, LTY, and SVAR), the $R^2_{OoS}$ changes from negative to positive and is statistically significant, highlighting the predictive power of our approach. We also find that the unconstrained equal-weighted forecast combination (EW-U) in (5) generates a positive $R^2_{OoS}$ of 0.342%, statistically significant at the 5% level. This result is consistent with Rapach, Strauss, and Zhou (2010) and Avramov (2002). In contrast, the constrained equal-weighted forecast combination (EW-C) in (6) yields an $R^2_{OoS}$ value of 0.957%, three times as large as the EW-U combination method, and statistically significant at the 1% level.

The last four columns of Table 2 report the forecasting performance of the unconstrained and constrained predictive regressions in two subsample periods, 1949–1981 and 1982–2015, obtained by splitting the full out-of-sample period into two halves. In the first subsample, our constrained method leads to an improved $R^2_{OoS}$ for seven of the 11 variables. The $R^2_{OoS}$ of the equal-weighted combination method increases from 0.939% to 0.984%. In the second subsample, the constrained approach improves the forecast accuracy for eight of the 11 variables. Interestingly, the $R^2_{OoS}$ of the EW-U combination method is now negative, implying that in this subsample the EW-U method fails to beat the historical average benchmark. Most notably, during the same period the EW-C combination approach yields an $R^2_{OoS}$ of 0.936%, very close to the forecast gains achieved by the same approach over the full out-of-sample period and the first subsample.

To further demonstrate the robustness of the constrained EW-C forecasts over time, we compute the Cumulative Sum of Squared prediction Error Difference (CSSED)

$$CSSED_{model,t} = \sum_{\tau=M+1}^{t} (e^2_{bench,\tau} - e^2_{model,\tau}) \quad t = M + 1, ..., T$$

(8)
where \textit{model} denotes either the unconstrained EW-U or constrained EW-C model combination, while \(e_{\text{model},\tau}\) and \(e_{\text{bench},\tau}\) denote the forecast error from time \(\tau\) forecast associated with either the EW-C, EW-U, or benchmark model. Note that an increase from \(CSSED_{\text{model},t-1}\) to \(CSSED_{\text{model},t}\) indicates that relative to the benchmark no-predictability model, the EW-U or EW-C model predicts more accurately at observation \(t\). Figure 3 plots the CSSEDs over time for both the EW-U and EW-C models. As can be seen from the figure, the relative predictive ability of the EW-U weakens considerably during the 1957–1969 and 1990–2000 periods. In sharp contrast, the EW-C displays a generally increasing pattern in the CSSEDs, a strong indication that the gains in forecast accuracy of the EW-C model reported in Table 2 are not the result of any specific and short-lived episode, but rather are built gradually over the entire out-of-sample period.

Figure 3 about here

4.2 Economic predictability

So far we have focused on the statistical performance of the unconstrained and constrained equity premia forecasts. We now turn to evaluating the economic significance of these predictions by considering the optimal portfolio decision of an investor who uses the return forecasts to guide her portfolio choices. We focus on an investor who allocates her wealth between a risky asset, which we proxy using the S&P 500 index, and a risk-free asset. The investor’s wealth at time \(t+1\) is,

\[
W_{t+1} = (1 - \omega_t) \exp(r_t^f) + \omega_t \exp(r_t^f + r_{t+1})
\]

where \(\omega_t\) is the share of wealth assigned to the risky asset, and \(r_{t+1}\) is the stock return in excess of the risk-free rate \(r_t^f\), both continuously compounded.

At time \(t\), the investor solves the following optimal asset allocation problem:

\[
\omega_t^* = \arg \max_{\omega_t} E_t[U(W_{t+1})]
\]

where \(E_t[\cdot]\) denotes the conditional expectation based on the investor’s information set at time \(t\). We follow Pettenuzzo, Timmermann, and Valkanov (2014) and focus on constant relative
risk aversion (CRRA) preferences,
\[ U(W_{t+1}) = \frac{W_{t+1}^{1-\gamma}}{1-\gamma}, \]  
(11)
where \( \gamma \) indicates the coefficient of risk aversion. We next rely on the Campbell and Viceira (2001) log-linearization, and obtain approximate optimal portfolio weights that are equal to
\[ \omega_t^* \approx \frac{forc_{t+1|t} + \hat{\sigma}_{t+1|t}^2/2}{\gamma \hat{\sigma}_{t+1|t}^2} \]  
(12)
where \( \hat{\sigma}_{t+1|t}^2 \) denotes the time \( t+1 \) variance forecast for the log excess returns made at time \( t \), and \( forc_{t+1|t} \) stands for the equity premia forecast produced at time \( t \) by either the unconstrained or constrained univariate model (or model combination).\(^{13}\) We restrict the optimal allocation to stocks between 0 and 1.5 to preclude short selling and excessive leverage, and set the risk aversion coefficient to \( \gamma = 3 \).\(^{14}\) Finally, we compute the certainty equivalent return (CER) as:
\[ CER = \left[ (1-\gamma)(T-M)^{-1} \sum_{t=M+1}^{T} U(W_t^*) \right]^{1/(1-\gamma)} - 1, \]  
(13)
where \( (T-M)^{-1} \sum_{t=M+1}^{T} U(W_t^*) \) denotes the average realized utility.

4.2.1 Results

Table 3 reports the differences (in percentage points) between the annualized CER of an investor who relies on the forecasts obtained using the model of interest (i.e., either the unconstrained or constrained univariate predictive regression or model combination) and the annualized CER of an investor who uses the benchmark historical average forecasts. We can interpret these values as the annualized performance fee that an investor would be willing to pay to have access to the return forecasts generated using the model of interest instead of

\(^{13}\)For all models considered, we set \( \hat{\sigma}_{t+1|t}^2 \) equal to the recursively estimated OLS residual variance associated with the model used to generate \( forc_{t+1|t} \). We also experimented with alternative measures and, as in Campbell and Thompson (2008), considered setting \( \hat{\sigma}_{t+1|t}^2 \) equal to the unconditional sample variance, computed using a five-year rolling window of historical log excess returns, i.e. \( \hat{\sigma}_{t+1|t}^2 = (1/59) \sum_{\tau=t-59}^{t} (r_{\tau} - \bar{r}_{t-59})^2 \), with \( \bar{r}_{t-59} = (1/60) \sum_{\tau=t-59}^{t} r_{\tau} \). The results of this alternative approach are qualitatively very similar to those reported here.

\(^{14}\)Our findings are robust to the choice of alternative bounds on the portfolio weights and risk aversion coefficients. We experimented with setting the bounds to \([0, 0.99]\), as suggested by Cenesizoglu and Timmermann (2012), and found the results to be qualitatively very similar to those reported here. We also considered different risk aversion coefficients, and summarize the results of this sensitivity in Section 5.
the benchmark forecasts. The numbers reported in the second and third columns of Table 3 denote the annualized CER gains based on the full evaluation period, 1949–2015. Starting with the individual models, we see that our constrained approach increases the CERs for all 11 predictors. The CER gains associated with eight of the constrained univariate regressions are all positive, supporting the existence of economic predictability. Moving on to the equal-weighted combination, we find that averaging over the univariate unconstrained and constrained regressions yields annualized CER gains of -6.4 bps in the case of the EW-U model, and 83.1 bps in the case of the EW-C model. Hence, as was the case with the univariate regressions, imposing our constraints in the forecast combination leads to sizable gains.

The remaining columns of Table 3 show the annualized CER gains separately for the two subsamples, i.e. 1949–1981 and 1982–2015. In the first subsample, imposing our constraints in the univariate predictive regressions leads to an increase in CERs for nine of the 11 predictors. With respect to the model combinations, the annualized CERs go from 25.5 basis points (bps) in the case of the EW-U method to 73.7 bps in the case of the EW-C approach. That is, incorporating our constraints in the equal-weighted model combination generates a CER gain of 48.2 bps. In the second subsample, applying the constraints to the univariate predictive regressions produces improvements in CERs for 9 of the 11 predictors. Notably, in the second subsample the EW-U model combination fails to improve over the no-predictability benchmark, as indicated by its negative CER figure. In contrast, the CER of the EW-C combination scheme remains positive, with a value of 93 basis points, which is higher than what the same model attained in the first subsample. Overall, these results provide strong evidence in support of our constrained approach, and particularly so for the constrained forecast combination method.

4.3 A comparison with existing constrained approaches

The results we have presented thus far show that the simple constraint we introduced in Section 2 helps boost the forecasting performance of both the univariate predictive regressions and the equal-weighted model combination. We now compare the performance of our method
to some of the most popular constrained methods that have been proposed in the literature. We focus on the approaches of Campbell and Thompson (2008) and Pettenuzzo, Timmermann, and Valkanov (2014). Campbell and Thompson (2008) consider simple predictive regressions of the type in (1) and show that either constraining the return forecasts to be non-negative or forcing the regression coefficients to have the theoretically expected signs leads to clear out-of-sample improvements. Similarly, Pettenuzzo, Timmermann, and Valkanov (2014) start with the regression model in (1) but modify it to allow for time-varying volatility, and propose a Bayesian approach to impose either non-negativity in the equity premia or bounds on the conditional Sharpe ratio. They find that both constraints lead to improved forecasts, and that the Sharpe ratio constraint is particularly successful at improving portfolio performance.

Table 4 presents the results of the comparisons, focusing on the economic evidence of predictability, as summarized by the CER measure. Along with the results from our approach, which we denote with PPW in the table, we include two versions of the Campbell and Thompson (2008) constrained approach, CT1 and CT2. The former imposes a non-negativity constraint on the equity premium forecasts, and the latter applies a sign restriction to the slope coefficient of each univariate predictive regression. We also include the Sharpe ratio constraint approach of Pettenuzzo, Timmermann, and Valkanov (2014), (PTV), which bounds the in-sample annualized Sharpe ratios between zero and one.\textsuperscript{15} As can be seen from the table, PPW and PTV are the two best performing methods, followed by CT1 and CT2. With respect to the univariate models, our approach and PTV perform similarly, with our approach achieving the largest CERs for seven of the 11 predictors in the full sample (and for eight of the 11 predictors in the two subsamples). With respect to the equal-weighted model combinations at the bottom of the table, we find that overall the

\textsuperscript{15}Note that in addition to enforcing bounds on the annualized Sharpe ratio, the approach of Pettenuzzo, Timmermann, and Valkanov (2014) imposes Bayesian shrinkage on the regression coefficients in (1). To better isolate the effect of the Sharpe ratio constraint and make the results more comparable to the other methods presented, we have opted for estimating the PTV constrained model using the approach of Johnson (2017). That is, for each of the $N$ predictors considered we estimate the coefficients $\hat{\alpha}_t$ and $\hat{\beta}_t$ in (1) via constrained least squares, subject to the constraint that

$$0 \leq \sqrt{12} \left( \hat{\alpha}_t + \hat{\beta}_t x_{\tau} \right) / \hat{\sigma}_{x_{\tau}} \leq 1, \quad \tau = 1, \ldots, t$$

and where $\hat{\sigma}_{x_{\tau}}$ is the empirical estimate of $\sigma_{x_{\tau}}$, obtained as in Johnson (2017, Section 2).
EW-C approach appears to be far more stable than all the alternatives considered. Although in the first subsample both the PTV and CT1 methods achieve a CER of 52 basis points (about 21 basis points lower than EW-C), their performance deteriorates significantly in the second half of the sample. These results are in line with the findings in Pettenuzzo et al. (2014, Table 6). In stark contrast, the EW-C approach delivers results that, across the two subsamples, are superior and remarkably more robust than the alternative methods.

Table 4 about here

5 Robustness and Extensions

In this section, we present a number of extensions and robustness checks to validate the main results presented in Section 4. We begin by evaluating the forecasting performance of our predictor-constrained approach over the business cycle. We next experiment with the use of alternative look-back periods \( n \) in the calculation of the constraint in (3), different risk aversion degrees, and various weighting schemes for the forecast combinations.

5.1 Forecasting performance over the business cycle

To better understand the sources of the statistical and economic predictability we have uncovered, we separately calculate the \( R^2_{OoS} \) during business recession and expansion periods as follows:

\[
R^2_{OoS,c} = 1 - \frac{\sum_{\tau=M+1}^{T} I^c_{\tau} (r_{\tau} - forc_{\tau|\tau-1})^2}{\sum_{\tau=M+1}^{T} I^c_{\tau} (r_{\tau} - \bar{r}_{\tau|\tau-1})^2}, \quad \text{for} \quad c = EXP, REC,
\]

where \( I^EXP_{\tau} \) (\( I^REC_{\tau} \)) is an indicator variable set equal to one when month \( \tau \) is within an NBER-dated expansion (recession), and \( forc_{\tau|\tau-1} \) denotes the equity premia forecast produced at time \( \tau-1 \) by either the unconstrained or constrained univariate model (or model combination).

Table 5 reports the results of this analysis. Consistent with the results in Rapach, Strauss, and Zhou (2010) and Henkel, Martin, and Nardari (2011), the unconstrained return prediction models perform better relative to the no-predictability benchmark during recessions than they do during expansions. This is true for both the univariate predictive regressions and the equal-weighted model combinations. Interestingly, we notice the same pattern in the constrained
regression models. During recessions, our constrained approach leads to better forecasts for
eight of the 11 variables, with the largest improvements in $R^2_{OoS}$ occurring for the LTY ($R^2_{OoS}$
went from -0.027% to 5.767%) and SVAR ($R^2_{OoS}$ goes from 0.487% to 4.247%) predictors. As
for the forecast combination method, the $R^2_{OoS}$ almost doubles when we apply our constraint,
going from 0.932% in the EW-U model to 1.753% in the EW-C model. The results during
expansions are weaker, but still notable. We find that our constrained approach leads to a
higher $R^2_{OoS}$ for seven of the 11 predictors. However, the $R^2_{OoS}$ is positive only in two of these
seven cases. The best results are once again coming from the constrained model combination
method, with the $R^2_{OoS}$ going from 0.119% in the EW-U model to 0.657% in the EW-C model.

Table 5 about here

Table 6 reports the CER results separately for recessions and expansions. During
recessions, our constrained approach leads to higher CERs for five of the 11 predictors,
whereas during expansions the improvements in CER from imposing our constraint occur
for nine out of the 11 univariate regressions. With respect to the model combination
methods, imposing our constraint in the regressions entering the model combinations
generates strong positive results during both recessions and expansions. In particular,
during recessions the CER of the model combination goes from 1.417 to 2.818, and it goes
from -0.330 to 0.474 during expansions.

Table 6 about here

5.2 Alternative look-back periods

As we discussed in Section 2, our constrained method works by setting the value of the
predictor to zero in all instances in which its value falls below its past $n$-month high. In
accordance with the literature on limited investor attention and psychological anchors, in our
main analysis we have set the value of the look-back period $n$ to 12 months. We now examine
the robustness of our results to this choice. We begin by noting that values of $n$ that are very
small tend not to work too well in practice. Our intuition for this result is that when $n$ is
small, the constraint in (3) is unable to separate the random noise from the true instances in
which unexpected information dominates the changes in the predictors. We next investigate three cases, namely $n = 6$, $n = 18$ and $n = 24$.

Table 7 shows the out-of-sample $R^2$ of the constrained univariate regressions and EW-C model combination for these three alternative look-back periods. Beginning with the full sample results, we find that for most of the predictors, our constrained method continues to improve over the unconstrained models. This is true for nine out of 11 variables when $n = 6$, for seven out of 11 variables when $n = 18$, and for eight out of 11 predictors when $n = 24$. The EW-C forecasts also appear to be robust to the choice of the look-back period, with $R^2_{OoS}$ values for $n = 18$ and $n = 24$ that are only slightly below the results reported in Table 2 for $n = 12$. In contrast, the $R^2_{OoS}$ of EW-C for $n = 6$ is almost half the size of its value when $n = 12$, but still above the EW-U $R^2_{OoS}$. Moving on to the subsample results in Panels B and C, by and large these tend to confirm the pattern found with the full sample. Most notably, in the first subsample the EW-C forecasts fail to improve over the unrestricted EW-U model combination. However, this is not the case in the second subsample, and for $n = 18$ the EW-C model produces an $R^2_{OoS}$ that is higher than the corresponding value for $n = 12$.

Table 7 about here

Table 8 reports the out-of-sample CER gains of the constrained univariate regressions and EW-C model combination for the three alternative look-back periods. Overall, the CER results are in line with the $R^2_{OoS}$ results reported in Table 7. In the full sample the CER of the individual models are always higher than their unconstrained counterparts, regardless of the choice of the look-back period. In addition, the EW-C model improves on the unconstrained EW-U model in all cases. Panels B and C of Table 8 confirm that these results are very robust across the two subsamples.

Table 8 about here

5.3 Alternative risk aversion choices

Our main analysis of the economic value of equity premium forecasts in Section 4 assumed a coefficient of relative risk aversion of $\gamma = 3$. To explore the sensitivity of our results to this
value, we consider lower ($\gamma = 2$) and higher ($\gamma = 4$, $\gamma = 6$) values of this parameter. Results for the EW-C model combination method are shown in Table 9.

Beginning with the full sample results in Panel A, we find that the portfolios constructed using the predictions from the EW-C model generate a large and positive CER for each of the three $\gamma$ values under consideration. In addition to improving over the benchmark no-predictability model in all three cases, the EW-C portfolios achieve annualized CER gains that are between 45 and 134 bps higher than the corresponding unconstrained EW-U model results. Results from the subsample analysis in Panels B and C are largely in line with the full sample findings from Panel A. In all cases, the EW-C generates CER gains that are higher than those given by the EW-U model combination method.

Table 9 about here

5.4 Alternative model combination schemes

As discussed in Section 2 and Section 4, our multivariate results have been based on equal-weighted combinations of unconstrained and constrained predictive regressions. We now explore the sensitivity of our results to this modeling choice by considering four alternative model combination schemes. Our first alternative is a trimmed-mean model combination, which is obtained by recursively dropping the worst performing model (as determined by the recursively calculated MSPEs), and applying equal weights to the surviving $N - 1$ models. The second and third methods are based on the Stock and Watson (2004) discounted MSPE (DMSPE) combination scheme, which dictates that the weight of each model in the combination (5) or (6) is given by either

\[
w_{i,t} = \frac{\phi_{i,t}^{-1}}{\sum_{j=1}^{N} \phi_{j,t}^{-1}},
\]

in the case of the unconstrained model combination in (5), or

\[
w_{i,t}^*(n) = \frac{\left[\phi_{i,t}^*(n)\right]^{-1}}{\sum_{j=1}^{N} \left[\phi_{j,t}^*(n)\right]^{-1}},
\]

in the case of the constrained model combination in (6). Here, $N$ is the number of individual models, $\phi_{j,t} = \sum_{\tau=M+1}^{t} \delta^{t-\tau}(r_{\tau} - \hat{r}_{j,\tau|\tau-1})^2$, and $\phi_{j,t}^*(n) = \sum_{\tau=M+1}^{t} \delta^{t-\tau}(r_{\tau} - \hat{r}_{j,\tau|\tau-1}(n))^2$. 20
We separately consider $\delta = 1$ and $\delta = 0.9$; in the former case we are treating all past observations equally when computing the MSFE, and in the latter case we are placing a greater emphasis on the most recent forecasting performance. Our last combination scheme follows Yang (2004), who shows that because of the large estimation uncertainty surrounding the combination weights, simple linear combinations of point forecasts can sometime lead to suboptimal performances. Yang introduces a non-linear combination scheme, with weights that, in the case of the unconstrained model combination in (5), are given by: \[ \omega_{i,t} = \frac{\pi_i \exp \left( -\lambda \sum_{\tau=M+1}^{t}(r_\tau - \hat{r}_{i,\tau|\tau-1})^2 \right)}{\sum_{j=1}^{N} \pi_j \exp \left( -\lambda \sum_{\tau=M+1}^{t}(r_\tau - \hat{r}_{j,\tau|\tau-1})^2 \right)}. \] \[ (17) \]

We follow Yang (2004) and set $\pi_i = \lambda = 1$.

Table 10 shows the out-of-sample $R^2$ for each of the four alternative combinations for both the full sample and the two sub-periods. With respect to the full sample results, we find values of $R^2_{OoS}$ for the unconstrained combination methods that range between 0.340% and 0.524%, depending on the weighting schemes used in the combination. Also, we find that regardless of the combination scheme adopted, the $R^2_{OoS}$ values from the constrained model combinations in the third column of the table are always above their unconstrained counterparts from the second column, and higher than the equal-weighted combination results reported in Table 2. Moving on to Panels B and C, we see that the superior forecasting performance of the constrained forecast combinations mainly come from the second subsample, whereas in the first subsample unconstrained and constrained forecast combinations yield very similar results.

Table 10 about here

6 Conclusions

We have proposed a novel methodology for implementing economically motivated constraints in predictive regressions of aggregate stock returns. Unlike other approaches in the literature, our restrictions are implemented directly on the predictor variables, setting their value to zero whenever they fall below their past 12-month high. Out-of-sample evidence indicates that

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\[ 16 \]The formula for the constrained model combination in (6) can be derived analogously.
our approach yields very accurate forecasts for a large host of macroeconomic and financial predictors of stock returns. We also show that a simple equal-weighted combination of the point forecasts obtained from the univariate constrained regressions further improves the out-of-sample predictability of our method, both in statistical and economic terms. Subsample analysis and a large battery of robustness checks reveal that our method is significantly more stable and robust than the existing constrained approaches in the literature.

References


Figure 1. Equity premia forecasts

This figure plots the time evolution of the constrained (solid blue lines) and unconstrained (dashed red lines) equity premia forecasts over the whole forecast evaluation period, 1949–2015. The forecasts are obtained using the constrained and unconstrained univariate regression models and the formulas in (2) and (3). We also include in the figure the forecasts obtained using a no-predictability benchmark (dashed-dotted yellow lines). The three panels of the figure use the dividend price ratio (DP), Treasury bill rate (TBL), and the long-term yield (LTY) as predictors, respectively.
This figure plots the time evolution of the constrained (solid blue lines) and unconstrained (dashed red lines) equity premia forecasts over the whole forecast evaluation period, 1949–2015. The forecasts are obtained by combining the $N$ univariate constrained and unconstrained forecasts as described in (5) and (6), respectively. We also include in the figure the forecasts obtained using a no-predictability benchmark (dashed-dotted yellow lines).
This figure plots the Cumulative Sum of Squared prediction Error Difference (CSSED) over time for both the constrained and unconstrained equal-weighted forecast combinations, relative to the benchmark no predictability model. We compute the CSSED as follows:

\[ CSSED_{model,t} = \sum_{\tau=M+1}^{t} (e_{bench,\tau}^2 - e_{model,\tau}^2) \quad t = M + 1, \ldots, T \]

where \( model \) denotes either the unconstrained or constrained model combination, while \( e_{model,\tau} \) and \( e_{bench,\tau} \) denote time \( \tau \) forecast errors associated with either the model combination or the benchmark model.
Table 1. Summary statistics of predictive variables and stock returns

<table>
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<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Max</th>
<th>Min</th>
<th>Std.Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>JB</th>
<th>Q(1)</th>
<th>Q(10)</th>
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<tr>
<td>RET</td>
<td>0.005</td>
<td>0.009</td>
<td>0.346</td>
<td>-0.339</td>
<td>0.055</td>
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<td>0.005</td>
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<td>&lt;0.001</td>
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<td>&lt;0.001</td>
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<td>0.030</td>
<td>0.163</td>
<td>0.000</td>
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<td>1.052</td>
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<td>&lt;0.001</td>
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<td>1.063</td>
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<td>16.517</td>
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</tbody>
</table>

This table provides summary statistics for the excess returns (RET) on the S&P 500 index and the 11 predictive variables used in our analysis. JB denotes the Jarque-Bera test statistic on the null hypothesis that the data come from a normal distribution, while Q(1) (Q(10)) denote the Ljung-Box Q test statistic on the null hypothesis that the first (first 10) autocorrelation coefficient is (are jointly) equal to zero. The sample period is January 1927 - December 2015.
Table 2. Forecasting performance evaluated by $R^2_{OoS}$

<table>
<thead>
<tr>
<th></th>
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<th></th>
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<td></td>
<td>Original variables</td>
<td>Truncated variables</td>
<td>Original variables</td>
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<tr>
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<td>-1.093</td>
<td>0.158**</td>
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<td>-1.194</td>
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<tr>
<td>INFL</td>
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<td>0.257</td>
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<tr>
<td>Forecast combination</td>
<td>0.342**</td>
<td>0.957***</td>
<td>0.939**</td>
</tr>
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</table>

This table reports the out-of-sample forecast performance of the 11 constrained and unconstrained univariate predictive regressions, as well as the equal-weighted combination methods, EW-U and EW-C. We evaluate the quality of the forecast using the Out-of-Sample $R^2$ ($R^2_{OoS}$) defined as the percentage reduction in MSPE of the model of interest relative to the benchmark model, and given by

$$R^2_{OoS} = 1 - \frac{MSPE_{model}}{MSPE_{bench}}$$

where $MSPE_{model} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_\tau - forc_{\tau|\tau-1})^2$ with $forc_{\tau|\tau-1}$ denoting the equity premia forecast produced at time $\tau-1$ by either the unconstrained or constrained univariate models (or model combination), while $MSPE_{bench} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_\tau - \bar{r}_{\tau|\tau-1})^2$. We next multiply the $R^2_{OoS}$ figures by 100, to denote percentage values. A positive $R^2_{OoS}$ implies that the model of interest generates more accurate forecasts than the benchmark model. Panel A reports the results for the full sample (1949-2015), while Panel B and Panel C report the results for the subsamples 1949-1981 and 1982-2015. Bold numbers indicates all instances in which the constrained $R^2_{OoS}$ is higher than its unconstrained counterpart. * , ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.
Table 3. Forecasting performance evaluated by CER

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original variables</td>
<td>Truncated variables</td>
<td>Original variables</td>
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<td>EP</td>
<td>0.051</td>
<td><strong>0.338</strong></td>
<td>0.208</td>
</tr>
<tr>
<td>SVAR</td>
<td>0.059</td>
<td><strong>0.863</strong></td>
<td>-0.170</td>
</tr>
<tr>
<td>BM</td>
<td>-1.930</td>
<td><strong>-0.345</strong></td>
<td>-1.700</td>
</tr>
<tr>
<td>NTIS</td>
<td>0.283</td>
<td><strong>0.539</strong></td>
<td>0.414</td>
</tr>
<tr>
<td>TBL</td>
<td>-0.691</td>
<td><strong>0.815</strong></td>
<td>-0.768</td>
</tr>
<tr>
<td>LTY</td>
<td>-0.389</td>
<td><strong>1.376</strong></td>
<td>-0.165</td>
</tr>
<tr>
<td>DFY</td>
<td>-0.271</td>
<td><strong>0.278</strong></td>
<td>-0.257</td>
</tr>
<tr>
<td>DFR</td>
<td>-0.069</td>
<td><strong>0.262</strong></td>
<td>-0.379</td>
</tr>
<tr>
<td>INFL</td>
<td>-0.159</td>
<td><strong>-0.131</strong></td>
<td>0.151</td>
</tr>
<tr>
<td>Forecast combination</td>
<td>-0.064</td>
<td><strong>0.831</strong></td>
<td>0.255</td>
</tr>
</tbody>
</table>

This table reports the out-of-sample forecast performance of the 11 constrained and unconstrained univariate predictive regressions, as well as the equal-weighted combination methods, EW-U and EW-C. Each period, the investor chooses the optimal weight to allocate to the risky asset in the portfolio by solving the following equation:

\[ \omega^*_t = \frac{\text{forc}_{t+1|t} + \hat{\sigma}^2_{t+1|t}/2}{\gamma \hat{\sigma}^2_{t+1|t}}, \quad \text{for } t = M, ..., T - 1 \]

where \( \gamma \) indicates the coefficient of risk aversion, \( \hat{\sigma}^2_{t+1|t} \) denotes the forecast for time \( t + 1 \) stock variance made at time \( t \), and \( \text{forc}_{t+1|t} \) stands for the equity premia forecast produced at time \( t \) by either the unconstrained or constrained univariate models/model combinations. The optimal weight of stock is restricted between 0 and 1.5, while the risk aversion coefficient is set to \( \gamma = 3 \). Each entry in the table represents the difference between the certainty equivalent return of a portfolio based on the forecasts from the model of interest and the no predictability model. These differences are multiplied by 1200 to denote annualized percentage values. Panel A reports results for the full sample (1949-2015), while Panel B and Panel C report the results for the subsamples 1949-1981 and 1982-2015. Bold numbers indicate all instances in which the constrained CER is higher than its unconstrained counterpart.
Table 4. Comparison with existing constrained methods

<table>
<thead>
<tr>
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<tr>
<td></td>
<td>CT1</td>
<td>CT2</td>
<td>PTV</td>
</tr>
<tr>
<td>DP</td>
<td>0.202</td>
<td>-0.787</td>
<td>-0.060</td>
</tr>
<tr>
<td>DY</td>
<td>0.460</td>
<td>-0.796</td>
<td>0.205</td>
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<tr>
<td>EP</td>
<td>-0.918</td>
<td>0.051</td>
<td>0.622</td>
</tr>
<tr>
<td>SVAR</td>
<td>-0.271</td>
<td>-0.176</td>
<td>-0.049</td>
</tr>
<tr>
<td>BM</td>
<td>-0.821</td>
<td>-1.930</td>
<td>0.063</td>
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<tr>
<td>NTIS</td>
<td>0.417</td>
<td>0.283</td>
<td>0.391</td>
</tr>
<tr>
<td>TBL</td>
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<td>-0.691</td>
<td>0.505</td>
</tr>
<tr>
<td>LTY</td>
<td>-0.889</td>
<td>-0.389</td>
<td>0.336</td>
</tr>
<tr>
<td>SVAR</td>
<td>0.529</td>
<td>-0.268</td>
<td>0.214</td>
</tr>
<tr>
<td>DFR</td>
<td>-0.17</td>
<td>-0.052</td>
<td>0.257</td>
</tr>
<tr>
<td>INFL</td>
<td>-0.338</td>
<td>-0.159</td>
<td>0.049</td>
</tr>
<tr>
<td>Forecast combination</td>
<td>0.183</td>
<td>-0.092</td>
<td>0.288</td>
</tr>
</tbody>
</table>

This table reports the out-of-sample performance of univariate predictive regressions and equal-weighted model combinations subject to different constraints. PPW refers to the constrained approach introduced in this paper. CT1 and CT2 denote two versions of the Campbell and Thompson (2008) constrained approach. The former imposes a non-negativity constraint on the equity premium predictions, while the latter requires the slope coefficients in the univariate predictive regressions to have the economically predicted sign. Lastly, PTV denotes the Sharpe ratio constraint approach of Pettenuzzo, Timmermann, and Valkanov (2014), which bounds the in-sample annualized Sharpe ratios between zero and one. Each period, the investor chooses the optimal weight to allocate to the risky asset in the portfolio by solving the following

\[
\omega^*_t = \frac{forc_{t+1|t} + \hat{\sigma}^2_{t+1|t}/\gamma}{\hat{\sigma}^2_{t+1|t}}, \quad t = M, ..., T - 1
\]

where \(\gamma\) indicates the coefficient of risk aversion, \(\hat{\sigma}^2_{t+1|t}\) denotes the forecast made at time \(t\) of time \(t + 1\) log excess return variance, and \(forc_{t+1|t}\) denotes the one-step-ahead forecast obtained from one of the constrained univariate predictive regressions or model combinations. The optimal weight of stock is restricted between 0 and 1.5, while the risk aversion coefficient is set to \(\gamma = 3\). Each entry in the table represents the difference between the certainty equivalent return of a portfolio based on the forecasts from the model of interest and the no predictability model. These differences are multiplied by 1200 to denote annualized percentage values. Panel A reports results for the full sample (1949-2015), while Panel B and Panel C report the results for the subsamples 1949-1981 and 1982-2015. Bold numbers indicates all instances in which the constrained CER is higher than its unconstrained counterpart.
Table 5. Forecasting performance over business cycles evaluated by $R^2_{OoS}$

<table>
<thead>
<tr>
<th></th>
<th>Recession periods</th>
<th>Expansion periods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original variables</td>
<td>Truncated variables</td>
</tr>
<tr>
<td>DP</td>
<td>1.656**</td>
<td>1.767*</td>
</tr>
<tr>
<td>DY</td>
<td>2.554***</td>
<td>-0.642</td>
</tr>
<tr>
<td>EP</td>
<td>-2.308</td>
<td>0.019</td>
</tr>
<tr>
<td>SVAR</td>
<td>0.487</td>
<td>4.247*</td>
</tr>
<tr>
<td>BM</td>
<td>-0.937</td>
<td>-0.859</td>
</tr>
<tr>
<td>NTIS</td>
<td>-4.605</td>
<td>-0.531</td>
</tr>
<tr>
<td>TBL</td>
<td>0.904</td>
<td>1.160</td>
</tr>
<tr>
<td>LTY</td>
<td>-0.027</td>
<td>5.767***</td>
</tr>
<tr>
<td>DFY</td>
<td>-0.259</td>
<td>-6.096</td>
</tr>
<tr>
<td>DFR</td>
<td>-0.609</td>
<td>-0.679</td>
</tr>
<tr>
<td>INFL</td>
<td>-0.462</td>
<td>-0.244</td>
</tr>
<tr>
<td>Forecast combination</td>
<td>0.932*</td>
<td>1.753**</td>
</tr>
</tbody>
</table>

This table reports the out-of-sample forecast performance of the 11 constrained and unconstrained univariate predictive regressions, as well as the equal-weighted combination methods, separately for recessions and expansions. We evaluate the quality of the forecast using the Out-of-Sample $R^2$ ($R^2_{OoS}$) defined as the percentage reduction in MSPE of the model of interest relative to the benchmark model, and given by

$$R^2_{OoS,c} = 1 - \frac{\sum_{\tau=M+1}^{T} I^c_{\tau}(r_{\tau} - f_{or c|\tau-1})^2}{\sum_{\tau=M+1}^{T} I^c_{\tau}(r_{\tau} - \bar{r}_{\tau-1})^2}, \quad \text{for} \quad c = EXP, REC,$$

where $I^c_{\tau}$ ($I^R_{\tau}$) is an indicator variable which is set equal to one when month $\tau$ is within an NBER-dated expansion (recession), and $f_{or c|\tau-1}$ denotes the equity premia forecast produced at time $\tau - 1$ by either the unconstrained or constrained univariate models (or model combination). We next multiply the $R^2_{OoS}$ figures by 100, to denote percentage values. A positive $R^2_{OoS}$ implies that the model of interest generates more accurate forecasts than the benchmark model. Panel A reports the results during recessions, while Panel B focuses on expansions. Bold numbers indicates all instances in which the constrained $R^2_{OoS,c}$ is higher than its unconstrained counterpart. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.
This table reports the out-of-sample forecast performance of the 11 constrained and unconstrained univariate predictive regressions, as well as the equal-weighted combination methods, separately for recessions and expansions. Each period, the investor chooses the optimal weight to allocate to the risky asset in the portfolio by solving the following:

$$\omega_t^* = \frac{forc_{t+1|t} + \hat{\sigma}_{t+1|t}^2/2}{\gamma \hat{\sigma}_{t+1|t}^2}, \quad t = M, \ldots, T - 1$$

where $\gamma$ indicates the coefficient of risk aversion, $\hat{\sigma}_{t+1|t}^2$ denotes the forecast for time $t + 1$ stock variance made at time $t$, and $forc_{t+1|t}$ stands for the equity premia forecast produced at time $t$ by either the unconstrained or constrained univariate models/model combinations. The optimal weight of stock is restricted between 0 and 1.5, while the risk aversion coefficient is set to $\gamma = 3$. Each entry in the table represents the difference between the certainty equivalent return of a portfolio based on the forecasts from the model of interest and the no predictability model. These differences are multiplied by 1200 to denote annualized percentage values. Panel A reports the results during recessions, while Panel B focuses on expansions. Bold numbers indicates all instances in which the constrained CER is higher than its unconstrained counterpart.
Table 7. Alternative look-back periods, $R_{OoS}^2$

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n = 6$</td>
<td>$n = 12$</td>
<td>$n = 18$</td>
</tr>
<tr>
<td>DP</td>
<td>-0.233</td>
<td>-0.038</td>
<td>0.183</td>
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<tr>
<td>DY</td>
<td>-0.734</td>
<td>-1.093</td>
<td>-1.223</td>
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<tr>
<td>EP</td>
<td>-0.159</td>
<td>-0.045</td>
<td>-0.396</td>
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<td>1.913**</td>
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<tr>
<td>BM</td>
<td>-0.487</td>
<td>-0.443</td>
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<tr>
<td>NTIS</td>
<td>-0.318</td>
<td>-0.067</td>
<td>0.379**</td>
</tr>
<tr>
<td>TBF</td>
<td>0.040*</td>
<td>0.371**</td>
<td>0.099*</td>
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<tr>
<td>LTY</td>
<td>0.442**</td>
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<td>1.410**</td>
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<tr>
<td>DFR</td>
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</tr>
<tr>
<td>INFL</td>
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<td>-0.603</td>
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<tr>
<td>Forecast combination</td>
<td>0.411***</td>
<td>0.957***</td>
<td>0.939***</td>
</tr>
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</table>

This table reports the out-of-sample forecast performance of the 11 constrained univariate predictive regressions and the EW-C model combination for four alternative look-back periods. The forecasting performance is evaluated using the Out-of-Sample $R^2$ ($R_{OoS}^2$) defined as the percentage reduction in MSPE of the model of interest relative to the benchmark model, and given by

$$ R_{OoS}^2 = 1 - \frac{MSPE_{model}}{MSPE_{bench}} $$

where $MSPE_{model} = \frac{1}{T-M} \sum_{t=M+1}^{T} (r_t - forct_{t\mid t-1})^2$ with $forct_{t\mid t-1}$ denoting either one of the constrained univariate models or the EW-C forecast combination, while $MSPE_{bench} = \frac{1}{T-M} \sum_{t=M+1}^{T} (r_t - \bar{r}_{t\mid t-1})^2$. We next multiply the $R_{OoS}^2$ figures by 100, to denote percentage values. A positive $R_{OoS}^2$ implies that the model of interest generates more accurate forecasts than the benchmark model. Panel A reports forecasting results for the full sample (1949-2015), while Panel B and Panel C report the results for the subsamples 1949-1981 and 1982-2015. We multiply $R_{OoS}^2$ by 100 to denote percent value. Bold numbers indicates all instances in which the constrained $R_{OoS}^2$ is higher than its unconstrained counterpart. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.
Table 8. Alternative look back periods, CER

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>n = 6</td>
<td>n = 12</td>
<td>n = 18</td>
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<tr>
<td>DP</td>
<td>-0.158</td>
<td>0.442</td>
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<td>DY</td>
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<td>-0.132</td>
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<td>SVAR</td>
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<td>0.863</td>
<td>0.776</td>
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<td>BM</td>
<td>-0.400</td>
<td>-0.345</td>
<td>-0.413</td>
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<td>NTIS</td>
<td>0.258</td>
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<td>TBL</td>
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<td>0.486</td>
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<td>0.944</td>
<td>1.376</td>
<td>1.573</td>
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<td>0.032</td>
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<td>-0.113</td>
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<td>Forecast</td>
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<tr>
<td>combination</td>
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<td>0.832</td>
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</table>

This table reports the out-of-sample forecast performance of the 11 constrained univariate predictive regressions and the EW-C model combination for three alternative look-back periods. Each period, the investor chooses the optimal weight to allocate to the risky asset in the portfolio by solving the following

\[
\omega^*_t = \frac{\text{forc}_{t+1} + \hat{\sigma}^2_{t+1}|t}/\gamma \hat{\sigma}^2_{t+1}|t}, \quad t = M, ..., T - 1
\]

where \(\gamma\) indicates the coefficient of risk aversion, \(\hat{\sigma}^2_{t+1}|t\) denotes the forecast for time \(t + 1\) stock variance made at time \(t\), and \(\text{forc}_{t+1} \mid t\) stands for the equity premia forecast produced at time \(t\) by either the unconstrained or constrained univariate models/model combinations. The optimal weight of stock is restricted between 0 and 1.5, while the risk aversion coefficient is set to \(\gamma = 3\). Each entry in the table represents the difference between the certainty equivalent return of a portfolio based on the forecasts from the model of interest and the no predictability model. These differences are multiplied by 1200 to denote annualized percentage values. Panel A reports results for the full sample (1949-2015), while Panel B and Panel C report the results for the subsamples 1949-1981 and 1982-2015. Bold numbers indicates all instances in which the constrained CER is higher than its unconstrained counterpart.
Table 9. Alternative risk aversion coefficients

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<tbody>
<tr>
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<td>Original variables</td>
<td>Truncated variables</td>
<td>Original variables</td>
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<td>$\gamma = 4$</td>
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<tr>
<td>$\gamma = 6$</td>
<td>-0.035</td>
<td>0.416</td>
<td>0.124</td>
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This table reports the out-of-sample forecast performance of the equal-weighted combination methods, EW-U and EW-C, for different degrees of risk aversion $\gamma$. Each period, the investor chooses the optimal weight to allocate to the risky asset in the portfolio by solving the following

$$
\omega_t^* = \frac{forc_{t+1|t} + \hat{\sigma}^2_{t+1|t}/2}{\gamma \hat{\sigma}^2_{t+1|t}}, \quad t = M, ..., T - 1
$$

where $\gamma$ indicates the coefficient of risk aversion, $\hat{\sigma}^2_{t+1|t}$ denotes the forecast for time $t + 1$ stock variance made at time $t$, and $forc_{t+1|t}$ stands for the equity premia forecast produced at time $t$ by either the unconstrained or constrained univariate models/model combinations. The optimal weight of stock is restricted between 0 and 1.5. Each entry in the table represents the difference between the certainty equivalent return of a portfolio based on the forecasts from the model of interest and the no predictability model. These differences are multiplied by 1200 to denote annualized percentage values. Panel A reports results for the full sample (1949-2015), while Panel B and Panel C report the results for the subsamples 1949-1981 and 1982-2015. Bold numbers indicates all instances in which the constrained CER is higher than its unconstrained counterpart.
Table 10. Alternative combination strategies

<table>
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</thead>
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<td>Truncated variables</td>
<td>Original variables</td>
</tr>
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<td>0.986***</td>
<td>1.240***</td>
</tr>
<tr>
<td>DMSPE(1)</td>
<td>0.355**</td>
<td>0.972***</td>
<td>0.965**</td>
</tr>
<tr>
<td>DMSPE(0.9)</td>
<td>0.426**</td>
<td>0.985***</td>
<td>1.037**</td>
</tr>
<tr>
<td>Yang</td>
<td>0.340**</td>
<td>0.965***</td>
<td>0.931**</td>
</tr>
</tbody>
</table>

This table reports the out-of-sample forecast performance of the unconstrained and constrained model combinations for four alternative combination schemes. The forecasting performance is evaluated using the Out-of-Sample $R^2$ ($R^{2}\text{OoS}$) defined as the percentage reduction in MSPE of the model of interest relative to the benchmark model, and given by

$$R^{2}\text{OoS} = 1 - \frac{MSE_{model}}{MSE_{bench}}$$

where $MSE_{model} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_{\tau} - forc_{r|\tau-1})^2$ with $forc_{r|\tau-1}$ denoting either one of the forecast combination schemes, while $MSE_{bench} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_{\tau} - \bar{r}_{\tau|\tau-1})^2$. We next multiply the $R^{2}\text{OoS}$ figures by 100, to denote percentage values. A positive $R^{2}\text{OoS}$ implies that the model of interest generates more accurate forecasts than the benchmark model. Panel A reports forecasting results for the full sample (1949-2015), while Panel B and Panel C report the results for the subsamples 1949-1981 and 1982-2015. We multiply $R^{2}\text{OoS}$ by 100 to denote percent value. Bold numbers indicates all instances in which the constrained $R^{2}\text{OoS}$ is higher than its unconstrained counterpart. *, ** and *** denote statistical significance at the 10%, 5% and 1% levels, respectively.