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To predict the equity market, consult economic theory

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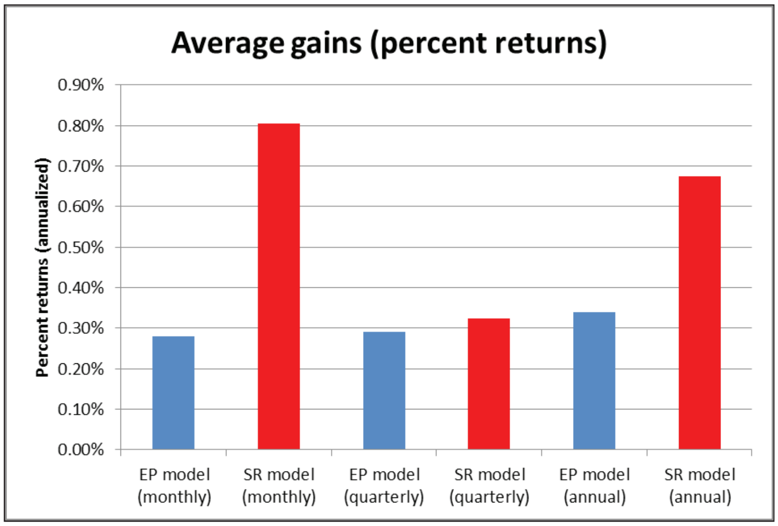
Despite more than half a century of research on forecasting stock market returns, most predictive models perform quite poorly when they are put to the test of actually predicting equity returns. In fact, many authors, including Bossaerts and Hillion (1999), Brennan and Xia (2005), and Welch and Goyal (2008) suggest that equity returns cannot be predicted at all. But what if your prediction model could produce average portfolio return gains like those below?

Figure 1: Average gains from imposing economic theory constraints to return forecasting models



Note: This figure displays the average annualized certainty equivalent returns (CER) for various models and horizons (monthly, quarterly, and annual). A given model's CER is the fee that an investor would be willing to pay in order to gain access to the forecasts generated by such model. EP model stands for equity premium constrained model, while SR model stands for Sharpe ratio constrained model. Additional details on these models are provided below.

This brief proposes a simple yet very effective solution to improve the quality of stock return predictions--take economic theory into account!

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Introduction

Stock return forecasts play a central role in areas as diverse as asset pricing, portfolio allocation, and performance evaluation of investment managers. Yet, research has yet to produce a model that can reliably and consistently forecast stock returns. This brief outlines how to apply plausible restrictions generated by economic theory to the behavior of key economic and financial variables that potentially can improve their forecasts. The challenge is that these restrictions can be quite hard to implement in practice. Our new approach helps to solve this problem. In the context of stock return predictions, we show how one can incorporate different economic constraints directly on the moments of the predictive distribution of the equity premium, and produce forecasts that are coherent with economic theory.

Imposing economic constraints on the return predictions

We start by specifying a simple linear model to forecast stock returns at time $T+1$,

$$r_{t+1} = \mu + \beta x_t + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \sigma^2_\varepsilon) \tag{1}$$

where r_{t+1} measures time $t+1$ stock return in excess of a risk-free rate (the so called excess return), x_t is a lagged predictor, and $t = 1,2,..., T\text{-}1$. We next describe our approach to combine the economic constraints with the forecasts generated by this simple model.

We focus on two types of economic constraints. The first, which we label the equity premium constraint, constrains the conditional mean of the equity premium to be non-negative. In fact, economic theory tells us that it would be hard to imagine an equilibrium setting where risk-averse investors would hold stocks if their expected compensations were negative.[1] To implement this idea, we estimate the parameters (μ, β) subject to the constraint that $(\mu + \beta x_t)$ is non-negative at all points in time. That is, the constraint in this case reads

$$\mu + \beta x_t \geq 0 \quad \text{for } t = 1,..., T \tag{2}$$

The second constraint we investigate operates directly on the Sharpe ratio (see Sharpe (1966)). The Sharpe ratio, also known as the reward-to-variability ratio, is a metric that is extensively used in finance, and, not surprisingly, economic theory can be quite useful in gauging bounds for this key statistics. For example, in the case of the market portfolio, an annualized Sharpe ratio lower than zero or higher than one is considered highly improbable.[2] To implement this idea, we estimate the parameters (μ, β) subject to the constraint that all points in time, the annualized Sharpe ratio lays between zero and one. That is,

$$0 \leq \frac{\sqrt{H}\,(\mu + \beta x_t)}{\sigma_{\varepsilon, t+1}} \leq 1 \quad \text{for } t = 1,2,..., T \tag{3}$$

where H denotes the number of observations per year (e.g., H = 12 in the case of monthly data)[3]

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1. The idea for this constraint was inspired by the work of Campbell and Thompson (2008). However, while we share with these authors the idea behind this constraint, our implementation differs significantly from theirs.

2. The lower bound of zero is consistent with the equity premium constraint.

3. We introduced $\sqrt{H}$ in the numerator to obtain an annualized Sharpe ratio. Note also that in this case we slightly modified the linear model above to allow for time-varying volatility of returns. Hence, we have added a time subscript in the term appearing in the denominator of the Sharpe ratio.

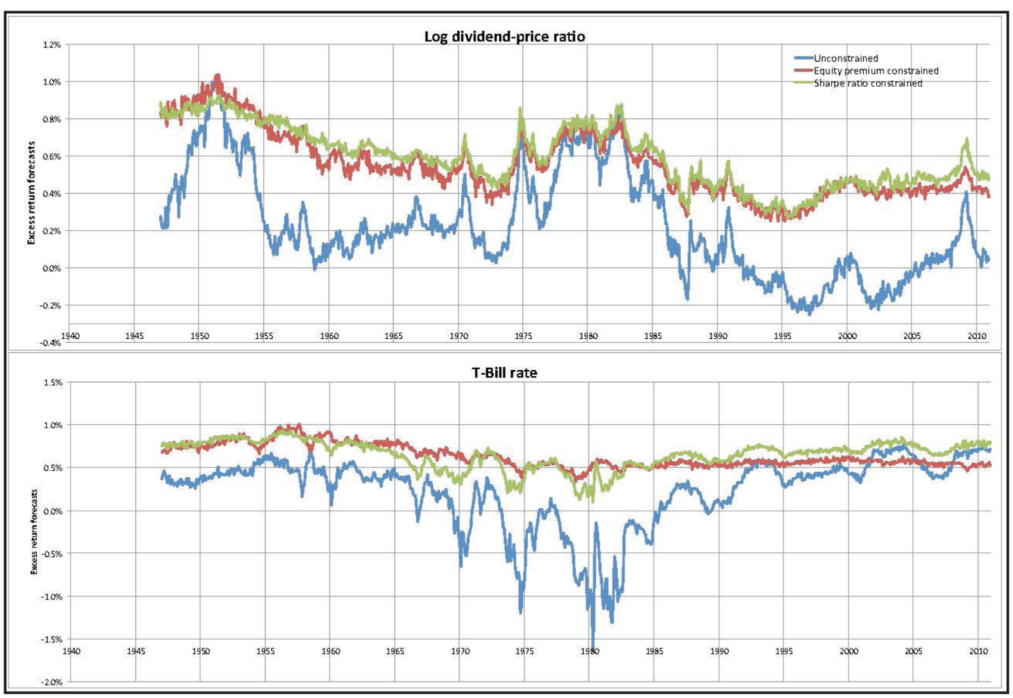
What we find

We investigate the merits of our method by forecasting the S&P 500 returns over the period 1947-2010. As for the returns, these are computed from the S&P 500 index and include dividends. A short T-bill rate is subtracted from the returns in order to obtain excess returns. As for the predictors of the S&P 500 excess returns, we consider a long list of variables that fall mainly into three broad categories: (i) valuation ratios, capturing some measure of fundamentals to equity market valuation; (ii) measures of bond yields, so as to capture level effects, slope effects, and default risk effects; (iii) estimates of equity risk. In addition to these, we include three corporate finance variables, namely the dividend payout ratio, net equity expansion, percent equity issuing, and a macroeconomic variable, inflation.

We begin by first evaluating the effect of the constraints described in equations (2) and (3) on the S&P 500 return forecasts. Next, we assess whether these constraints would have helped an investor to obtain positive and significant portfolio returns over the period we are investigating.

Figure 2 sheds lights as to the effect of the economic constraints on the stock return forecasts. As it can be seen, over the period 1947-2010 the economic constraints clearly make a substantial impact on the S&P 500 return forecasts. For example, the unconstrained model forecasts (eq.1) based on the logarithm of the dividend-price ratio (top panel) are lower and far more volatile than their constrained counterparts from equations (2) and (3) and turn negative for most of the period between 1990 and 2005. Also, the economic constraints in (2) and (3) lead to predicted excess returns whose differences from the unconstrained counterparts can last very long, e.g., from 1955 through 1975 and again from around 1985 to the end of the sample. Large and persistent differences in predicted mean returns are also found for the return model based on the T-bill rate (bottom panel). For this model, negative values of the unconstrained forecasts occur most of the time between 1970 and 1985, whereas the constrained forecasts hover around small, but positive values throughout the sample. The Sharpe ratio constrained forecasts are smaller than the equity premium constrained forecasts for long periods of time, and both series are notably more stable than the unconstrained equity premium forecasts.

Figure 2: Stock return forecasts for constrained and unconstrained models



We next turn to investigate whether the effects of the constraints that we saw in the return forecasts translate to positive and significant portfolio returns for an investor trying to exploit them. Table 1 presents the annualized certainty equivalent returns (CER) for the forecasts generated using the predictors and the constraints described above, and relative to a simple historical average forecast.[4] The CER can be interpreted as the average fee that an investor would be willing to pay in order to gain access to the forecasts generated by either of the constrained models. Table 1 below shows that both economic constraints lead to higher CER-values, relative to the unconstrained model, at all horizons and across practically all predictors.[5] Specifically, the EP constraint results in a higher CER (relative to the unconstrained case) of about 50 basis points per year, whereas for the SR-constrained models the increase is about 100 basis points per year.

Table 1: Certainty equivalent returns of portfolios based on different models and horizons

Variables	Panel A: Monthly			Panel B: Quarterly			Panel C: Annual		
	No constraint	Equity premium constraint	Sharpe ratio constraint	No constraint	Equity premium constraint	Sharpe ratio constraint	No constraint	Equity premium constraint	Sharpe ratio constraint
Log dividend price ratio	-0.28%	**0.32%**	**0.74%**	-0.18%	**0.38%**	**0.50%**	-0.11%	**0.39%**	**0.92%**
Log dividend yield	-0.26%	**0.41%**	**0.76%**	-0.08%	**0.39%**	**0.62%**	-0.04%	**0.50%**	**0.90%**
Log earning price ratio	0.13%	**0.49%**	**0.84%**	0.18%	**0.54%**	**0.33%**	0.18%	**0.47%**	**0.79%**
Log smooth earning price ratio	-0.29%	**0.38%**	**0.81%**	-0.16%	**0.44%**	**0.36%**	0.03%	**0.58%**	**1.03%**
Log dividend-payout ratio	0.20%	**0.37%**	**0.70%**	0.15%	**0.35%**	**0.24%**	-0.09%	**0.31%**	**0.63%**
Book-to-market ratio	-0.76%	**0.28%**	**0.63%**	-0.54%	**0.38%**	**0.19%**	-0.27%	**0.44%**	**0.53%**
T-Bill rate	-0.15%	**0.25%**	**1.06%**	-0.13%	**0.24%**	**0.85%**	-0.37%	**0.27%**	**0.63%**
Long-term yield	-0.26%	**0.25%**	**1.00%**	-0.22%	**0.26%**	**0.70%**	-0.25%	**0.26%**	**0.76%**
Long-term return	-0.03%	**0.35%**	**1.41%**	-0.03%	**0.33%**	**0.01%**	0.22%	**0.44%**	**0.75%**
Term spread	0.21%	**0.29%**	**0.96%**	0.15%	**0.28%**	**0.52%**	-0.07%	**0.32%**	**0.56%**
Default yield spread	-0.11%	**0.10%**	**0.56%**	-0.19%	**0.12%**	**0.00%**	-0.04%	**0.24%**	**0.49%**
Default return spread	-0.06%	**0.31%**	**0.65%**	-0.74%	**0.03%**	**0.30%**	-0.46%	**0.22%**	**0.53%**
Stock variance	0.17%	-0.14%	**0.61%**	0.00%	0.00%	-0.20%	-0.02%	**0.21%**	**0.55%**
Net equity expansion	-0.05%	**0.25%**	**0.77%**	-0.26%	**0.29%**	**0.04%**	-1.75%	**0.19%**	**0.50%**
Inflation	-0.09%	**0.26%**	**0.72%**	0.02%	**0.31%**	**0.39%**	-0.16%	**0.27%**	**0.54%**
Log total net payout yield	-0.30%	**0.30%**	**0.68%**						
Percent equity issuing							-0.57%	**0.25%**	**0.58%**

Summary

This brief shows how economic theory can be used to improve the forecasts of stock returns. To do so, we have developed a new methodology for imposing constraints that rule out negative excess returns and bound the conditional Sharpe ratio from above and below. We have found that imposing economic constraints on the equity premium forecast improves the predictive accuracy for nearly all of the prediction models we consider. Most importantly, when used to make investment decisions, the constrained forecasts are found to yield higher certainty equivalent returns than their unconstrained counterparts.

4. In particular, each row of Table 1 corresponds to a different univariate prediction model. So for example, the first row refers to a model where excess returns are predicted by means of the lagged log dividend-price ratio. Red entries highlight all instances in which the constrained models in (2) and (3) perform better than the unconstrained model in (1).

5. These CER are calculated for an investor with power utility and a coefficient of relative risk aversion of five.

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