

Maurice Auslander

1926–1994

Maurice Auslander died of cancer on November 18, 1994, in Trondheim, Norway.

Auslander has made fundamental contributions in many central parts of algebra. It would be senseless to try to describe his work within a given specialty or to present it under a particular title. Quite the contrary, he liked to attack problems by surprise, from apparently nowhere. This approach resulted in many original theorems in commutative and noncommutative ring theory, for orders and Brauer groups, and in the representation theory of Artin algebras as well as in the theory of singularities. Among the main characteristics of his work one will particularly remember the extreme elegance of the methods he liked to introduce and develop and also his ability to present and explain the crucial points. These qualities were also typical of his personality.

The importance of his scientific influence is clearly established for many years to come.

Maurice Auslander was born on August 3, 1926, in Brooklyn, New York. He received his B.S. in 1949 and his Ph.D. in 1954, both from Columbia University. His thesis was within group theory and was written under the direction of Robert L. Taylor. He spent the years 1953–1957

at the University of Chicago, the University of Michigan, and at the Institute for Advanced Study in Princeton. He joined Brandeis University in 1957, where he was chairman 1960–1961 and 1976–1978. He held visiting positions in Paris, Urbana, London, Trondheim, Austin, and Blacksburg. He had fellowships from Sloan,

Guggenheim, and Fulbright, and a few weeks before his death he was awarded a Senior Humboldt Research Prize. Auslander was a Fellow of the American Academy of Arts and Sciences and a member of the Royal Norwegian Society of Sciences and Letters.

In commutative ring theory, Auslander was immediately attracted by homological methods. The title of his first joint work with Buchsbaum was “Homological dimension in noetherian rings” (1956). It soon became “Homological dimension in noetherian rings I” for

all algebraists, since their collaboration on this topic was long and fruitful. Progressively, they organized and formalized what became “Homological commutative algebra with a view toward algebraic geometry”. The characterization of regular rings as the rings with finite homological dimension (Auslander-Buchsbaum-Serre) as well as the factoriality of regular local rings

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(Auslander-Buchsbaum) are now classic results. The interest of Auslander in this theme culminated with “Modules over unramified regular local rings” (1961). This was also the title of his talk at the World Congress of 1962. In this work he started a research program for thirty years to come. The list of contributors to this program includes the names of his students Peskine and Szpiro and also of Hochster and Fulton (via the local Riemann-Roch theorem). This program was essentially completed with the answers of Roberts to the “Homological Conjectures”. Ramification theory was, during the same years, Auslander’s other favorite topic. He collaborated there with Buchsbaum, Goldman, and Rim. The new methods were first developed for noetherian rings. Later on, a general theory of separable algebras over a commutative ring was proposed. The success of this theory and particularly of the homological difference of an algebra over a ring is now well established. In 1962, Auslander gave a new proof of “the purity of the branch locus”. It was very much in his style, completely surprising for the specialists and strikingly beautiful. First he reduced the question to proving that if R is a regular local ring and M a reflexive finitely generated R -module such that $\text{Hom}(M, M)$ is isomorphic to a direct sum of copies of M , then M is free. His proof of this last statement is really exhilarating.

With his work on coherent functors in the mid-sixties, Auslander started to develop systematically a functorial point of view, in the spirit of

his favorite book by Cartan-Eilenberg. The monograph on stable module theory (with Bridger) is rather typical of this evolution. A central role is played by the transpose of a module, one of Auslander’s favorite tools. Although this work is still influenced by algebraic geometry, through dualities, the representation theory of Artin algebras is a developing theme. In this monograph are also the roots of Auslander’s successful version of noncommutative Gorenstein rings, giving rise to what is now known as Auslander-Gorenstein rings and Auslander regular rings.

The new phase for Auslander took a clearer form around 1970, during his visits to Urbana and London, with a decided interest in the module theory of Artin algebras. Modular group representation theory was a first source of inspiration. Higman had much earlier described the group algebras of finite (representation) type in terms of p -Sylow subgroups. Also the Brauer-Thrall conjectures had their origin from group algebras, the first one having been proven by Roiter for algebras over a field. Auslander started by homologically characterizing finite type in terms of endomorphism algebras, now called Auslander algebras. Then he proved the first Brauer-Thrall conjecture for left Artin rings, and with an elegant application of functors he characterized infinite type in terms of existence of indecomposable modules which are not finitely generated. This work, together with the work of Roiter and Gabriel’s work on representations of quivers, belongs to the start of the modern phase of representation theory of Artin algebras, where Auslander was a main contributor until the end of his life.

Starting also in the early seventies, Auslander developed with Reiten the theory of almost-split sequences, also called Auslander-Reiten sequences, and the related irreducible maps. The transpose of a module plays a major role in this work. Almost-split sequences and the associated Auslander-Reiten quivers are now central in representation theory. They are important for classification theorems, provide a useful combinatorial invariant for an algebra, and criteria for finite type. They also play an important role in group representation theory, in particular through the work of Webb and Erdmann.

By interpreting work of Bernstein-Gelfand-Ponomarev, Auslander (with Platzeck and Reiten) gave the first module theoretic construction of what is now, through the work of Brenner-Butler and Happel-Ringel, known as tilting theory. A few years ago Auslander and Reiten had also established a close connection between tilting and the theory of contravariantly finite subcategories, as developed by Auslander-Smalo around 1980. This connection has beautiful applications to quasihereditary algebras, as dis-

covered by Ringel, and inspired applications of tilting to algebraic groups by Donkin. The contravariantly finite subcategories also were important in Auslander's recent work (with Solberg) on relative homological algebra in representation theory.

Even though the representation theory of Artin algebras, including the interplay with group representation theory, was Auslander's main field of research during the last 20–25 years, he always strived to carry his ideas and proofs to their utmost generality.

Through these efforts he obtained interesting applications to the theory of commutative and noncommutative noetherian rings. In the mid-seventies, before the theory of almost-split sequences was generally accepted in representation theory of Artin algebras, he generalized the basic existence theorem to a setting including lattices over orders and (maximal) Cohen-Macaulay modules over complete local Cohen-Macaulay rings. Auslander several years later discovered connections with singularity theory, thus opening up a new, exciting area of research. He showed that his methods were particularly well suited in dimension two, in fact in some sense better than for Artin algebras. He saw that the Auslander-Reiten quivers for two-dimensional invariant rings are isomorphic to the corresponding McKay quivers, which for the special linear group are given by (extended) Dynkin diagrams. This provided a surprising link to the resolution graph of the isolated singularity and was a main topic of his invited address at the 1986 International Congress in Berkeley. Several people were involved in identifying the hypersurfaces of finite (representation) type as the simple isolated singularities in the sense of Arnold, and Auslander found (with Reiten) further examples of finite (representation) type for Cohen-Macaulay rings which are not hypersurfaces. Last but not least was the theory of Cohen-Macaulay approximations which he developed in the late eighties in collaboration with Buchweitz, with strong ties to the theory of contravariantly finite subcategories and the work with Bridger.



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Maurice Auslander had a warm and sensitive personality. His door was always open to his friends. He enjoyed discussions, often provocative ones, about mathematics and its philosophy in particular. He had considerable administrative talent, doubling the size of the department during his first chairmanship at Brandeis. His interests outside mathematics included art, poetry and music, and he enjoyed playing the violin. He was himself “un homme libre”, free of all influences, and wanted others to be the same. He

would pin down clichés on the spot. In mathematics he had a sense for beauty; he truly enjoyed some results and their proofs. He had a special interest and concern for young researchers, including his thirty or so doctoral students, who loved and admired him. He would frequently visit their offices or wake them up by phone calls in the early mornings, inquiring about the latest progress. One of his students once complained that Maurice did the easy work. When things were getting harder, he would leave the subject. He answered goodheartedly that the student was welcome to do the same.

Maurice loved to travel. On his travels he attracted students and collaborators for himself and visitors to Brandeis. Through his extensive travels and collaborations, he had a positive influence on the development of algebra in many foreign countries. In the sixties he had French students in Paris whom he invited to work near him at Brandeis. Through collaboration he became a frequent visitor to Trondheim during the past twenty years, and in recent years he also had a formal affiliation. Half of his research publications, including a book on the representation theory of algebras which appeared shortly after his death, have coauthors from Trondheim. With his visit to Mexico in 1975 and the subsequent research stays of Mexicans at Brandeis, he was responsible for the start of the successful Mexican research group on the representation theory of Artin algebras. Through his visit in China in 1986 as the first Western visitor, he helped the Chinese group in representation theory get off

to a good start. He also had strong impact in many other countries, in particular Germany, and he had longer research visits in Brazil, Israel, Switzerland, and Uruguay.

Despite declining health, Maurice managed to continue with his favorite occupations during the last year of his life. He revisited China, seeing the results of his influence and the changes in the society. He enjoyed the spectacular fjords and glaciers of Norway and put the finishing touch to the manuscript for his last book. He attended a conference in Utrecht and saw the impact of the theory of Cohen-Macaulay approximations. He gave his final public lecture, pushing homological methods in representation theory, at ICRA VII in Mexico. With cancer in bones, liver, and lungs he planned his last nostalgic tour, putting his faith in what money and willpower could do. He enjoyed the company of old friends; wandered through the streets and gardens of Paris; appreciated for the last time his favorite painting, a self portrait of Rembrandt, in London; and enjoyed the Munch museum in Oslo. Shortly after arriving in Trondheim he was hospitalized. He died a week and a half later, amongst close friends and colleagues, in the middle of the European meeting on “Invariants and Representations of Algebras” which he had looked forward to attending. He died the way he lived and worked—elegantly.

—*Christian Peskine, University of Paris VI*
—*Idun Reiten, University of Trondheim, AVH*

Editor's note: During my brief and naive tenure as chair of the Department of Mathematics at Brandeis, Maurice was my close friend and mentor. These were disturbing times (1968–1970), and Maurice was ever the steadying influence. For him, the difference between pragmatics and theory was only theoretical: his complete comprehension of complex situations, and only that, guided his actions. There never was any suspicion of compromise with principle: in his life, his politics, his mathematics. He always delivered his message with charm, elegance, and humor; because of this he was surprisingly effective...I never heard Maurice play the violin...I wish I had.

—*Hugo Rossi*