

# Forecasting stock returns: A predictorconstrained approach

Zhiyuan Pan, Southwestern Univesity of Finance and Economics, Institute of

**Chinese Financial Studies** 

Davide Pettenuzzo, Department of Economics, Brandeis University

Yudong Wang, School of Economics and Management, Nanjing University of

Science and Technology

**Working Paper Series** 

# Forecasting stock returns: A predictor-constrained approach\*

Zhiyuan Pan<sup>†</sup> Davide Pettenuzzo<sup>‡</sup> Yudong Wang<sup>§</sup>

February 13, 2018

#### Abstract

We develop a novel method to impose constraints on univariate predictive regressions of stock returns. Unlike the previous approaches in the literature, we implement our constraints directly on the predictor, setting it to zero whenever its value falls within the variable's past 24-month high and low. Empirically, we find that relative to standard unconstrained predictive regressions, our approach leads to significantly larger forecasting gains. We also show how a simple equal-weighted combination of our constrained forecasts leads to further improvements in forecast accuracy, generating forecasts that are more accurate than those obtained using existing constrained methods. Further analysis confirms that these findings are robust to the presence of model instabilities and structural breaks.

Keywords: Equity premium; Predictive regressions; Predictor constraints; 24-month high and low; Model combinations.

JEL Classifications: C11; C22; G11; G12

<sup>\*</sup>This work was supported by the Chinese National Science Foundation through grant number 71501095 and 71722015 (Yudong Wang), 71601161 (Zhiyuan Pan).

<sup>&</sup>lt;sup>†</sup>Institute of Chinese Financial Studies, Southwestern University of Finance and Economics, China; Collaborative Innovation Center of Financial Security, China. Email: panzhiyuancd@126.com

 $<sup>^{\</sup>ddagger}$ Brandeis University, Sachar International Center, 415 South St, Waltham, MA. Tel: +1 (781) 736-2834 Email: dpettenu@brandeis.edu

<sup>§</sup>Corresponding author. School of Economics and Management, Nanjing University of Science and Technology, Xiaolingwei 200, Xuanwu District, Nanjing 210094, China. Tel: +86-13681663442 Email: wangyudongnj@126.com

# 1 Introduction

Stock return forecasts affect many areas in finance ranging from asset pricing to portfolio allocation and risk management. Not surprisingly, over the years, the quest for accurate and reliable return forecasts has attracted significant interest by both practitioners and academics. A large number of earlier studies have shown convincing evidence of in-sample predictability (Campbell and Shiller, 1988; Fama and French, 1988, 1989; Ferson and Harvey, 1993; Keim and Stambaugh, 1986; Lettau and Ludvigson, 2001; Pontiff and Schall, 1998). However, as noted by Welch and Goyal (2008), it remains difficult to find models that can improve on even the most naive benchmarks out-of-sample.<sup>1</sup>

Recently, it has been shown that constraining in some ways the parameters or the forecasts implied by otherwise standard predictive regressions can lead to sharper predictions of aggregate returns. For example, Pástor and Stambaugh (2009, 2012) propose a Bayesian approach with priors that are set in a way to ensure that shocks to unexpected and expected returns are negatively correlated. Campbell and Thompson (2008) (CT hereafter) show that either constraining the return forecasts to be non-negative or forcing the regression coefficients to have the theoretically expected signs leads to clear gains in out-of-sample predictability. Along the same lines, Pettenuzzo, Timmermann, and Valkanov (2014) (PTV hereafter) propose a Bayesian approach to impose non-negative equity premia and bounds on the conditional Sharpe ratio of univariate predictive regressions, and find that their approach leads to very accurate forecasts.<sup>2</sup>

We build on the above-mentioned work and propose a novel approach to imposing constraints on univariate predictive regressions of stock returns. Unlike the previous

<sup>&</sup>lt;sup>1</sup>In this paper, we focus on the standard and widely used predictors suggested by Welch and Goyal (2008) because their data span a time period long enough for us to reliably test our method. There is also a branch of studies that has uncovered new predictive variables for aggregate stock returns, including short interest (Rapach, Ringgenberg, and Zhou, 2016), variance risk premia (Bollerslev, Tauchen, and Zhou, 2009), oil price changes (Driesprong, Jacobsen, and Maat, 2008), technical indicators (Neely, Rapach, Tu, and Zhou, 2014), news implied volatility (Manela and Moreira, 2017), economic policy uncertainty (Brogaard and Detzel, 2015), aligned investor sentiment (Huang, Jiang, Tu, and Zhou, 2014), and skewness in expected macro fundamentals (Colacito, Ghysels, Meng, and Siwasarit, 2016). These predictors tend to span a much shorter time period, and for this reason we do not focus on them in our study. In addition, we restrict our attention to U.S. stock returns and leave the question of international stock return predictability (see, e.g., Jordan, Vivian, and Wohar, 2014; Lawrenz and Zorn, 2017) to future research.

<sup>&</sup>lt;sup>2</sup>Chib and Zeng (2016) extend the PTV approach to a multivariate regression using a large set of return predictors, and find similar gains in forecast accuracy.

methods in the literature, we implement our restrictions directly on the predictor, setting it to zero whenever its value falls within the predictor's rolling 24-month high and low. Our main motivation for this approach comes from the idea that investors are likely to pay extra attention to stock return fundamentals and macroeconomic indicators when these undergo extreme changes, and are therefore more likely to react to the arrival of such information. In this, we draw heavily from the recent work on limited attention in behavioral finance, in which a growing body of evidence has uncovered the fact that investors' attention changes over time in a way that affects stock returns (Da, Engelberg, and Gao, 2011; Barber and Odean, 2008; Barber, Odean, and Zhu, 2009; DellaVigna and Pollet, 2009). For example, Huberman and Regev (2001) show that stock prices respond to new information only when investors pay attention to it. Along the same lines, Barber and Odean (2008) report that investors have limited attention, and only pay attention to specific categories of stocks and/or extreme returns. Similarly, George and Hwang (2004) suggest that traders use the stock's 52-week high as an anchor in evaluating whether a recent increase in stock price implies new information. Li and Yu (2012) propose using the proximity of a stock price to its 52-week high as a measure of investor under-reaction, and find that their indicator has predictive power for stock returns.<sup>3</sup>

A second motivation for our approach comes directly from the notion that asset prices change in response to unexpected, rather than expected, fundamental information (see, e.g., Ball and Brown, 1968; Aharony and Swary, 1980). For example, Dew-Becker, Giglio, Le, and Rodriguez (2017) find that only the unexpected and transitory stock return variance is priced, and investors are not willing to pay to hedge risks due to expected economic uncertainty. In the context of stock return predictability, it is well known that most of the popular predictors (e.g., the dividend/price ratio or interest rates) display strong persistent behavior (see, e.g. Stambaugh, 1999), which implies that a large fraction of the future variance of these predictors is dominated by their persistent time series behaviors, rather than their unexpected shocks. It follows that identifying times when unexpected information dominates the changes in the predictors can offer a potential new way to improve the forecasting performance of predictive

<sup>&</sup>lt;sup>3</sup>In the context of oil prices, Hamilton (1996, 2011) finds that the effects of oil price changes on U.S. GDP are significant only when the current oil price exceeds the nearby past summit.

regressions.

To evaluate our approach empirically, we use 13 well-known monthly predictors of stock returns, covering a time period spanning January 1927 to December 2015. We use the first 20 years of data as a warm-up period to train the various models, and generate recursive forecasts of Standard and Poor (S&P) 500 excess returns starting in January 1947 and continuing through December 2015. Overall, we find that imposing our novel constraint on the predictors improves the statistical accuracy of the univariate predictive models, as measured by the Outof-Sample  $R^2$  ( $R^2_{OoS}$ ), for 11 of the 13 predictors, with particularly large gains occurring for the Earning price ratio ( $R_{OoS}^2$  goes from -1.306% to 0.210%), and stock variance ( $R_{OoS}^2$  goes from -0.769% to 1.449%). In both cases, the return forecasts from the constrained regression models are significantly more accurate than the benchmark no-predictability forecasts, as indicated by the Clark and West (2007) test. We also investigate the economic value of using the returns forecasts generated using our constrained regressions to make portfolio decisions. We consider an investor with constant relative risk aversion (CRRA) utility who allocates his/her wealth between the S&P 500 and the short-term Treasury bill, and evaluate the portfolio performance using certainty equivalent returns (CERs). Our results show that imposing our constraint on the predictors leads to large CER gains for the same 11 of 13 predictors.

The previous results refer to univariate regression models with a single predictor variable. Although our constrained approach is quite successful in the univariate setting, it does not offer a way to deal with the issue of model uncertainty, which has been documented to be quite pervasive in the context of stock return predictability. In addition, it is very well known that predictive regressions of the sort considered here are plagued by model instability, and that the predictive accuracy of these regressions fluctuates over time. For example, Pettenuzzo, Timmermann, and Valkanov (2014, see Table 2) show that their univariate constrained models outperform the naive no-predictability benchmark in a statistically significant way for 8 of their 16 predictors in the full sample, but the evidence in favor of predictability weakens significantly in the second half of their sample. To tackle

<sup>&</sup>lt;sup>4</sup>Our main results rely on a look-back period of 24 months when constraining the predictors. As a robustness check, we experiment with alternative look-back periods of 12, 18, 30 and 36 months. For comparability across the specifications, we omit the first 36 months of data (January 1927–December 1929), and as a result, the sample period starts in January 1930 for all our analyses.

these issues, we extend our analysis to a multivariate setting by considering equal-weighted combinations of univariate constrained predictive regressions.<sup>5</sup> We separately generate equal-weighted combinations of either unconstrained (EW-U) or constrained (EW-C) models, and find that the combination of constrained models yields an  $R_{OoS}^2$  of 0.626%, almost two times as large as the  $R_{OoS}^2$  obtained using an equal-weighted combination of unconstrained forecasts. Most notably, the gains in forecasting performance of the EW-C combination are highly robust to the subsample considered. After splitting the entire sample into two halves, we find that in the first subsample (1947-1981) the EW-C performs slightly better than EW-U combination, with  $R_{OoS}^2$  of 0.936% and 0.836% respectively. In the second subsample, the superior performance of EW-C over its unconstrained counterpart is even stronger, with an improvement in  $R_{OoS}^2$  of 0.454%. In terms of economic predictability, over the entire sample the CER gains of the equal-weighted combination methods improve from -2.1 basis points to 41.9 basis points after implementing our constraints. As for the subsample performance, we find that in the first subsample the EW-C leads to portfolios with a CER that is 19.4 bps higher than its unconstrained counterpart. As with the statistical predictability results, the superior performance of the EW-C is more impressive in the second subsample, with a CER gain of 43.3 basis points, whereas EW-U leads to a negative CER gain of -26.1 basis points. We also compare the forecasting performance of our EW-C approach against the constrained approaches of Campbell and Thompson (2008) and Pettenuzzo, Timmermann, and Valkanov (2014). Our results indicate that our EW-C model combination performs better than the equal-weighted combinations of either CT or PTV constrained univariate regressions, with gains that are particularly large in the second subsample.

The remainder of this paper is organized as follows. Section 2 introduces the econometric methodology we rely on to estimate and forecast the equity premium subject to our constraint on the predictor variable. Section 3 describes the data, and Section 4 presents results on both the statistical and economic predictability of our approach. Section 5 reports the results of a number of extensions and robustness checks, and Section 6 provides some concluding remarks.

<sup>&</sup>lt;sup>5</sup>As pointed out by Baumeister and Kilian (2015), forecast combination methods also provide insurance against possible model misspecification.

<sup>&</sup>lt;sup>6</sup>In particular, for the PTV approach we focus on their Sharpe-ratio constrained model.

# 2 Methodology

In this section, we introduce our predictor-constrained approach and show how to estimate and forecast the equity premium subject to this constraint.

### 2.1 Predictive regression

As is customary in the literature on stock return predictability, we model  $r_{\tau+1}$ , the stock returns in excess of the risk free rate at time  $\tau+1$ , as a linear function of a lagged predictor variable  $x_{\tau}$ :

$$r_{\tau+1} = \alpha + \beta x_{\tau} + \epsilon_{\tau+1}, \qquad \tau = 1, ..., t - 1$$
  

$$\epsilon_{\tau+1} \sim \mathcal{N}\left(0, \sigma_{\epsilon}^{2}\right)$$
(1)

We consider a sample of T observations and use the first M observations in the sample as a training period. Next, we forecast the excess stock returns over the remaining P observations, where P = T - M. Out-of-sample forecasts are easily obtained as

$$\hat{r}_{t+1|t} = \hat{\alpha}_t + \hat{\beta}_t x_t \tag{2}$$

where  $\hat{\alpha}_t$  and  $\hat{\beta}_t$  are the ordinary least squares (OLS) estimates of  $\alpha$  and  $\beta$  from (1), obtained by regressing  $\{r_{\tau+1}\}_{\tau=1}^{t-1}$  on a constant and  $\{x_{\tau}\}_{\tau=1}^{t-1}$ . Repeating this procedure for t=M,...,T-1 yields a time series of P one-step-ahead (unconstrained) forecasts, which we denote with  $\{\hat{r}_{t+1|t}\}_{t=M}^{T-1}$ .

# 2.2 Constrained predictive regression

We now introduce a simple truncation of the predictor variable  $x_{\tau}$  and define the following non-linear transformation of the original variable:

$$x_{\tau}^{*}(n) = \begin{cases} x_{\tau} & \text{if } x_{\tau} > \max(x_{\tau-1}, x_{\tau-2}, \cdots, x_{\tau-n}) & \text{or } x_{\tau} < \min(x_{\tau-1}, x_{\tau-2}, \cdots, x_{\tau-n}) \\ 0 & \text{otherwise} \end{cases}$$
(3)

where  $\tau = n + 1, ..., T - 1$  and n is the "look-back" period. Next, a constrained out-of-sample forecast can be easily generated as follows,

$$\hat{r}_{t+1|t}^*(n) = \hat{\alpha}_t^*(n) + \hat{\beta}_t^*(n)x_t^*(n) \tag{4}$$

where  $\hat{\alpha}_t^*(n)$  and  $\hat{\beta}_t^*(n)$  are the ordinary least squares (OLS) estimates of  $\alpha$  and  $\beta$  obtained by regressing  $\{r_{\tau+1}\}_{\tau=n+1}^{t-1}$  on a constant and  $\{x_{\tau}^*(n)\}_{\tau=n+1}^{t-1}$ . Repeating this procedure for t=M,...,T-1 yields a time series of P one-step-ahead (constrained) forecasts, which we denote with  $\{\hat{r}_{t+1}^*|_{t}(n)\}_{t=M}^{T-1}$ .

It is important to note that the newly defined predictor variable  $x_t^*(n)$  and, as a byproduct, the constrained forecast  $\hat{r}_{t+1|t}^*(n)$  depend on the number of periods used to compute
the past n-period highs and or lows. In particular, a larger n leads to more zeros in the time
series of the constrained predictor,  $\{x_{\tau}^*(n)\}_{\tau=n+1}^{T-1}$ . This is because as n increases, the value
of the unconstrained predictor  $x_{\tau}$  ( $\tau=n+1,...,T-1$ ) is compared to a longer sequence of
past observations. Our main results below are obtained by setting n=24. However, as a
robustness check we also experimented with alternative look-back periods, and include the
results of this sensitivity analysis in Section 5.

A drawback of the truncation approach in (2) and (3) is that the regression coefficients  $\hat{\alpha}_t^*(n)$  and  $\hat{\beta}_t^*(n)$  in (3) are trained only using the information contained in the "abnormal" periods (i.e., those instances where the predictor variable falls outside its past lows and highs), ignoring the potentially useful information coming from the "normal" times. To alleviate this concern, we propose using as our constrained forecast a simple average between  $\hat{r}_{t+1|t}$  and  $\hat{r}_{t+1|t}^*(n)$ ,

$$\hat{r}_{t+1|t}^{\sharp}(n) = 0.5\hat{r}_{t+1|t} + 0.5\hat{r}_{t+1|t}^{*}(n). \tag{5}$$

### 2.3 Forecasting with multivariate information

So far, we have followed much of the finance literature on return predictability and focused on (unconstrained and constrained) univariate prediction models. However, the prevailing view (Campbell and Thompson, 2008; Rapach, Strauss, and Zhou, 2010; Pettenuzzo, Timmermann, and Valkanov, 2014) is that univariate predictive regressions tend to generate less accurate and less stable forecasts than even the simplest benchmark models. In this section, we extend the previous analysis to a multivariate setting, in which instead of conditioning on a single predictor variable we exploit the information contained in N predictors. In particular, we combine the predictive information contained in the different variables by using forecast combination methods (see for example Rapach et al., 2010; Dangl

and Halling, 2012) that pool together the individual forecasts from the N univariate models. Our unconstrained forecast combination is given by:

$$\hat{r}_{t+1|t}^{comb} = \sum_{i=1}^{N} w_{i,t} \times \hat{r}_{i,t+1|t},\tag{6}$$

where  $\hat{r}_{i,t+1|t}$  denotes the unconstrained return forecast in (1) generated using predictor i at time t, t = M, ..., T - 1, and N is the total number of predictors. Along the same lines, our constrained forecast combination is given by:

$$\hat{r}_{t+1|t}^{\sharp,comb}(n) = \sum_{i=1}^{N} w_{i,t}^{\sharp}(n) \times \hat{r}_{i,t+1|t}^{\sharp}(n), \tag{7}$$

where  $\hat{r}_{i,t+1|t}^{\sharp}(n)$  denotes the constrained return forecast in (5) generated using predictor i at time t, i = 1, ..., N and t = M, ..., T - 1, while n is the length of the look-back period.

The choice of weights in pooling together the individual univariate forecasts (i.e,  $w_{i,t}$  and  $w_{i,t}^{\sharp}(n)$ ) is of key importance. A popular approach is to use the equal-weighted average of forecasts (i.e.,  $w_{i,t} = w_{i,t}^{\sharp}(n) = 1/N$ ). Recent empirical studies (Rapach, Strauss, and Zhou, 2010; Claeskens, Magnus, Vasnev, and Wang, 2016) have shown that in the context of return predictability, the equal-weighted combination method is a very competitive model, and often outperforms more sophisticated combination schemes. Accordingly, we rely on this simple combination method in our analysis. As a robustness check, in Section 5 we consider the sensitivity of our results to the use of more sophisticated model combination schemes.

# 3 Data

Our empirical analysis uses data on stock returns along with a set of 13 predictor variables originally analyzed by Welch and Goyal (2008) and subsequently extended to 2015 by the same authors.<sup>7</sup> Stock returns are computed from the S&P 500 index and include dividends. A short T-bill rate is subtracted from stock returns to capture excess returns. The data on these series are readily available on Amit Goyal's website.<sup>8</sup> Below we provide a list of the

<sup>&</sup>lt;sup>7</sup>A number of recent papers investigating predictability in stock returns and stock return volatility also use the same data. See Dangl and Halling, 2012; Neely, Rapach, Tu, and Zhou, 2014; Rapach, Strauss, and Zhou, 2010 for examples of the former, and Christiansen, Schmeling, and Schrimpf, 2012 as an example of the latter.

<sup>8</sup>We thank Amit Goyal for making his data available at http://www.hec.unil.ch/agoyal/.

predictors we relied on in our analysis, along with a short description:<sup>9</sup>

- Dividend-price ratio (DP): the log of dividends on the S&P 500 index minus the log of stock prices;
- Dividend yield (DY): the log of dividends on the S&P 500 index minus the log of lagged stock prices;
- Earning-price ratio (EP): the log of earnings on the S&P 500 index minus the log of stock prices;
- Dividend-Payout ratio (DE): the difference between DP and EP;
- Stock variance (SVAR): the sum of squared daily returns on the S&P 500 index;
- Book-to-market ratio (BM): the ratio between book value at the end of the previous year and the end-of-month market value of the Dow-Jones Industrial Average (DJIA) index:
- Net equity expansion (NITS): the ratio between the twelve-month moving sum of net equity issues by NYSE-listed stocks and the total end-of-year market capitalization of NYSE stocks;
- Treasury bill rate (TBL): the 3-month US Treasury bill rate from the secondary market rate;
- Long-term yield (LTY): the long-term government bond yield;
- Term Spread (TMS): the difference between LTY and TBL;
- Default yield spread (DFY): the difference between Moody's corporate BAA- and AAA- rated bond yields;
- Default return spread (DFR): the difference between the returns on long-term corporate and long-term government bonds;

 $<sup>^{9}</sup>$ Compared to the original work of Welch and Goyal (2008), we exclude the long-term bond return because it is not highly persistent.

• Inflation (INFL): the change in the Consumer Price Index (CPI) for all urban consumers.<sup>10</sup>

Our sample spans from January 1927 through December 2015, for a total of 1,068 monthly observations. The summary statistics for both predictors and stock returns are provided in Table 1. As can be easily seen by inspecting the Ljung-Box Q statistics Q(1) and Q(10) at the bottom of the table, all the predictors we focus on are highly persistent.

#### Table 1 about here

# 4 Equity premia forecasts

We now turn our attention to the out-of-sample performance of the constrained and unconstrained predictive regressions described in Section 2. As previously mentioned, we use a look-back period (n) of 24 months to implement the constraints on the predictor variables in (3). We also experiment with a look-back period of 12, 18, 30 and 36 months, and separately reports the results of this sensitivity analysis in Section 5. We initially estimate our unconstrained and constrained regression models over the period February 1930-December 1946, and use the estimated coefficients to forecast excess returns for January 1947. We next include January 1947 in the estimation sample, giving a sample of February 1930-January 1947, and use the corresponding estimates to predict excess returns for February 1947. We proceed in this recursive fashion until we reach the last observation in the sample, producing a time series of unconstrained and constrained one-step-ahead forecasts spanning the period from January 1947 to December 2015.

Figure 1 plots the time evolution of the unconstrained and constrained equity premia forecasts over the full forecast evaluation period, 1947-2015. For completeness, we also include in the figure the forecasts obtained using a no-predictability benchmark,  $\bar{r}_{t+1|t} = (1/(t-n)) \sum_{\tau=n+1}^{t} r_{\tau}$ . In the interest of space we focus our discussion here on three predictor variables, namely DP, TBL, and LTY. In all three cases, our constrained forecasts  $\hat{r}_{t+1|t}^{\sharp}(n)$  display a marked switching pattern whose timing is dictated by the level

 $<sup>^{10}</sup>$ To account for the delayed release of the CPI index, we lag the inflation one extra month.

<sup>&</sup>lt;sup>11</sup>To keep our main results consistent with the robustness analysis presented later in Section 5, we exclude the first 36 months of data from the estimation sample, and begin estimating all our models in 1930.

of the predictor variable, and in particular whether or not it falls within its past n-month high and low.

# Figure 1 about here

# 4.1 Statistical Predictability

We now turn to evaluating the predictive accuracy of the constrained and unconstrained predictive regressions. As is customary in the literature, the predictive performance of each model is evaluated against the no-predictability forecasts,  $\{\bar{r}_{t+1|t}\}_{t=M}^{T-1}$ . We rely on the out-of-sample  $R^2$   $(R^2_{OoS})$ , which is defined as the percent reduction in the mean squared predictive error (MSPE) of the model of interest  $(MSPE_{model})$ , relative to the MSPE of the benchmark model  $(MSPE_{bench})$ , i.e.

$$R_{OoS}^2 = 1 - \frac{MSPE_{model}}{MSPE_{bench}} \tag{8}$$

where  $MSPE_{model} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_{\tau} - forc_{\tau|\tau-1})^2$  with  $forc_{\tau|\tau-1}$  denoting the equity premia forecast produced at time  $\tau-1$  by either the unconstrained or constrained univariate model (or model combination), and  $MSPE_{bench} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_{\tau} - \bar{r}_{\tau|\tau-1})^2$ . A positive  $R_{OoS}^2$  implies that the model of interest generates more accurate forecasts than the benchmark model. We also evaluate the statistical significance of the changes in  $R_{OoS}^2$  using the Clark and West (2007) (CW hereafter) test statistic, which allows us to test the null hypothesis that the benchmark forecast MSPE is less than or equal to the competing forecast MSPE against the one-sided (upper-tail) alternative hypothesis that the benchmark forecast MSPE is greater than the competing forecast MSPE. This statistic is a correction of the Diebold and Mariano (1995) statistic and is demonstrated to be more suitable for nested models.  $^{12}$ 

### 4.1.1 Results

The second and third columns of Table 2 reports the  $R_{OoS}^2$  of the unconstrained and constrained methods (that is  $\hat{r}_{t+1|t}$  and  $\hat{r}_{t+1|t}^{\sharp}(n)$ ), as well as their equal-weighted combinations ( $\hat{r}_{t+1|t}^{comb}$  and  $\hat{r}_{t+1|t}^{\sharp,comb}(n)$ ). Starting with the forecasting performance of the univariate models, we notice that constraining the predictor variables as in (5) leads to

<sup>&</sup>lt;sup>12</sup>As discussed in Diebold (2015), the p-values resulting from such test statistics should be interpreted as a measure of the relative accuracy of the sequence of forecasts.

 $R_{OoS}^2$  improvements in 11 out of 13 cases. We also find that the unconstrained equal-weighted forecast combination (EW-U) generates a positive  $R_{OoS}^2$  of 0.334%, statistically significant at the 5% level. This result is consistent with Rapach, Strauss, and Zhou (2010) and Avramov (2002). In contrast, the constrained equal-weighted forecast combination (EW-C) in (7) yields an  $R_{OoS}^2$  value of 0.626%, almost two times as large, and statistically significant at the 1% level.

### Table 2 about here

The last four columns of Table 2 report the forecasting performance of the unconstrained and constrained predictive regressions during two subsample periods, 1947–1981 and 1982–2015, obtained by splitting the full out-of-sample period into two halves. In the first subsample, our constrained method leads to an improved  $R_{OoS}^2$  for seven of the 13 variables (the forecast improvements are statistically significant for six of the seven cases). During the same period, the  $R_{OoS}^2$  of the equal-weighted combination method also increases, going from 0.836% to 0.936% when the constraints are implemented. In the second subsample, the constrained approach improves the forecast accuracy for 11 of the 13 variables. Interestingly, the  $R_{OoS}^2$  of the EW-U combination method is now negative, implying that in this subsample the EW-U method fails to beat the historical average benchmark. Most notably, during the same period the EW-C combination approach yields an  $R_{OoS}^2$  of 0.363%, statistically significant at 5% level.

To investigate the robustness of the constrained EW-C forecasts over time, we compute the Cumulative Sum of Squared prediction Error Difference (CSSED)

$$CSSED_{model,t} = \sum_{\tau=M+1}^{t} \left( e_{bench,\tau}^2 - e_{model,\tau}^2 \right) \qquad t = M+1, ..., T$$

$$(9)$$

where model denotes either the unconstrained EW-U or constrained EW-C model combination, while  $e_{model,\tau}$  and  $e_{bench,\tau}$  denote the forecast error from time  $\tau$  forecast associated with either the EW-C, EW-U, or benchmark model. Note that an increase from  $CSSED_{model,t-1}$  to  $CSSED_{model,t}$  indicates that relative to the benchmark no-predictability model, the EW-U or EW-C model predicts more accurately at observation

t. Figure 2 plots the CSSEDs over time for both the EW-U and EW-C models. As can be seen from the figure, the relative predictive ability of the EW-U weakens considerably during the 1957–1969 and 1990–2000 periods. In sharp contrast, the EW-C displays a generally increasing pattern in the CSSEDs, a strong indication that the gains in forecast accuracy of the EW-C model reported in Table 2 are not the result of any specific and short-lived episode, but rather are built gradually over the entire out-of-sample period.

### Figure 2 about here

### 4.2 Economic predictability

So far we have focused on the statistical performance of the unconstrained and constrained equity premia forecasts. We now turn to evaluating the economic significance of these predictions by considering the optimal portfolio decision of an investor who uses the return forecasts to guide her portfolio choices. We focus on an investor who allocates her wealth between a risky asset, which we proxy using the S&P 500 index, and a risk-free asset. The investor's wealth at time t+1 is,

$$W_{t+1} = (1 - \omega_t) \exp(r_t^f) + \omega_t \exp(r_t^f + r_{t+1})$$
(10)

where  $\omega_t$  is the share of wealth assigned to the risky asset, and  $r_{t+1}$  is the stock return in excess of the risk-free rate  $r_t^f$ , both continuously compounded.

At time t, the investor solves the following optimal asset allocation problem:

$$\widehat{\omega}_t = \arg\max_{\omega_t} E_t[U(W_{t+1})] \tag{11}$$

where  $E_t[\cdot]$  denotes the conditional expectation based on the investor's information set at time t. We follow Pettenuzzo, Timmermann, and Valkanov (2014) and focus on constant relative risk aversion (CRRA) preferences,

$$U(W_{t+1}) = \frac{W_{t+1}^{1-\gamma}}{1-\gamma},\tag{12}$$

where  $\gamma$  indicates the coefficient of risk aversion. We next rely on the Campbell and Viceira (2001) log-linearization, and obtain approximate optimal portfolio weights that are equal to

$$\widehat{\omega}_t \approx \frac{forc_{t+1|t} + \widehat{\sigma}_{t+1|t}^2/2}{\gamma \widehat{\sigma}_{t+1|t}^2}$$
(13)

where  $\hat{\sigma}_{t+1|t}^2$  denotes the time t+1 variance forecast for the log excess returns made at time t, and  $forc_{t+1|t}$  stands for the equity premia forecast produced at time t by either the unconstrained or constrained univariate model (or model combination).<sup>13</sup> We restrict the optimal allocation to stocks between 0 and 1.5 to preclude short selling and excessive leverage, and set the risk aversion coefficient to  $\gamma = 3.^{14}$  Finally, we compute the certainty equivalent return (CER) as:

$$CER = \left[ (1 - \gamma)(T - M)^{-1} \sum_{t=M+1}^{T} U(\widehat{W}_t) \right]^{1/(1-\gamma)} - 1, \tag{14}$$

where  $(T-M)^{-1} \sum_{t=M+1}^T U(\widehat{W}_t)$  denotes the average realized utility.

### 4.2.1 Results

Table 3 reports the differences (in percentage points) between the annualized CER of an investor who relies on the forecasts obtained using the model of interest (i.e., either the unconstrained or constrained univariate predictive regression or model combination) and the annualized CER of an investor who uses the benchmark historical average forecasts. We can interpret these values as the annualized performance fee that an investor would be willing to pay to have access to the return forecasts generated using the model of interest instead of the benchmark forecasts. The numbers reported in the second and third columns of Table 3 denote the annualized CER gains based on the full evaluation period, 1947–2015. Starting with the individual models, we see that our constrained approach increases the CERs for 11 out of 13 predictors. The CER gains associated with seven of the constrained univariate regressions are all positive, supporting the existence of economic predictability. Moving on to the equal-weighted combination, we find that averaging over the univariate unconstrained and constrained regressions yields annualized CER gains of -2.1 bps in the case of the EW-U

The results of this alternative approach are qualitatively very similar to those reported here.

<sup>&</sup>lt;sup>14</sup>Our findings are robust to the choice of alternative bounds on the portfolio weights and risk aversion coefficients. We experimented with setting the bounds to [0, 0.99], as suggested by Cenesizoglu and Timmermann (2012), and found the results to be qualitatively very similar to those reported here. We also considered different risk aversion coefficients, and summarize the results of this sensitivity in Section 5.

model, and 41.9 bps in the case of the EW-C model. Hence, as was the case with the univariate regressions, imposing our constraints in the multivariate forecast combination leads to sizable gains.

The remaining columns of Table 3 show the annualized CER gains separately for the two subsamples, i.e. 1947–1981 and 1982–2015. In the first subsample, imposing our constraints in the univariate predictive regressions leads to an increase in CERs for eight of the 13 predictors. With respect to the model combinations, the annualized CERs go from 21.2 basis points (bps) in the case of the EW-U method to 40.6 bps in the case of the EW-C approach. That is, incorporating our constraints in the equal-weighted model combination generates a CER gain of 19.4 bps. In the second subsample, applying the constraints to the univariate predictive regressions produces improvements in CERs for 11 of the 13 predictors. Notably, in the second subsample the EW-U model combination fails to improve over the no-predictability benchmark, as indicated by its negative CER figure. In contrast, the CER of the EW-C combination scheme remains positive, with a value of 43.3 basis points, which is higher than what the same model attained in the first subsample. Overall, these results provide strong evidence in support of our constrained approach, and particularly so for the constrained forecast combination method.

#### Table 3 about here

# 4.3 A comparison with existing constrained approaches

The results we have presented thus far show that the simple constraint we introduced in Section 2 helps boost the forecasting performance of both the univariate predictive regressions and the equal-weighted model combination. We now compare the performance of our method to some of the most popular constrained methods that have been proposed in the literature. In particular, we focus on the approaches of Campbell and Thompson (2008) and Pettenuzzo, Timmermann, and Valkanov (2014). Campbell and Thompson (2008) consider simple predictive regressions of the type in (1) and show that either constraining the return forecasts to be non-negative or forcing the regression coefficients to have the theoretically expected signs leads to clear out-of-sample improvements. Similarly, Pettenuzzo, Timmermann, and Valkanov (2014) start with the regression model in (1) but

modify it to allow for time-varying volatility, and propose a Bayesian approach to impose either non-negativity in the equity premia or bounds on the conditional Sharpe ratio. They find that both constraints lead to improved forecasts, and that the Sharpe ratio constraint is particularly successful at improving portfolio performance.

Table 4 presents the results of the comparisons, focusing on the statistical evidence of predictability, as summarized by the  $R_{OoS}^2$  measure. Along with the results from our approach, which we denote with PPW in the table, we include two versions of the Campbell and Thompson (2008) constrained approach, CT1 and CT2. The former imposes a non-negativity constraint on the equity premium forecasts, and the latter applies a sign restriction to the slope coefficient of each univariate predictive regression. We also include the Sharpe ratio constraint approach of Pettenuzzo, Timmermann, and Valkanov (2014), (PTV), which bounds the in-sample annualized Sharpe ratios between zero and one. <sup>15</sup> As can be seen from the table, PPW and PTV are the two best performing methods, followed by CT1 and CT2. With respect to the univariate models, the relative ranking between PPW and PTV depends on the predictor variable. With respect to the equal-weighted model combinations at the bottom of the table, we find that overall the EW-C approach appears to be far more stable than all the alternatives considered, including PTV. In the full sample, CT1 and PTV methods achieve an  $R_{OoS}^2$  of 0.350% and 0.463%, respectively. While both methods improve upon the original EW-U combination method ( $R_{OoS}^2$  of 0.334%), they both fall below the EW-C method, which delivers an  $R_{OoS}^2$  of 0.626%. As for the subsample analysis, we find that in the first subsample PTV method achieves an  $R_{OoS}^2$  of 0.901%, almost as high as our EW-C approach. However, the performance of all three alternative constrained approaches, including the PTV method, deteriorates significantly in the second

$$0 \le \frac{\sqrt{12} \left( \widehat{\alpha}_t + \widehat{\beta}_t x_\tau \right)}{\widehat{\sigma}_\tau} \le 1, \quad \tau = 1, ...., t$$

and where  $\hat{\sigma}_{\tau}$  is the empirical estimate of  $\sigma_{\tau}$ , obtained as in Johnson (2017, Section 2).

<sup>&</sup>lt;sup>15</sup>Note that in addition to enforcing bounds on the annualized Sharpe ratio, the approach of Pettenuzzo, Timmermann, and Valkanov (2014) imposes Bayesian shrinkage on the regression coefficients in (1). To better isolate the effect of the Sharpe ratio constraint and make the results more comparable to the other methods presented here, we have opted for estimating the PTV constrained model using the approach of Johnson (2017). That is, for each of the N predictors considered we estimate the coefficients  $\hat{\alpha}_t$  and  $\hat{\beta}_t$  in (1) via constrained least squares, subject to the restriction that

half of the sample. These results are in line with the findings in Pettenuzzo et al. (2014, Table 6). In stark contrast, the EW-C approach delivers results that, across the two subsamples, are superior and remarkably more robust than all alternative approaches.

### Table 4 about here

Table 5 shows the portfolio performance of the three alternative constrained approaches, and once again compares them to our proposed method. Overall, we find results that are highly consistent with the  $R_{OoS}^2$  reported in Table 4. For the univariate models, PTV and PPW perform better than CT1 and CT2. For the equal-weighted combination methods, CT1 and CT2 display CER gains of approximatively 7.5 bps, while the PTV method delivers significantly higher CER gains, 23.3 bps. Most notably, both CT and PTV methods produce CER gains that are significantly below those achieved by the PPW approach, 41.9 bps. As for the subsample analysis, we find that in the first subsample, PTV produces CER gains that are slightly higher than those obtained by our method. However, in the second subsample the constrained approach generates CERs that are virtually equal to zero while our proposed approach is still able to deliver sizable CER gains, near the levels we observed in the full sample.

### Table 5 about here

## 5 Robustness and Extensions

In this section, we present a number of extensions and robustness checks to validate the main results presented in Section 4. We begin by evaluating the forecasting performance of our predictor-constrained approach over the business cycle. We next experiment with the use of alternative look-back periods (n) in the calculation of the constrained forecasts in (5) and (7), different risk aversion degrees, and various weighting schemes for the forecast combinations.

### 5.1 Forecasting performance over the business cycle

To better understand the sources of the statistical and economic predictability we have uncovered, we separately calculate the  $R_{OoS}^2$  during business recession and expansion

periods as follows:

$$R_{OoS,c}^{2} = 1 - \frac{\sum_{\tau=M+1}^{T} I_{\tau}^{c} (r_{\tau} - forc_{\tau|\tau-1})^{2}}{\sum_{\tau=M+1}^{T} I_{\tau}^{c} (r_{\tau} - \bar{r}_{\tau|\tau-1})^{2}}, \quad \text{for} \quad c = EXP, REC,$$
 (15)

where  $I_{\tau}^{EXP}$  ( $I_{\tau}^{REC}$ ) is an indicator variable set equal to one when month  $\tau$  is within an NBERdated expansion (recession), and  $forc_{\tau|\tau-1}$  denotes the equity premia forecast produced at time  $\tau-1$  by either the unconstrained or constrained univariate model (or model combination).

Table 6 reports the results of this analysis. Consistent with the results in Rapach, Strauss, and Zhou (2010) and Henkel, Martin, and Nardari (2011), the unconstrained return prediction models perform better relative to the no-predictability benchmark during recessions than they do during expansions. This is true for both the univariate predictive regressions and the equal-weighted model combinations. Interestingly, we notice the same pattern in the constrained methods. During recessions, our constrained approach leads to better forecasts for nine of the 13 variables, with the largest improvements in  $R_{OoS}^2$  occurring for the LTY ( $R_{OoS}^2$  goes from 0.153% to 4.023%) and SVAR ( $R_{OoS}^2$  goes from -0.453% to 2.588%) predictors. As for the forecast combination method, the  $R_{OoS}^2$  improves from 0.680% in the EW-U model to 0.943% in the EW-C model. The results during expansions are weaker, but still notable. We find that our constrained approach leads to a higher  $R_{OoS}^2$  for ten of the 13 predictors. However, the  $R_{OoS}^2$  is positive only in three of these seven cases. The best results are once again coming from the constrained model combination method, with the the  $R_{OoS}^2$  going from 0.208% in the EW-U model to to 0.510% in the EW-C model.

### Table 6 about here

Table 7 reports the CER results separately for recessions and expansions. During recessions, our constrained approach leads to higher CERs for nine of the 13 predictors, whereas during expansions the improvements in CER from imposing our constraint occur for ten out of the 13 univariate regressions. With respect to the model combination methods, imposing our constraint in the regressions entering the model combinations generates strong positive results during both recessions and expansions. In particular, during recessions the CER of the model combination goes from 88.2 to 164.9 bps, and it goes from -17.9 to 20.4 bps during expansions.

#### Table 7 about here

# 5.2 Alternative look-back periods

As we discussed in Section 2, our constrained method works by setting the value of the predictor to zero in all instances in which its value falls between its past n-month high and low. In our main analysis we have set the value of the look-back period n to 24 months. We now examine the robustness of our results to this choice. We begin by noting that values of n that are very small tend not to work too well in practice. Our intuition for this result is that when n is small, the constraint in (3) is unable to separate the random noise from the true instances in which unexpected information dominates the changes in the predictors. We next investigate four cases, namely n = 12, n = 18, n = 30 and n = 36.

Table 8 shows the out-of-sample  $R^2$  of the constrained univariate regressions and EW-C model combination for these four alternative look-back periods. Beginning with the full sample results, we find that for most of the predictors, our constrained method continues to improve over the unconstrained models. This is true for ten out of 13 variables using each of these alternative look-back periods under consideration. Moving on to the subsample results in Panels B and C, by and large these tend to confirm the pattern found with the full sample. Most notably, in the first subsample the EW-C forecasts fail to improve over the unrestricted EW-U model combination when n = 12, but this is not the case when n = 18, 30 and 36. In the second subsample, the superior performance of EW-C over EW-U combination is found for all the choices of look-back periods and the  $R^2_{OoS}$  values are close for various values of n.

#### Table 8 about here

Table 9 reports the out-of-sample CER gains of the constrained univariate regressions and EW-C model combination for the four alternative look-back periods. Overall, the CER results are in line with the  $R_{OoS}^2$  results reported in Table 8. In the full sample the CER of the individual models are always higher than their unconstrained counterparts, regardless of the choice of the look-back period. In addition, the EW-C model improves on the unconstrained EW-U model in all cases. Panels B and C of Table 9 confirm that these results are very robust across the two subsamples.

#### Table 9 about here

### 5.3 Alternative risk aversion choices

Our main analysis of the economic value of equity premium forecasts in Section 4 assumed a coefficient of relative risk aversion of  $\gamma = 3$ . To explore the sensitivity of our results to this value, we consider lower ( $\gamma = 2$ ) and higher ( $\gamma = 4$ ,  $\gamma = 6$ ) values of this parameter. Results for the EW-C model combination method are shown in Table 10.

Beginning with the full sample results in Panel A, we find that the portfolios constructed using the predictions from the EW-C model generate a large and positive CER for each of the three  $\gamma$  values under consideration. In addition to improving over the benchmark no-predictability model in all three cases, the EW-C portfolios achieve annualized CER gains that are between 24.3 and 59.5 bps higher than the corresponding unconstrained EW-U model results. Results from the subsample analysis in Panels B and C are largely in line with the full sample findings from Panel A. In all cases, the EW-C generates CER gains that are higher than those given by the EW-U model combination method.

#### Table 10 about here

### 5.4 Alternative model combination schemes

As discussed in Section 2 and Section 4, our multivariate results have been based on equal-weighted combinations of (unconstrained and constrained) univariate predictive regressions. We now explore the sensitivity of our results to this modeling choice by considering four alternative model combination schemes. Our first alternative is a trimmed-mean model combination, which is obtained by recursively dropping the worst performing model (as determined by the recursively calculated MSPEs), and applying equal weights to the surviving N-1 models. The second and third methods are based on the Stock and Watson (2004) discounted MSPE (DMSPE) combination scheme, which dictates that the weight of each model in the combination (6) or (7) is given by either

$$w_{i,t} = \frac{\phi_{i,t}^{-1}}{\sum_{j=1}^{N} \phi_{j,t}^{-1}},\tag{16}$$

in the case of the unconstrained model combination in (6), or

$$w_{i,t}^{\sharp}(n) = \frac{\left[\phi_{i,t}^{\sharp}(n)\right]^{-1}}{\sum_{j=1}^{N} \left[\phi_{j,t}^{\sharp}(n)\right]^{-1}},\tag{17}$$

in the case of the constrained model combination in (7). Here, N is the number of individual models,  $\phi_{j,t} = \sum_{\tau=M+1}^{t} \delta^{t-\tau} (r_{\tau} - \hat{r}_{j,\tau|\tau-1})^2$ , and  $\phi_{j,t}^{\sharp}(n) = \sum_{\tau=M+1}^{t} \delta^{t-\tau} (r_{\tau} - \hat{r}_{j,\tau|\tau-1}^{\sharp}(n))^2$ . We separately consider  $\delta = 1$  and  $\delta = 0.9$ ; in the former case we are treating all past observations equally when computing the MSFE, and in the latter case we are placing a greater emphasis on the most recent forecasting performance. Our last combination scheme follows Yang (2004), who shows that because of the large estimation uncertainty surrounding the combination weights, simple linear combinations of point forecasts can sometime lead to suboptimal performances. Yang introduces a non-linear combination scheme, with weights that, in the case of the unconstrained model combination in (6), are given by:  $^{16}$ 

$$\omega_{i,t} = \frac{\pi_i \exp\left(-\lambda \sum_{\tau=M+1}^t (r_\tau - \hat{r}_{i,\tau|\tau-1})^2\right)}{\sum_{j=1}^N \pi_j \exp\left(-\lambda \sum_{\tau=M+1}^t (r_\tau - \hat{r}_{j,\tau|\tau-1})^2\right)}.$$
(18)

We follow Yang (2004) and set  $\pi_i = \lambda = 1$ .

Table 11 shows the out-of-sample  $R^2$  for each of the four alternative combinations for both the full sample and the two sub-periods. With respect to the full sample results, we find values of  $R_{OoS}^2$  for the unconstrained combination methods that range between 0.330% and 0.432%, depending on the weighting schemes used in the combination. Also, we find that regardless of the combination scheme adopted, the  $R_{OoS}^2$  values from the constrained model combinations in the third column of the table are always above their unconstrained counterparts from the second column, and higher than the equal-weighted combination results reported in Table 2. Moving on to Panels B and C, we see that the superior forecasting performance of the constrained forecast combinations mainly come from the second subsample, whereas in the first subsample unconstrained and constrained forecast combinations yield similar results.

### Table 11 about here

<sup>&</sup>lt;sup>16</sup>The formula for the constrained model combination in (7) can be derived analogously.

# 6 Conclusions

We have proposed a novel methodology for implementing economically motivated constraints in predictive regressions of aggregate stock returns. Unlike other approaches in the literature, our restrictions are implemented directly on the predictor variables, setting their value to zero whenever it falls within the predictor's rolling past 24-month high and low. Out-of-sample evidence indicates that our approach yields very accurate forecasts for a large host of macroeconomic and financial predictors of stock returns. We also show that a simple equal-weighted combination of the point forecasts obtained from the univariate constrained regressions further improves the out-of-sample predictability of our method, both in statistical and economic terms. Subsample analysis and a large battery of robustness checks reveal that our method is significantly more stable and robust than the existing constrained approaches in the literature.

# References

- Aharony, J. and I. Swary (1980). Quarterly dividend and earnings announcements and stockholders' returns: An empirical analysis. *The Journal of Finance* 35(1), 1–12.
- Avramov, D. (2002). Stock return predictability and model uncertainty. *Journal of Financial Economics* 64(3), 423–458.
- Ball, R. and P. Brown (1968). An empirical evaluation of accounting income numbers. *Journal* of accounting research 6(2), 159–178.
- Barber, B. M. and T. Odean (2008). All that glitters: The effect of attention and news on the buying behavior of individual and institutional investors. *The Review of Financial Studies* 21(2), 785–818.
- Barber, B. M., T. Odean, and N. Zhu (2009). Systematic noise. *Journal of Financial Markets* 12(4), 547–569.
- Baumeister, C. and L. Kilian (2015). Forecasting the real price of oil in a changing world: a forecast combination approach. *Journal of Business & Economic Statistics* 33(3), 338–351.

- Bollerslev, T., G. Tauchen, and H. Zhou (2009). Expected stock returns and variance risk premia. *The Review of Financial Studies* 22(11), 4463–4492.
- Brogaard, J. and A. Detzel (2015). The asset-pricing implications of government economic policy uncertainty. *Management Science* 61(1), 3–18.
- Campbell, J. Y. and R. J. Shiller (1988). The dividend-price ratio and expectations of future dividends and discount factors. *The Review of Financial Studies* 1(3), 195–228.
- Campbell, J. Y. and S. B. Thompson (2008). Predicting excess stock returns out of sample: Can anything beat the historical average? *The Review of Financial Studies* 21(4), 1509–1531.
- Campbell, J. Y. and L. M. Viceira (2001). Who should buy long-term bonds. *American Economic Review* 91(2), 99C–127.
- Cenesizoglu, T. and A. Timmermann (2012). Do return prediction models add economic value. *Journal of Banking & Finance* 11(36), 2974–2987.
- Chib, S. and X. Zeng (2016). Bayesian strategy for improved forecasts of the equity premium.

  Technical report, Working paper, Olin Business School, Washington University.
- Christiansen, C., M. Schmeling, and A. Schrimpf (2012). A comprehensive look at financial volatility prediction by economic variables. *Journal of Applied Econometrics* 27(6), 956–977.
- Claeskens, G., J. R. Magnus, A. L. Vasnev, and W. Wang (2016). The forecast combination puzzle: A simple theoretical explanation. *International Journal of Forecasting* 32(3), 754–762.
- Clark, T. E. and K. D. West (2007). Approximately normal tests for equal predictive accuracy in nested models. *Journal of econometrics* 138(1), 291–311.
- Colacito, R., E. Ghysels, J. Meng, and W. Siwasarit (2016). Skewness in expected macro fundamentals and the predictability of equity returns: Evidence and theory. The Review of Financial Studies 29(8), 2069–2109.

- Da, Z., J. Engelberg, and P. Gao (2011). In search of attention. *The Journal of Finance* 66(5), 1461–1499.
- Dangl, T. and M. Halling (2012). Predictive regressions with time-varying coefficients. *Journal* of Financial Economics 106(1), 157–181.
- Della Vigna, S. and J. M. Pollet (2009). Investor inattention and friday earnings announcements. *The Journal of Finance* 64(2), 709–749.
- Dew-Becker, I., S. Giglio, A. Le, and M. Rodriguez (2017). The price of variance risk. *Journal of Financial Economics* 123(2), 225–250.
- Diebold, F. X. (2015). Comparing predictive accuracy, twenty years later: A personal perspective on the use and abuse of Diebold? Mariano tests. *Journal of Business & Economic Statistics* 33(1), 1–1.
- Diebold, F. X. and R. S. Mariano (1995). Comparing predictive accuracy. *Journal of Business & Economic Statistics* 13(3), 253–263.
- Driesprong, G., B. Jacobsen, and B. Maat (2008). Striking oil: Another puzzle? *Journal of Financial Economics* 89(2), 307–327.
- Fama, E. F. and K. R. French (1988). Dividend yields and expected stock returns. *Journal* of financial economics 22(1), 3–25.
- Fama, E. F. and K. R. French (1989). Business conditions and expected returns on stocks and bonds. *Journal of financial economics* 25(1), 23–49.
- Ferson, W. E. and C. R. Harvey (1993). The risk and predictability of international equity returns. *Review of financial Studies* 6(3), 527–566.
- George, T. J. and C.-Y. Hwang (2004). The 52-week high and momentum investing. *The Journal of Finance* 59(5), 2145–2176.
- Hamilton, J. D. (1996). This is what happened to the oil price-macroeconomy relationship.

  Journal of Monetary Economics 38(2), 215–220.

- Hamilton, J. D. (2011). Nonlinearities and the macroeconomic effects of oil prices.

  Macroeconomic dynamics 15(S3), 364–378.
- Henkel, S. J., J. S. Martin, and F. Nardari (2011). Time-varying short-horizon predictability.

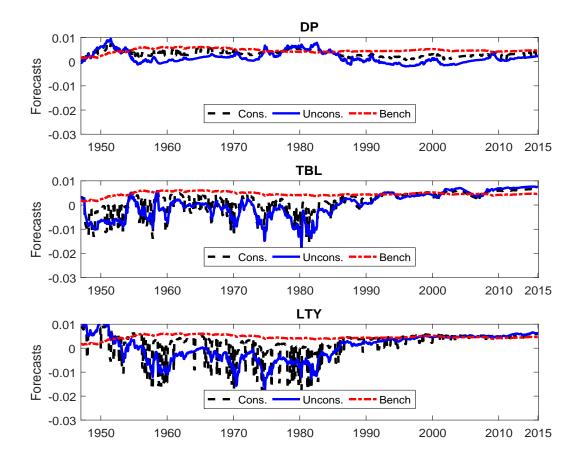
  Journal of Financial Economics 99(3), 560 580.
- Huang, D., F. Jiang, J. Tu, and G. Zhou (2014). Investor sentiment aligned: A powerful predictor of stock returns. *The Review of Financial Studies* 28(3), 791–837.
- Huberman, G. and T. Regev (2001). Contagious speculation and a cure for cancer: A nonevent that made stock prices soar. The Journal of Finance 56(1), 387-396.
- Johnson, T. L. (2017). Return predictability revisited using weighted least squares. Working paper, University of Texas at Austin.
- Jordan, S. J., A. J. Vivian, and M. E. Wohar (2014). Forecasting returns: new european evidence. *Journal of Empirical Finance* 26, 76–95.
- Keim, D. B. and R. F. Stambaugh (1986). Predicting returns in the stock and bond markets.

  Journal of financial Economics 17(2), 357–390.
- Lawrenz, J. and J. Zorn (2017). Predicting international stock returns with conditional priceto-fundamental ratios. *Journal of Empirical Finance* 43, 159–184.
- Lettau, M. and S. Ludvigson (2001). Consumption, aggregate wealth, and expected stock returns. *The Journal of Finance* 56(3), 815–849.
- Li, J. and J. Yu (2012). Investor attention, psychological anchors, and stock return predictability. *Journal of Financial Economics* 104(2), 401–419.
- Manela, A. and A. Moreira (2017). News implied volatility and disaster concerns. *Journal of Financial Economics* 123(1), 137–162.
- Neely, C. J., D. E. Rapach, J. Tu, and G. Zhou (2014). Forecasting the equity risk premium: the role of technical indicators. *Management Science* 60(7), 1772–1791.

- Pástor, L. and R. F. Stambaugh (2009). Predictive systems: Living with imperfect predictors. The Journal of Finance 64(4), 1583–1628.
- Pástor, L. and R. F. Stambaugh (2012). Are stocks really less volatile in the long run? The Journal of Finance 67(2), 431–478.
- Pettenuzzo, D., A. Timmermann, and R. Valkanov (2014). Forecasting stock returns under economic constraints. *Journal of Financial Economics* 114(3), 517–553.
- Pontiff, J. and L. D. Schall (1998). Book-to-market ratios as predictors of market returns.

  Journal of Financial Economics 49(2), 141–160.
- Rapach, D. E., M. C. Ringgenberg, and G. Zhou (2016). Short interest and aggregate stock returns. *Journal of Financial Economics* 121(1), 46–65.
- Rapach, D. E., J. K. Strauss, and G. Zhou (2010). Out-of-sample equity premium prediction: Combination forecasts and links to the real economy. *The Review of Financial Studies* 23(2), 821–862.
- Stambaugh, R. F. (1999). Predictive regressions. Journal of Financial Economics 54(3), 375 421.
- Stock, J. H. and M. W. Watson (2004). Combination forecasts of output growth in a seven-country data set. *Journal of Forecasting* 23(6), 405–430.
- Welch, I. and A. Goyal (2008). A comprehensive look at the empirical performance of equity premium prediction. *The Review of Financial Studies* 21(4), 1455–1508.
- Yang, Y. (2004). Combining forecasting procedures: some theoretical results. *Econometric Theory* 20(1), 176–222.

Figure 1. Excess return forecasts from univariate models



Notes: This figure plots the time evolution of the unconstrained (solid blue lines) and constrained (dashed black lines) equity premia forecasts as well as the benchmark forecasts (dashed-dotted red lines) over the whole forecast evaluation period, 1947-2015. The forecasts are obtained using one of the 13 predictors considered in this study, and as described in (2) and (5), respectively. The three panels show results based on the Dividend-price ratio (DP), Treasury bill rate (TBL), and Long-term yield (LTY), respectively.

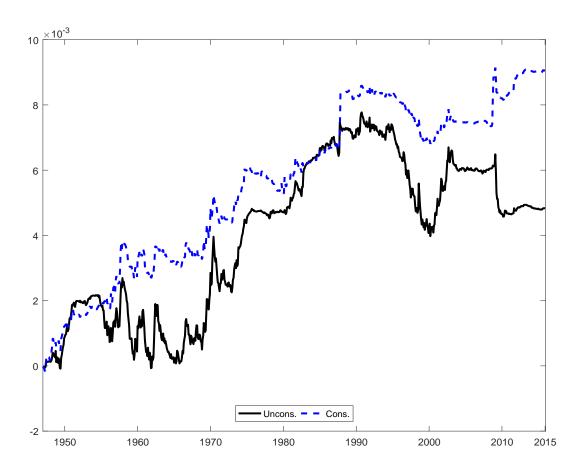


Figure 2. Cumulative Sum of Squared Prediction Error Differences

Notes: This figure plots the Cumulative Sum of Squared prediction Error Difference (CSSED) over time for both the constrained and unconstrained equal-weighted forecast combinations, relative to the benchmark no predictability model. We compute the CSSED as follows:

$$CSSED_{model,t} = \sum_{\tau=M+1}^{t} \left( e_{bench,\tau}^2 - e_{model,\tau}^2 \right) \qquad t = M+1,...,T$$

where model denotes either the unconstrained or constrained model combination, while  $e_{model,\tau}$  and  $e_{bench,\tau}$  denote time  $\tau$  forecast errors associated with either the model combination or the benchmark model.

Table 1. Summary statistics of predictive variables and stock returns

	Mean	Median	Max.	Min.	Std.Dev.	Skewness	Kurtosis	JB	Q(1)	Q(10)
RET	0.005	0.009	0.346	-0.339	0.055	-0.422	10.753	< 0.001	0.005	< 0.001
DP	-3.361	-3.332	-1.873	-4.524	0.460	-0.268	2.738	0.002	< 0.001	< 0.001
DY	-3.357	-3.328	-1.913	-4.531	0.458	-0.297	2.719	< 0.001	< 0.001	< 0.001
EP	-2.728	-2.783	-1.775	-4.836	0.416	-0.661	5.781	< 0.001	< 0.001	< 0.001
DE	-0.633	-0.624	1.380	-1.244	0.332	1.484	8.806	< 0.001	< 0.001	< 0.001
SVAR	0.003	0.001	0.071	0.000	0.006	5.728	45.549	< 0.001	< 0.001	< 0.001
$_{\mathrm{BM}}$	0.575	0.550	2.028	0.121	0.265	0.755	4.474	< 0.001	< 0.001	< 0.001
NTIS	0.018	0.018	0.177	-0.058	0.025	1.746	11.779	< 0.001	< 0.001	< 0.001
TBL	0.035	0.030	0.163	0.000	0.031	1.052	4.232	< 0.001	< 0.001	< 0.001
LTY	0.052	0.043	0.148	0.018	0.028	1.063	3.555	< 0.001	< 0.001	< 0.001
TMS	0.017	0.018	0.046	-0.037	0.014	-0.281	3.095	0.001	< 0.001	< 0.001
DFY	0.011	0.009	0.056	0.003	0.007	2.449	11.610	< 0.001	< 0.001	< 0.001
DFR	0.000	0.001	0.074	-0.098	0.014	-0.389	10.997	< 0.001	< 0.001	< 0.001
INFL	0.002	0.002	0.059	-0.021	0.005	1.064	16.517	< 0.001	< 0.001	< 0.001

Notes: This table provides summary statistics for the excess returns (RET) on the S&P 500 index and the 11 predictive variables used in our analysis. JB denotes the Jarque-Bera test statistic on the null hypothesis that the data come from a normal distribution, while Q(1) (Q(10)) denote the Ljung-Box Q test statistic on the null hypothesis that the first (first 10) autocorrelation coefficient is (are jointly) equal to zero. The sample period is January 1927 - December 2015.

Table 2. Forecasting performance evaluated by  $R_{OoS}^2$ 

	Panel A:	Full sample	Panel B: I	First subsample	Panel C: Se	econd subsample
	(194	7-2015)	(19	47-1981)	(19)	82-2015)
	Original	Truncated	Original	Truncated	Original	Truncated
	variables	variables	variables	variables	variables	variables
DP	-0.195	0.114	0.733**	0.486*	-0.980	-0.200
DY	-0.509	-0.113	0.530**	$0.434^{*}$	-1.387	-0.575
EP	-1.306	$0.210^*$	-1.405	$0.621^{**}$	-1.223	-0.137
DE	-1.274	-0.465	-2.111	-1.227	-0.566	0.179
SVAR	-0.769	$1.449^{**}$	-0.689	0.032	-0.836	$2.648^{**}$
BM	-3.140	-1.093	-3.143	-1.145	-3.137	-1.050
NTIS	-1.180	-0.804	-0.182	-0.588	-2.024	-0.987
TBL	-1.960	-0.936	-3.443	-1.412	-0.706	-0.535
LTY	-1.309	-0.090	-1.893	1.456***	-0.815	-1.397
TMS	-0.545	0.076	-0.406	0.137	-0.663	0.025
DFY	-0.408	-0.170	-0.731	-1.258	-0.134	$\boldsymbol{0.751}^*$
DFR	-0.181	-0.205	-0.395	-0.517	-0.001	0.060
INFL	-0.075	-0.130	0.294	0.187	-0.387	-0.398
Forecast combination	0.334**	0.626***	0.836**	0.936**	-0.091	0.363**

Notes: This table reports the out-of-sample forecast performance of the 13 univariate constrained and unconstrained predictors, as well as the equal-weighted combination methods, EW-U and EW-C. The constrained forecasts are obtained by regressing stock excess returns on a predictor whose value is set to zero whenever its value falls between the variable's past 24-month high and low, and further combining the resulting forecasts,  $\hat{r}_{t+1|t}^*(n)$ , as follows

$$\hat{r}_{t+1|t}^{\sharp}(n) = 0.5 \times \hat{r}_{t+1|t} + 0.5 \times \hat{r}_{t+1|t}^{*},$$

where  $\hat{r}_{t+1|t}$  denotes the unconstrained forecasts. We evaluate the quality of the forecast using the Out-of-Sample  $R^2$  ( $R^2_{OoS}$ ) defined as the percentage reduction in MSPE of the model of interest relative to the benchmark model, and given by

$$R_{OoS}^2 = 1 - \frac{MSPE_{model}}{MSPE_{bench}}$$

where  $MSPE_{model} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_{\tau} - forc_{\tau|\tau-1})^2$  with  $forc_{\tau|\tau-1}$  denoting the equity premia forecast produced at time  $\tau-1$  by either the unconstrained or constrained univariate models (or model combination), while  $MSPE_{bench} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_{\tau} - \bar{r}_{\tau|\tau-1})^2$ . We next multiply the  $R_{OoS}^2$  figures by 100, to denote percentage values. A positive  $R_{OoS}^2$  implies that the model of interest generates more accurate forecasts than the benchmark model. Panel A reports the results for the full sample (1947-2015), while Panel B and Panel C report the results for the subsamples 1947-1981 and 1982-2015. Bold numbers indicate all instances in which the constrained  $R_{OoS}^2$  is higher than its unconstrained counterpart. \*, \*\* and \*\*\* denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table 3. Forecasting performance evaluated by CER

	Panel A:	Full sample	Panel B: I	First subsample	Panel C: Se	econd subsample
	(194	7-2015)	(19	47-1981)	(198	82-2015)
	Original	Truncated	Original	Truncated	Original	Truncated
	variables	variables	variables	variables	variables	variables
DP	-0.791	-0.249	0.039	0.022	-1.645	-0.529
DY	-0.929	-0.562	0.008	-0.098	-1.893	-1.039
EP	-0.144	0.333	-0.250	0.335	-0.034	0.330
DE	0.292	0.668	0.031	0.197	0.561	1.154
SVAR	-0.523	0.304	-0.614	-0.035	-0.430	0.654
$_{\mathrm{BM}}$	-2.221	-1.405	-2.090	-1.229	-2.356	-1.587
NTIS	-0.084	0.094	-0.008	0.128	-0.163	0.059
$\mathrm{TBL}$	-1.039	-0.534	-1.370	-0.565	-0.696	-0.503
LTY	-0.575	-0.033	-0.486	0.720	-0.668	-0.807
TMS	0.124	0.333	0.437	0.327	-0.198	0.339
DFY	-0.327	0.276	-0.351	-0.490	-0.301	1.069
DFR	-0.093	-0.137	-0.288	-0.175	0.109	-0.098
INFL	-0.156	-0.163	0.173	0.142	-0.495	-0.477
Forecast combination	-0.021	0.419	0.212	0.406	-0.261	0.433

Notes: This table reports the out-of-sample forecast performance of the 13 univariate constrained and unconstrained predictors, as well as the equal-weighted combination methods, EW-U and EW-C. The constrained forecasts are obtained by regressing stock excess returns on a predictor whose value is set to zero whenever its value falls between the variable's past 24-month high and low, and further combining the resulting forecasts,  $\hat{r}_{t+1|t}^*(n)$ , as follows

$$\hat{r}_{t+1|t}^{\sharp}(n) = 0.5 \times \hat{r}_{t+1|t} + 0.5 \times \hat{r}_{t+1|t}^{*},$$

where  $\hat{r}_{t+1|t}$  denotes the unconstrained forecasts. Each period, the investor chooses the optimal weight to allocate to the risky asset in the portfolio by solving the following

$$\widehat{\omega}_t = \frac{forc_{t+1|t} + \widehat{\sigma}_{t+1|t}^2/2}{\gamma \widehat{\sigma}_{t+1|t}^2}, \qquad t = M, ..., T-1$$

where  $\gamma$  indicates the coefficient of risk aversion,  $\hat{\sigma}_{t+1|t}^2$  denotes the forecast for time t+1 stock variance made at time t, and  $forc_{t+1|t}$  stands for the equity premia forecast produced at time t by either the unconstrained or constrained univariate models/model combinations. The optimal weight of stock is restricted between 0 and 1.5, while the risk aversion coefficient is set to  $\gamma=3$ . Each entry in the table represents the difference between the certainty equivalent return of a portfolio based on the forecasts from the model of interest and the no predictability model. These differences are multiplied by 1200 to denote annualized percentage values. Panel A reports results for the full sample (1947-2015), while Panel B and Panel C report the results for the subsamples 1947-1981 and 1982-2015. Bold numbers indicate all instances in which the constrained CER is higher than its unconstrained counterpart.

Table 4. Comparison with existing constrained methods:  $R_{OoS}^2$ 

		Panel A:	Full sample			Panel B: Fi	rst subsample	e	I	Panel C: Sec	ond subsan	ple
		(1947	7-2015)			(1947	7-1981)		(1982-2015)			
	CT1	CT2	PTV	PPW	CT1	CT2	PTV	PPW	CT1	CT2	PTV	PPW
DP	-0.028	-0.195	0.322**	0.114	0.792**	0.733**	1.036***	0.486*	-0.721	-0.980	-0.282	-0.200
DY	0.065**	-0.509	0.568**	-0.113	1.056**	0.530**	$1.412^{***}$	$0.434^{*}$	-0.772	-1.387	-0.147	-0.575
EP	-0.273	-1.306	0.651**	0.210*	-0.756	-1.405	0.926**	0.621**	0.135	-1.223	0.418	-0.137
DE	-0.961	0.000	-0.059	-0.465	-2.111	0.000	-0.233	-1.227	0.011	0.000	0.089	0.179
SVAR	-0.712	-0.802	-0.691	1.449**	-0.578	-0.689	-0.477	0.032	-0.825	-0.897	-0.872	2.648**
BM	-1.455	-3.140	0.181*	-1.093	-1.857	-3.143	0.893*	-1.145	-1.115	-3.137	-0.421	-1.050
NTIS	-1.084	-1.180	0.129	-0.804	-0.077	-0.182	0.306	-0.588	-1.935	-2.024	-0.022	-0.987
TBL	-0.452	-1.960	$0.741^{***}$	-0.936	-0.359	-3.443	1.587***	-1.412	-0.530	-0.706	0.026	-0.535
LTY	0.032**	-1.309	0.706***	-0.090	0.541**	-1.893	1.644**	1.456***	-0.398	-0.815	-0.087	-1.397
TMS	-0.066	-0.527	0.301*	0.076	0.614*	-0.366	0.626**	0.137	-0.641	-0.663	0.027	0.025
DFY	-0.259	-0.397	-0.220	-0.170	-0.407	-0.708	-0.320	-1.258	-0.134	-0.134	-0.136	0.751*
DFR	-0.279	-0.091	0.092	-0.205	-0.397	-0.197	-0.181	-0.517	-0.180	-0.001	0.322	0.060
INFL	-0.001	-0.075	0.056	-0.130	0.438*	0.294	0.181***	0.187	-0.373	-0.387	-0.049	-0.398
Forecast combination	0.350**	0.295**	0.463**	0.626***	0.763**	0.768**	0.901***	0.936***	0.001	-0.104	0.092	0.363*

This table reports the out-of-sample performance of univariate predictive regressions and equal-weighted model combinations subject to different constraints. PPW refers to the constrained approach introduced in this paper. CT1 and CT2 denote two versions of the Campbell and Thompson (2008) constrained approach. The former imposes a non-negativity constraint on the equity premium predictions, while the latter requires the slope coefficients in the univariate predictive regressions to have the economically predicted sign. Lastly, PTV denotes the Sharpe ratio constraint approach of Pettenuzzo, Timmermann, and Valkanov (2014), which bounds the in-sample annualized Sharpe ratios between zero and one. We evaluate the quality of the forecast using the Out-of-Sample  $R^2$  ( $R^2_{OoS}$ ) defined as the percentage reduction in MSPE of the model of interest relative to the benchmark model, and given by

$$R_{OoS}^2 = 1 - \frac{MSPE_{model}}{MSPE_{bench}}$$

where  $MSPE_{model} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_{\tau} - forc_{\tau|\tau-1})^2$  with  $forc_{\tau|\tau-1}$  denoting the equity premia forecast produced at time  $\tau-1$  by either the unconstrained or constrained univariate models (or model combination), while  $MSPE_{bench} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_{\tau} - \bar{r}_{\tau|\tau-1})^2$ . We next multiply the  $R_{OoS}^2$  figures by 100, to denote percentage values. A positive  $R_{OoS}^2$  implies that the model of interest generates more accurate forecasts than the benchmark model. Panel A reports the results for the full sample (1947-2015), while Panel B and Panel C report the results for the subsamples 1947-1981 and 1982-2015. Bold numbers indicates all instances in which the constrained  $R_{OoS}^2$  is higher than its unconstrained counterpart. \*, \*\* and \*\*\* denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table 5. Comparison with existing constrained methods: CER gains

	F	Panel A: I	Full samp	le	Par	nel B: Fir	st subsan	nple	Pane	el C: Seco	nd subsa	mple
		(1947 -	-2015)			(1947-	-1981)		(1982-2015)			
	CT1	CT2	PTV	PPW	CT1	CT2	PTV	PPW	CT1	CT2	PTV	PPW
DP	-0.618	-0.791	-0.212	-0.249	0.079	0.039	0.285	0.022	-1.336	-1.645	-0.724	-0.529
DY	-0.531	-0.930	0.019	-0.562	0.303	0.006	0.554	-0.098	-1.390	-1.894	-0.533	-1.039
EP	0.010	-0.145	0.458	0.333	0.025	-0.252	0.466	0.335	-0.005	-0.035	0.451	0.330
DE	0.330	-0.001	0.408	0.668	0.030	-0.001	0.294	0.197	0.640	-0.001	0.525	1.154
SVAR	-0.486	-0.581	-0.495	0.304	-0.552	-0.613	-0.501	-0.035	-0.417	-0.548	-0.489	0.654
$_{\mathrm{BM}}$	-1.618	-2.223	-0.187	-1.405	-1.669	-2.094	0.484	-1.229	-1.565	-2.356	-0.879	-1.587
NTIS	-0.033	-0.087	0.305	0.094	0.025	-0.015	0.449	0.128	-0.094	-0.160	0.156	0.059
$\mathrm{TBL}$	-0.644	-1.039	0.397	-0.534	-0.753	-1.370	0.671	-0.565	-0.532	-0.696	0.114	-0.503
LTY	-0.312	-0.576	0.265	-0.033	-0.139	-0.487	0.637	0.720	-0.490	-0.667	-0.118	-0.807
TMS	0.241	0.135	0.378	0.333	0.643	0.457	0.479	0.327	-0.174	-0.198	0.275	0.339
DFY	-0.250	-0.322	-0.229	0.276	-0.202	-0.343	-0.159	-0.490	-0.300	-0.300	-0.302	1.069
DFR	-0.130	0.003	0.163	-0.137	-0.287	-0.101	-0.142	-0.175	0.031	0.110	0.478	-0.098
INFL	-0.102	-0.155	0.053	-0.163	0.264	0.175	0.138	0.142	-0.481	-0.495	-0.036	-0.477
Forecast	0.075	0.076	0.233	0.419	0.250	0.252	0.416	0.406	-0.104	-0.104	0.045	0.433
combination	0.075	0.076	0.233	0.419	0.230	0.232	0.410	0.400	-0.104	-0.104	0.045	0.455

Notes: This table reports the out-of-sample performance of univariate predictive regressions and equal-weighted model combinations subject to different constraints. PPW refers to the constrained approach introduced in this paper. CT1 and CT2 denote two versions of the Campbell and Thompson (2008) constrained approach. The former imposes a non-negativity constraint on the equity premium predictions, while the latter requires the slope coefficients in the univariate predictive regressions to have the economically predicted sign. Lastly, PTV denotes the Sharpe ratio constraint approach of Pettenuzzo, Timmermann, and Valkanov (2014), which bounds the in-sample annualized Sharpe ratios between zero and one. Each period, the investor chooses the optimal weight to allocate to the risky asset in the portfolio by solving the following

$$\widehat{\omega}_t = \frac{forc_{t+1|t} + \widehat{\sigma}_{t+1|t}^2/2}{\gamma \widehat{\sigma}_{t+1|t}^2}, \qquad t = M, ..., T-1$$

where  $\gamma$  indicates the coefficient of risk aversion,  $\hat{\sigma}_{t+1|t}^2$  denotes the forecast made at time t of time t+1 log execess return variance, and  $forc_{t+1|t}$  denotes the one-step-ahead forecast obtained from one of the constrained univariate predictive regressions or model combinations. The optimal weight of stock is restricted between 0 and 1.5, while the risk aversion coefficient is set to  $\gamma=3$ . Each entry in the table represents the difference between the certainty equivalent return of a portfolio based on the forecasts from the model of interest and the no predictability model. These differences are multiplied by 1200 to denote annualized percentage values. Panel A reports results for the full sample (1947-2015), while Panel B and Panel C report the results for the subsamples 1947-1981 and 1982-2015. Bold numbers indicate all instances in which the constrained CER is higher than its unconstrained counterpart.

Table 6. Forecasting performance over business cycles evaluated by  $R_{OoS}^2$ 

	Recessio	n periods	Expansio	on periods
-	Original	Truncated	Original	Truncated
	variables	variables	variables	variables
DP	1.605**	0.870**	-0.850	-0.161
DY	$2.417^{***}$	1.063**	-1.573	-0.540
EP	-1.837	-0.073	-1.113	$0.313^*$
DE	-2.564	-2.512	-0.805	$0.279^{**}$
SVAR	-0.453	2.588	-0.883	1.036
BM	-0.900	-0.504	-3.954	-1.308
NTIS	-4.962	-2.762	$0.195^{*}$	-0.092
$\mathrm{TBL}$	0.062	0.526	-2.695	-1.468
LTY	0.153	$4.023^{**}$	-1.840	-1.585
TMS	0.821	0.295	-1.042	-0.004
DFY	-0.320	-0.523	-0.440	-0.041
DFR	-0.660	-0.016	-0.007	-0.273
INFL	-0.455	-0.527	0.063	0.014
Forecast combination	0.680*	0.943**	0.208	0.510***

Notes: This table reports the out-of-sample forecast performance of the 13 univariate constrained and unconstrained predictors, separately for recessions and expansions. The constrained forecasts are obtained by regressing stock excess returns on a predictor whose value is set to zero whenever its value falls between the variable's past 24-month high and low, and further combining the resulting forecasts,  $\hat{r}_{t+1|t}^*(n)$ , as follows

$$\hat{r}_{t+1|t}^{\sharp}(n) = 0.5 \times \hat{r}_{t+1|t} + 0.5 \times \hat{r}_{t+1|t}^{*},$$

where  $\hat{r}_{t+1|t}$  denotes the unconstrained forecasts. We evaluate the quality of the forecast using the Out-of-Sample  $R^2$  ( $R^2_{OoS}$ ) defined as the percentage reduction in MSPE of the model of interest relative to the benchmark model, and given by

$$R_{OoS,c}^2 = 1 - \frac{\sum_{\tau=M+1}^{T} I_{\tau}^c (r_{\tau} - forc_{\tau|\tau-1})^2}{\sum_{\tau=M+1}^{T} I_{\tau}^c (r_{\tau} - \bar{r}_{\tau|\tau-1})^2}, \quad \text{for} \quad c = EXP, REC,$$

where  $I_{\tau}^{EXP}$  ( $I_{\tau}^{REC}$ ) is an indicator variable which is set equal to one when month  $\tau$  is within an NBER-dated expansion (recession), and  $forc_{\tau|\tau-1}$  denotes the equity premia forecast produced at time  $\tau-1$  by either the unconstrained or constrained univariate models (or model combination). We next multiply the  $R_{OoS}^2$  figures by 100, to denote percentage values. A positive  $R_{OoS}^2$  implies that the model of interest generates more accurate forecasts than the benchmark model. Panel A reports the results during recessions, while Panel B focuses on expansions. Bold numbers indicate all instances in which the constrained  $R_{OoS,c}^2$  is higher than its unconstrained counterpart. \*, \*\* and \*\*\* denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table 7. Forecasting performance over business cycles evaluated by CER

	Recessio	n periods	Expansio	n periods
-	Original	Truncated	Original	Truncated
	variables	variables	variables	variables
DP	2.810	1.592	-1.419	-0.572
DY	4.362	2.021	-1.851	-1.013
EP	0.860	1.885	-0.319	0.061
DE	-1.796	-2.790	0.659	1.278
SVAR	-0.819	1.073	-0.471	0.169
BM	-0.858	-0.724	-2.459	-1.525
NTIS	-6.400	-5.096	1.034	1.012
$\mathrm{TBL}$	-0.307	0.074	-1.167	-0.641
LTY	0.862	1.639	-0.827	-0.325
TMS	1.624	0.206	-0.139	0.355
DFY	-0.509	-0.177	-0.295	0.356
DFR	-0.851	-0.049	0.040	-0.153
INFL	-1.409	-1.158	0.064	0.012
Forecast combination	0.882	1.649	-0.179	0.204

Notes: This table reports the out-of-sample forecast performance of the 13 univariate constrained and unconstrained predictors, separately for recessions and expansions. The constrained forecasts are obtained by regressing stock excess returns on a predictor whose value is set to zero whenever its value falls between the variable's past 24-month high and low, and further combining the resulting forecasts,  $\hat{r}_{t+1|t}^*(n)$ , as follows

$$\hat{r}_{t+1|t}^{\sharp}(n) = 0.5 \times \hat{r}_{t+1|t} + 0.5 \times \hat{r}_{t+1|t}^{*},$$

where  $\hat{r}_{t+1|t}$  denotes the unconstrained forecasts. Each period, the investor chooses the optimal weight to allocate to the risky asset in the portfolio by solving the following

$$\widehat{\omega}_{t} = \frac{forc_{t+1|t} + \widehat{\sigma}_{t+1|t}^{2}}{\gamma \widehat{\sigma}_{t+1|t}^{2}}, \quad t = M, ..., T - 1$$

where  $\gamma$  indicates the coefficient of risk aversion,  $\hat{\sigma}_{t+1|t}^2$  denotes the forecast for time t+1 stock variance made at time t, and  $forc_{t+1|t}$  stands for the equity premia forecast produced at time t by either the unconstrained or constrained univariate models/model combinations. The optimal weight of stock is restricted between 0 and 1.5, while the risk aversion coefficient is set to  $\gamma=3$ . Each entry in the table represents the difference between the certainty equivalent return of a portfolio based on the forecasts from the model of interest and the no predictability model. These differences are multiplied by 1200 to denote annualized percentage values. Panel A reports the results during recessions, while Panel B focuses on expansions. Bold numbers indicate all instances in which the constrained CER is higher than its unconstrained counterpart.

Table 8. Alternative look-back periods,  $R_{OoS}^2$ 

		Pan	el A: Full sa	mple			Panel	B: First sub	sample			Panel (	C: Second su	bsample	
			(1947-2015)	)				(1947-1981)			(1982-2015)				
	n = 12	n = 18	n = 24	n = 30	n = 36	n = 12	n = 18	n = 24	n = 30	n = 36	n = 12	n = 18	n = 24	n = 30	n = 36
DP	0.116	0.065	0.114	0.145	0.124	0.566**	$0.490^*$	0.486*	0.481*	$0.429^*$	-0.265	-0.294	-0.200	-0.139	-0.133
DY	-0.242	-0.274	-0.113	-0.099	-0.036	$0.113^{*}$	0.361*	0.434*	0.349*	$0.443^{*}$	-0.542	-0.811	-0.575	-0.477	-0.441
EP	$0.230^{*}$	0.276**	$0.210^{*}$	$0.305^{**}$	0.258**	$0.627^{**}$	$0.692^{**}$	0.621**	0.748**	0.706**	-0.105	-0.076	-0.137	-0.070	-0.120
DE	-0.562	-0.598	-0.465	-0.479	-0.779	-1.081	-1.109	-1.227	-1.107	-1.667	-0.123	-0.166	0.179	0.052	-0.027
SVAR	1.605**	$1.507^{**}$	1.449**	$1.472^{*}$	$1.243^{*}$	0.145	0.131	0.032	0.205	0.091	2.840**	2.670**	2.648**	2.544*	$2.217^{*}$
BM	-1.232	-1.009	-1.093	-1.145	-1.161	-1.401	-0.923	-1.145	-1.162	-1.197	-1.089	-1.081	-1.050	-1.131	-1.131
NTIS	-0.489	-0.492	-0.804	-1.204	-1.277	-0.312	-0.236	-0.588	-1.153	-1.034	-0.639	-0.708	-0.987	-1.246	-1.483
TBL	-0.894	-0.831	-0.936	-0.826	-0.774	-1.750	-1.209	-1.412	-1.405	-1.368	-0.169	-0.512	-0.535	-0.337	-0.272
LTY	0.065**	-0.014	-0.090	0.487***	0.382**	1.326**	1.006***	1.456***	2.094***	1.872***	-1.001	-0.878	-1.397	-0.873	-0.878
TMS	-0.207	-0.225	0.076	0.008	0.069	-0.148	-0.184	0.137	0.125	0.195	-0.256	-0.260	0.025	-0.092	-0.038
DFY	-0.789	-0.572	-0.170	-0.277	-0.345	-1.477	-1.781	-1.258	-1.159	-1.232	-0.206	0.451	0.751*	0.468	0.405
DFR	-0.284	-0.191	-0.205	-0.315	-0.298	-0.790	-0.556	-0.517	-0.584	-0.602	0.143	0.117	0.060	-0.089	-0.041
INFL	-0.156	-0.102	-0.130	-0.084	-0.083	0.289	0.231	0.187	0.301	0.373	-0.532	-0.384	-0.398	-0.409	-0.469
Forecast combination	0.511***	0.594***	0.626***	0.671***	0.575***	0.753***	0.917***	0.936***	0.992***	0.888***	0.306	0.321	0.363*	0.401*	0.309

Notes: This table reports the out-of-sample forecast performance of the 13 constrained predictors and the EW-C combination for alternative look-back periods. The constrained forecasts are obtained by regressing stock excess returns on a predictor whose value is set to zero whenever its value falls between the variable's past 24-month high and low, and further combining the resulting forecasts,  $\hat{r}_{t+1|t}^*(n)$ , as follows

$$\hat{r}_{t+1|t}^{\sharp}(n) = 0.5 \times \hat{r}_{t+1|t} + 0.5 \times \hat{r}_{t+1|t}^{*},$$

where  $\hat{r}_{t+1|t}$  denotes the unconstrained forecasts. The forecasting performance is evaluated using the Out-of-Sample  $R^2$  ( $R^2_{OoS}$ ) defined as the percentage reduction in MSPE of the model of interest relative to the benchmark model, and given by

$$R_{OoS}^2 = 1 - \frac{MSPE_{model}}{MSPE_{bench}}$$

where  $MSPE_{model} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_{\tau} - forc_{\tau|\tau-1})^2$  with  $forc_{\tau|\tau-1}$  denoting either one of the constrained univariate models or the EW-C forecast combination, while  $MSPE_{bench} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_{\tau} - \bar{r}_{\tau|\tau-1})^2$ . We next multiply the  $R_{OoS}^2$  figures by 100, to denote percentage values. A positive  $R_{OoS}^2$  implies that the model of interest generates more accurate forecasts than the benchmark model. Panel A reports forecasting results for the full sample (1949-2015), while Panel B and Panel C report the results for the subsamples 1949-1981 and 1982-2015. We multiply  $R_{OOS}^2$  by 100 to denote percent value. Bold numbers indicate all instances in which the constrained  $R_{OoS}^2$  is higher than its unconstrained counterpart. \*, \*\* and \*\*\* denote statistical significance at the 10%, 5% and 1% levels, respectively.

Table 9. Alternative look-back periods, CER gains

		Panel	A: Full s	ample			Panel B	: First su	bsample			Panel C:	Second s	ubsample	
		(	1947-2015	5)			(	1947-1981	1)		(1982-2015)				
	n = 12	n = 18	n = 24	n = 30	n = 36	n = 12	n = 18	n = 24	n = 30	n = 36	n = 12	n = 18	n = 24	n = 30	n = 36
DP	-0.251	-0.306	-0.249	-0.213	-0.167	0.103	0.031	0.022	0.016	0.093	-0.617	-0.653	-0.529	-0.448	-0.435
DY	-0.515	-0.652	-0.562	-0.544	-0.484	-0.081	-0.011	-0.098	-0.177	-0.107	-0.962	-1.313	-1.039	-0.922	-0.871
EP	0.339	0.383	0.333	0.454	0.451	0.355	0.384	0.335	0.460	0.493	0.323	0.382	0.330	0.448	0.407
DE	0.401	0.435	0.668	0.601	0.385	0.182	0.274	0.197	0.208	0.003	0.626	0.602	1.154	1.007	0.780
SVAR	0.478	0.407	0.304	0.254	0.160	0.094	0.079	-0.035	0.018	-0.047	0.875	0.745	0.654	0.497	0.373
$_{\mathrm{BM}}$	-1.494	-1.307	-1.405	-1.497	-1.506	-1.355	-0.993	-1.229	-1.317	-1.336	-1.637	-1.630	-1.587	-1.683	-1.681
NTIS	-0.141	0.139	0.094	0.008	-0.015	-0.124	0.117	0.128	0.194	0.286	-0.159	0.162	0.059	-0.184	-0.326
TBL	-0.350	-0.511	-0.534	-0.370	-0.283	-0.662	-0.571	-0.565	-0.424	-0.335	-0.027	-0.450	-0.503	-0.314	-0.229
LTY	-0.146	0.051	-0.033	0.089	-0.131	0.261	0.443	0.720	0.817	0.503	-0.565	-0.353	-0.807	-0.661	-0.784
TMS	0.029	-0.015	0.333	0.247	0.297	0.124	0.095	0.327	0.319	0.373	-0.069	-0.128	0.339	0.172	0.218
DFY	-0.272	-0.033	0.276	0.107	0.044	-0.545	-0.762	-0.490	-0.434	-0.487	0.010	0.721	1.069	0.667	0.592
DFR	-0.217	-0.107	-0.137	-0.178	-0.172	-0.309	-0.119	-0.175	-0.280	-0.289	-0.121	-0.095	-0.098	-0.074	-0.052
INFL	-0.202	-0.147	-0.163	-0.150	-0.102	0.220	0.167	0.142	0.187	0.229	-0.637	-0.469	-0.477	-0.497	-0.444
Forecast combination	0.327	0.390	0.419	0.427	0.363	0.282	0.386	0.406	0.453	0.407	0.372	0.395	0.433	0.400	0.318

Notes: This table reports the out-of-sample forecast performance of the 13 constrained predictors and the EW-C combination for alternative look-back periods. The constrained forecasts are obtained by regressing stock excess returns on a predictor whose value is set to zero whenever its value falls between the variable's past 24-month high and low, and further combining the resulting forecasts,  $\hat{r}_{t+1|t}^*(n)$ , as follows

$$\hat{r}_{t+1|t}^{\sharp}(n) = 0.5 \times \hat{r}_{t+1|t} + 0.5 \times \hat{r}_{t+1|t}^{*},$$

where  $\hat{r}_{t+1|t}$  denotes the unconstrained forecasts. Each period, the investor chooses the optimal weight to allocate to the risky asset in the portfolio by solving the following

$$\widehat{\omega}_t = \frac{forc_{t+1|t} + \widehat{\sigma}_{t+1|t}^2/2}{\gamma \widehat{\sigma}_{t+1|t}^2}, \quad t = M, ..., T - 1$$

where  $\gamma$  indicates the coefficient of risk aversion,  $\hat{\sigma}_{t+1|t}^2$  denotes the forecast for time t+1 stock variance made at time t, and  $forc_{t+1|t}$  stands for the equity premia forecast produced at time t by either the unconstrained or constrained univariate models/model combinations. The optimal weight of stock is restricted between 0 and 1.5, while the risk aversion coefficient is set to  $\gamma=3$ . Each entry in the table represents the difference between the certainty equivalent return of a portfolio based on the forecasts from the model of interest and the no predictability model. These differences are multiplied by 1200 to denote annualized percentage values. Panel A reports results for the full sample (1947-2015), while Panel B and Panel C report the results for the subsamples 1947-1981 and 1982-2015. Bold numbers indicate all instances in which the constrained CER is higher than its unconstrained counterpart.

Table 10. Alternative risk aversion coefficients

		Full sample		rst subsample		ond subsample
_		-2015)	\	7-1981)	\	2-2015)
	Original	Truncated	Original	Truncated	Original	Truncated
	variables	variables	variables	variables	variables	variables
			$\gamma = 2$			
DP	-1.006	-0.265	0.241	0.142	-2.290	-0.685
DY	-1.216	-0.733	0.185	-0.060	-2.658	-1.428
EP	0.027	0.514	-0.092	$\boldsymbol{0.472}$	0.150	0.557
DE	0.405	1.005	-0.161	0.654	0.990	1.368
SVAR	-0.584	0.440	-0.798	-0.043	-0.363	0.938
$_{\mathrm{BM}}$	-2.656	-1.917	-1.992	-1.608	-3.340	-2.236
NTIS	0.235	0.457	0.042	0.150	0.435	0.773
$\mathrm{TBL}$	-1.390	-0.666	-1.713	-0.586	-1.057	-0.749
LTY	-0.727	0.023	-0.484	1.219	-0.977	-1.209
TMS	0.647	0.590	0.746	0.520	0.544	0.662
DFY	-0.453	0.274	-0.498	-0.927	-0.408	1.516
DFR	-0.061	-0.044	-0.424	-0.220	0.315	0.138
INFL	-0.128	-0.184	0.293	0.231	-0.562	-0.612
Forecast	0.071	0.000	0.479	0.697	0.242	0.644
combination	0.071	0.666	0.472	0.687	-0.343	0.644
-			$\gamma = 4$			
DP	-0.638	-0.214	-0.017	-0.012	-1.278	-0.423
DY	-0.742	-0.449	-0.039	-0.096	-1.465	-0.812
EP	-0.052	0.252	-0.059	0.271	-0.044	0.233
DE	0.339	0.596	0.204	0.284	0.478	0.918
SVAR	-0.485	0.233	-0.491	-0.028	-0.478	0.502
$_{\mathrm{BM}}$	-1.733	-1.078	-1.602	-0.933	-1.868	-1.227
NTIS	-0.381	0.000	-0.006	0.106	-0.768	-0.109
$\mathrm{TBL}$	-0.821	-0.435	-1.116	-0.492	-0.516	-0.377
LTY	-0.466	-0.043	-0.428	0.504	-0.506	-0.605
TMS	0.110	0.228	0.318	0.237	-0.104	0.219
DFY	-0.255	0.244	-0.272	-0.317	-0.237	0.824
DFR	-0.135	-0.144	-0.218	-0.124	-0.050	-0.164
INFL	-0.123	-0.125	0.121	0.101	-0.375	-0.359
Forecast combination	-0.042	0.305	0.119	0.284	-0.208	0.326

Table 10 Continued: Alternative risk aversion coefficients

	Panel A: 1	Full sample	Panel B: Fi	rst subsample	Panel C: Sec	ond subsample
	(1947)	-2015)	(194)	7-1981)	(1982	2-2015)
-	Original	Truncated	Original	Truncated	Original	Truncated
	variables	variables	variables	variables	variables	variables
DP	-0.457	-0.161	-0.044	-0.027	-0.882	-0.300
DY	-0.526	-0.317	-0.058	-0.079	-1.007	-0.563
EP	-0.009	0.171	0.022	0.195	-0.041	0.146
DE	0.308	0.464	0.261	0.284	0.358	0.649
SVAR	-0.457	0.159	-0.349	-0.019	-0.568	0.343
BM	-1.144	-0.738	-1.025	-0.639	-1.266	-0.839
NTIS	-0.413	-0.038	-0.004	0.077	-0.834	-0.156
$\mathrm{TBL}$	-0.576	-0.314	-0.807	-0.376	-0.339	-0.250
LTY	-0.334	-0.041	-0.330	0.311	-0.339	-0.403
TMS	0.082	0.154	0.203	0.151	-0.042	0.158
DFY	-0.177	0.190	-0.187	-0.174	-0.166	0.565
DFR	-0.094	-0.167	-0.146	-0.078	-0.040	-0.258
INFL	-0.087	-0.086	0.074	0.063	-0.252	-0.240
Forecast combination	-0.046	0.197	0.052	0.176	-0.147	0.219

Notes: This table reports the out-of-sample forecast performance of the 13 constrained and unconstrained predictors, as well as the equal-weighted combination methods, EW-C and EW-U, for alternative risk aversion  $\gamma$ . The constrained forecasts are obtained by regressing stock excess returns on a predictor whose value is set to zero whenever its value falls between the variable's past 24-month high and low, and further combining the resulting forecasts,  $\hat{r}_{t+1|t}^*(n)$ , as follows

$$\hat{r}_{t+1|t}^{\sharp}(n) = 0.5 \times \hat{r}_{t+1|t} + 0.5 \times \hat{r}_{t+1|t}^{*},$$

where  $\hat{r}_{t+1|t}$  denotes the unconstrained forecasts. Each period, the investor chooses the optimal weight to allocate to the risky asset in the portfolio by solving the following

$$\widehat{\omega}_t = \frac{forc_{t+1|t} + \widehat{\sigma}_{t+1|t}^2/2}{\gamma \widehat{\sigma}_{t+1|t}^2}, \quad t = M, ..., T - 1$$

where  $\gamma$  indicates the coefficient of risk aversion,  $\hat{\sigma}_{t+1|t}^2$  denotes the forecast for time t+1 stock variance made at time t, and  $forc_{t+1|t}$  stands for the equity premia forecast produced at time t by either the unconstrained or constrained univariate models/model combinations. The optimal weight of stock is restricted between 0 and 1.5. Each entry in the table represents the difference between the certainty equivalent return of a portfolio based on the forecasts from the model of interest and the no predictability model. These differences are multiplied by 1200 to denote annualized percentage values. Panel A reports results for the full sample (1947-2015), while Panel B and Panel C report the results for the subsamples 1947-1981 and 1982-2015. Bold numbers indicate all instances in which the constrained CER is higher than its unconstrained counterpart.

Table 11. Alternative combination strategies

		Full sample 7-2015)		rst subsample 7-1981)	Panel C: Second subsample (1982-2015)		
-	Original Truncated		Original	Truncated	Original	Truncated	
	variables variables		variables variables		variables	variables	
Trimmed Mean	0.432**	0.658***	1.010**	0.989***	-0.057	0.378*	
DMSPE(1)	$0.315^{**}$	$0.623^{***}$	$0.796^{**}$	$0.928^{***}$	-0.092	$0.365^*$	
DMSPE(0.9)	$0.377^{**}$	$0.652^{***}$	$0.870^{**}$	$0.955^{***}$	-0.040	$\boldsymbol{0.395^*}$	
Yang	0.330** <b>0.626</b> ***		0.830**	$0.935^{***}$	-0.093	$\boldsymbol{0.365^*}$	

This table reports the out-of-sample forecast performance of the unconstrained and constrained model combinations for four alternative combination schemes. The forecasting performance is evaluated using the Out-of-Sample  $R^2$  ( $R^2_{OoS}$ ) defined as the percentage reduction in MSPE of the model of interest relative to the benchmark model, and given by

$$R_{OoS}^2 = 1 - \frac{MSPE_{model}}{MSPE_{bench}}$$

where  $MSPE_{model} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_{\tau} - forc_{\tau|\tau-1})^2$  with  $forc_{\tau|\tau-1}$  denoting either one of the forecast combination schemes, while  $MSPE_{bench} = \frac{1}{T-M} \sum_{\tau=M+1}^{T} (r_{\tau} - \bar{r}_{\tau|\tau-1})^2$ . We next multiply the  $R_{OoS}^2$  figures by 100, to denote percentage values. A positive  $R_{OoS}^2$  implies that the model of interest generates more accurate forecasts than the benchmark model. Panel A reports forecasting results for the full sample (1947-2015), while Panel B and Panel C report the results for the subsamples 1947-1981 and 1982-2015. We multiply  $R_{OOS}^2$  by 100 to denote percent value. Bold numbers indicate all instances in which the constrained  $R_{OoS}^2$  is higher than its unconstrained counterpart. \*, \*\* and \*\*\* denote statistical significance at the 10%, 5% and 1% levels, respectively.