Background for the day:
For the Teacher Forum, we will be experiencing a form of professional development called “lesson study.” In lesson study, teachers study curriculum materials and then design a lesson together. The team observes one of its members teaching the lesson, and the observers gather data of student learning during the lesson. Afterwards, they analyze the student work from the lesson, and reflect on student learning and their own learning from the cycle. The cycle then begins again. (See the diagram on pg. 2.) To learn about lesson study, we will experience a truncated cycle, by using a lesson developed from a previous lesson study group. That lesson is found here; please read the lesson thoroughly in advance of the Teacher Forum on Sunday.

Background about students in this class:
Students have started their school year working with youcubed.org’s “Week of Inspirational Math” (see https://www.youcubed.org/week-inspirational-math/).
Students are working in Investigations 3 for their mathematics curriculum. They are currently working on arrays.
Students are working with number fluency but often do not have enough time to get to that with all they need to cover in school.
Students in this class tend to want to get everything right, so the class is working on developing more bandwidth for uncertainty and error.

Goals for student learning:
Students will work with arrays of square numbers and determine the perimeter of each, ultimately finding a generalizeable way to find the perimeter of any square.
Students will have the opportunity to share an incorrect answer to a problem, explain why it seemed to make sense, and then give reasons for why it wasn’t right.
Students will work on a complex nonroutine problem that is related to the current curriculum goals (arrays) but is not isometric with their class work.
Students will listen to and contribute multiple strategies for determining solutions to this problem.

Related Common Core State Standards for 4th grade mathematics:
Use place value understanding and properties of operations to perform multi-digit arithmetic.
CCSS.MATH.CONTENT.4.NBT.B.5
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
CCSS.MATH.CONTENT.4.NBT.B.6
Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Goals for observers:
Observers will focus on hearing and seeing students as they work, and collect relevant data to share during the lesson study.
Observers will get a taste of lesson study, by participating in an abbreviated form of lesson study and by hearing about the practice.
Observers will hear multiple perspectives about instruction in a single lesson.
Phases of the lesson study cycle adapted from “Implementing Japanese Lesson Study in Foreign Countries: Misconceptions Revealed” by T. Fujii, 2014, Mathematics Teacher Education and Development, 16(1), p. 4. Copyright 2014 MERGA.

**Math4theNines**

**The Border Problem**

Without counting one by one, determine how many blue squares are in this 10x10 grid

1. What would your answer be if it were a 12 X 12 grid?
2. What would you answer be if it were a 100 X 100 grid?

Boaler & Humphreys, 2005

## Lesson Plan for October 21, 2018

“The Border Problem:” Supporting Equitable Learning through Students’ Ownership of Academic Ideas

<table>
<thead>
<tr>
<th>STUDENT LEARNING ACTIVITIES AND TIMING</th>
<th>ANTICIPATED STUDENT RESPONSES AND TEACHER RESPONSE</th>
<th>POINTS TO NOTICE (EVALUATION)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Teacher Intro</strong>&lt;br&gt;- 30 seconds</td>
<td><em>Teacher will introduce themselves and distribute problem activity to whole group.</em>&lt;br&gt;<em>T will call on someone to read the question aloud.</em>&lt;br&gt;<em>T will allow individual think time.</em>&lt;br&gt;<em>T will signal students to share answer and method with a partner.</em>&lt;br&gt;<em>T writes down all answers (do not indicate any correct/incorrect responses.)</em>&lt;br&gt;<em>Possible incorrect responses:</em>&lt;br&gt;100 (10 x 10)&lt;br&gt;40 (10 x 4 or 10 + 10 + 10 + 10)&lt;br&gt;38 (students may take off 2 corners and not 4 corners)&lt;br&gt;<em>T: “Did someone think 40?”</em>&lt;br&gt;<em>To promote active learning (how to ensure equity):</em>&lt;br&gt;1. Did students understand the question?&lt;br&gt;2. Are the students engaged?&lt;br&gt;3. Are the students counting one by one around the entire border?&lt;br&gt;4. Are students sharing their work with a partner?&lt;br&gt;5. What values are being shared?&lt;br&gt;6. If answers are different is that stimulating conversation among the students?</td>
<td>&lt;br&gt;<strong>Problem of the Day:</strong>&lt;br&gt;(Border Problem)&lt;br&gt;see attached sheet&lt;br&gt;<strong>Pass out papers 10x10 problem</strong>&lt;br&gt;<strong>THINK</strong> (1-2 mins)&lt;br&gt;<strong>Pair Share</strong> (3-4 mins)&lt;br&gt;*T-monitor progress/write down student’s names with different strategies to call upon during discussion/board time</td>
</tr>
</tbody>
</table>
Wrong answer(s) = (40) *shown on board (1-2 min)

That’s an understandable mistake. Can someone come up and explain how someone may have gotten this answer.”
(Any student can be used to come to the board to show thinking. T or S will use different colors to demonstrate on chart paper so that student’s explanation and the error can be seen. From demonstration and explanations 36 should be the answers most students agree on.

**OPTIONAL: (if someone says/interjects 100)
T) “Just so you know, originally I came up with the answer of 100. Does anyone see how I came up with that answer?” (student will explain from seat)

**Transition to 36. INDICATE that 36 is the right answer**

T asks students: “Would anyone like to come to the board and explain your strategy on how you got 36 shaded squares as an answer? Now can you use these markers to show us your method?”
Call on Ss to suggest various ways to determine 36.

**Teacher movement:**
T will call on students to come up to the projected image to explain his/her method while teacher writes matching expression on the board. T then writes the equivalent mathematical expression of students’ verbal response on chart paper with student’s name, removes paper when next student is called, puts up new chart up. T will continue to move students through the process for all possible methods/strategies

T will use a chart that will display student’s name, 10x10 expressions, 12x12, and nxn expression on white board in various colors.

**Teacher dialogue:**
T calls Student A to the board to explain method and will ask another student to re-explain the written mathematical expression from his/her seat or at the board. “so this number came from…and this from…”

Ask “Who else thought the same way as Student A?” Look for students to raise hands as a response. T: “Thank you for that.”

7. Are students changing their answers based on discussion?
8. Do all the students share out?
9. What responses were provided?
10. Are students willing to share their thinking?
11. Does the student explain that the corners cannot be counted more than once?
<table>
<thead>
<tr>
<th>Strategies on board (for solution of 36) (5 mins)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DEMONSTRATE 1 WRONG ANSWER on board (1-2 mins)</strong></td>
</tr>
<tr>
<td><em>(Two ways to pose question)</em></td>
</tr>
<tr>
<td>T: <em>Is there another strategy/method that someone came up with to show that there are 36?</em></td>
</tr>
<tr>
<td><strong>OR</strong></td>
</tr>
<tr>
<td><strong>Did anyone see it differently?</strong></td>
</tr>
<tr>
<td><strong>IF THERE AREN’T ENOUGH METHODS/STRATEGIES GIVEN:</strong></td>
</tr>
<tr>
<td>T will give students more time to Think/Pair/Share to come up with a few more strategies. (2-3 mins)</td>
</tr>
<tr>
<td><strong>OR</strong></td>
</tr>
<tr>
<td>Teacher will say, “In another class a student came up with this method. <em>(select method not given)</em> Can someone explain how they came up with this method? (call on a student to explain/show on board)</td>
</tr>
<tr>
<td>Possible responses:</td>
</tr>
<tr>
<td>10 + 10 + 8 + 8*</td>
</tr>
<tr>
<td>10 x 2 + 8 x 2</td>
</tr>
<tr>
<td>10 + 9 + 9 + 8</td>
</tr>
<tr>
<td>9 + 9 + 9 + 9*</td>
</tr>
<tr>
<td>9 x 4</td>
</tr>
<tr>
<td>8 x 4 + 4</td>
</tr>
<tr>
<td>10 x 10 - 8 x 8*</td>
</tr>
<tr>
<td>(2(10) + 2(10)) - 4</td>
</tr>
<tr>
<td>10 x 4 - 4*</td>
</tr>
<tr>
<td>100-64 <em>(T will demonstrate her understanding of using the area model to get 36)</em></td>
</tr>
<tr>
<td>12. Are students explaining their strategies?</td>
</tr>
<tr>
<td>13. Are students demonstrating their strategies?</td>
</tr>
<tr>
<td>14. Are other students in class attentive?</td>
</tr>
<tr>
<td>15. Are students beginning to look for more strategies as new strategies are demonstrated.</td>
</tr>
<tr>
<td>16. Could another student explain the first student’s method?</td>
</tr>
</tbody>
</table>
**Transition to 12x12**

Read

**Think Time (1-2 mins)**

*T- monitor progress/write down students with same/different strategies*

**Pair Share (3-4 mins)**

HAVE A STUDENT READ THE PROBLEM ALoud

“**So we are looking to find how many shaded squares there would be in a similarly shaded 12 x 12 grid?**”

(T puts the 12x12 grid on the board)

Take (1-2 minutes) by yourself to think about it and record your answer.

I have a 12 x 12 grid here on the board for you to look at while you think about this new problem.

Ok, now I want you to take (2-3 mins) to share your answer. Remember to discuss the strategy/method you used to find the answer with a partner. (listen for discussion that student is providing both answer and expression) Be prepared to share your answers with the class.

**Teacher movement during 3-4 minutes:**

T will walk around room looking for strategies similar to the ones used in the 10x10 and will walk around looking for NEW strategies not written on the board.

T will ask “What is our answer class? (class will shout out answer) Yes, 44 is the number of shaded squares in the 12x12 grid. Let’s look at some of the different strategies/methods used to come up with that answer.”

T will strategically call on (3-4 student) to come to share their methods/answer. T will (choose students with similar methods first, and will then call on students with different methods) [WHICH METHOD WILL YOU LOOK FOR TO PRESENT FIRST?]

17. Were there various strategies shared?
18. Are students using more than one operation to solve?
19. Are students using more than one set of number
20. Are students actively coming up with other strategies?
21. Were students able to explain strategy for teacher’s example?
22. Were students able to show the removing of the corners in a mathematical expression?

**Expected outcome:**

12 x 12 and n x n discussions will be shorter than 10x10 discussion because students will realize a pattern with 10 x 10. If they don’t teacher will lead students into thinking about how they found patterns for 10x10.
Strategies on board (for solution of 44) (5 mins)

Teacher movement:
T will call student to state/show method, and will write each numeric equation on board. Student will demonstrate their answer while T writes down expressions on the grid paper. 
T says: Can you explain what I saw you doing (method) to get this answer?

Possible (S) answers:

\[
\begin{align*}
12+12+10+10** & \quad 12x4-4** & \quad 11+11+11+11 \\
12(2) +10(2) & \quad 2(12) +2(12) -4 & \quad 11(4)**
\end{align*}
\]

\[
\begin{align*}
144-100=(area\ model)*** & \quad 12 + 11 + 11 + 10 \\
12x12- 10x10
\end{align*}
\]

TRANSITION to \( n \times n \)
“Ok, so we have come up with methods for how to get the number of shaded squares on a 10x10 and a 12x12 grid. What if we presented you with a grid that is represented by an unknown number. (T will write “n” on the board to signify unknown number).

“Based on the work you have already done, can you come up with a general rule/method that will help us solve for the shaded number of units on the grid. Work with your group to solve.

(\text{another way to explain}) **How would you explain how to find the number of units WITHOUT counting, regardless of what size the grid is? I want you to work with your \text{group} to solve.” (3-5mins)

T will walk around to monitor progress and listen for discussion.

“Ok, Let’s hear what you’re thinking…”
T will select students with different methods to come to the board and explain.

REMEMBER to write specific student’s names down during 3-5 mins teacher monitoring. Tell students that they can refer to the previous examples posted on the walls if needed.

If students get partial answer OR no methods are generated help direct their thinking by asking:

---

Transition to \( n \times n \) read (1-2 min)

---

Pair/Share with group (5 mins)

---

Are the students engaged?

Are students referencing the 10x10 methods/strategies without teachers assistance?

What answers were shared among the students?

What different answers did you hear shared?

Were students able to help each other make corrections?
### Strategies on the board
(10 mins)
**help students work through methods/strategies as needed**

#### SHARE STRATEGIES ON BOARD

<table>
<thead>
<tr>
<th>Possible Responses</th>
<th>AREA MODEL FORMULA</th>
</tr>
</thead>
<tbody>
<tr>
<td>10+10+8+8</td>
<td>10 x 10 - 8 x 8</td>
</tr>
<tr>
<td>12+12+10+10</td>
<td>12x12 - 10x10</td>
</tr>
<tr>
<td>N + N + (N-2)+ (N-2)*</td>
<td>(nxn) - [(n-2)x(n-2)]</td>
</tr>
<tr>
<td>10x2 + 8x2</td>
<td>(2(10) + 2(10)) - 4</td>
</tr>
<tr>
<td>12x2 + 10x2</td>
<td>[2(12) +2(12)]-4</td>
</tr>
<tr>
<td>Nx2 + (n-2)x2</td>
<td>[2(N) + 2(N)]-4</td>
</tr>
<tr>
<td>10 + 9 + 9 + 8</td>
<td>10 x 4 - 4*</td>
</tr>
<tr>
<td>12+11+11+11</td>
<td>12 x 4 -4</td>
</tr>
<tr>
<td>(N) + (N-1) + (N-1) + (N-2)</td>
<td>N x 4 -4</td>
</tr>
<tr>
<td>9 + 9 + 9 + 9*</td>
<td></td>
</tr>
<tr>
<td>11+11+11+11</td>
<td></td>
</tr>
<tr>
<td>(N-1) + (N-1) + (N-1) + (N-1)</td>
<td></td>
</tr>
<tr>
<td>9 x 4</td>
<td></td>
</tr>
<tr>
<td>11x4</td>
<td></td>
</tr>
<tr>
<td>(N-1)x4</td>
<td></td>
</tr>
<tr>
<td>8 x 4 + 4</td>
<td></td>
</tr>
<tr>
<td>10x4+4</td>
<td></td>
</tr>
<tr>
<td>(N-2) x4 +4</td>
<td></td>
</tr>
</tbody>
</table>

**EXIT TICKET:**
Ok, we have done one together. THIS is your exit ticket. I would like you to pick one of these strategies that you have seen today but has NOT been used to solve how many shaded squares are in the border of an nxn grid.

Are the students able to use the 10x10/12x12 strategies/methods to solve for nxn?

Are students able use algebraic symbols to come up with an nxn method?