Syllabi for Required Courses

Math 201a: Algebra I

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) Group theory:
   Quick review of the basic theory (subgroups, homomorphisms, etc.).
   Group actions
   Sylow theorems.
   Solvable and nilpotent groups.
   Free groups, presentations.

2) Category theory
   Basic notions of categories and functors
   Example of categories, basic constructions (products), universal objects
   Use of Category language when treating the different part of the course
   Natural transformations

3) Rings and Modules:
   Review of basic theory (subrings, ideals, fields, homomorphisms, etc.)
   PID’s, UFD’s, Polynomial rings.
   Modules (over a commutative ring)
   Tensor products, exterior and symmetric powers, determinants.
   Finitely generated modules over a PID and applications.

4) Field theory:
   Field extensions, splitting fields, finite fields.
   Separable and inseparable extensions, algebraic closure.
   Fundamental theorem of Galois theory, solvability by radicals.

Additional topics (if time permits):
   • Field theory (trace and norm, transcendental extensions, purely inseparable extensions, infinite Galois extensions, Kummer theory).
   • Category theory (adjoint functors, Yoneda’s lemma, limits).

Possible Texts:
   • Lang: Algebra
   • Jacobson: Basic Algebra
Math 201b: Algebra II

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) **Homological algebra**:
   - Exact sequences
   - Complexes and homology
   - Projective and injective modules
   - Ext and Tor

2) **Commutative algebra**:
   - Chain conditions
   - Hilbert basis theorem
   - Localization
   - Nullstellensatz

3) **Representation theory (of finite groups)**:
   - Maschke’s theorem
   - Schur’s Lemma
   - Fundamental isomorphism theorem for the group algebra
   - Characters
   - Frobenius reciprocity

**Additional topics (if time permits):**
- Non-commutative algebra (Semisimple rings, Wedderburn’s theorem).
- Additional representation theory (representations of Sn, Brauer’s theorem, representations in finite characteristic, representations of Lie algebras and Lie groups).
- Commutative algebra/number theory (integrality, completion, DVR’s, Dedekind domains).
- Commutative algebra/algebraic geometry (dimension theory, Noether normalization, the ideal-variety correspondence, primary decomposition).

**Possible Texts:**
- Lang: Algebra
- Jacobson: Basic Algebra

225a: Geometry of Manifolds

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) **Manifolds**:
Change of coordinates
Differential structure

2) Tangent vectors
Tangent bundle
Derivations
Vector fields
Lie bracket
Tensors

3) Vector bundles
Basics of vector bundles
Normal bundles
Pullback construction

4) Differential topology
Inverse and implicit function theorems—as assigned reading
Transversality
Sard’s theorem—discussion without proof

5) Differential equations and systems
Frobenius Theorem
Existence and uniqueness theorems for ODE’s—discussion without proof

6) Differential forms:
Closed and exact
Poincaré Lemma

7) Integration
Basics of Integration
Stokes Theorem
Orientations and volume elements

Additional topics (if time permits).
• Basic Lie Groups: Lie algebra, one parameter subgroups, structural equations, left and right
  invariant vector fields.
• Principal bundles; connections on vector bundles
• Frobenius Theorem in differential form version
• de Rham cohomology and theorem

Possible Texts:
• Lee: Introduction to Smooth Manifolds
• Hitchin’s Oxford Notes: Differentiable Manifolds
  (http://people.maths.ox.ac.uk/hitchin/files/LectureNotes/Differentiable_manifolds/manifolds2014.pdf)
• Spivak: A Comprehensive Introduction to Differential Geometry, vol. I
Math 211a: Real Analysis

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) General topology
   Basic axioms of topology, continuous maps
   Compact spaces
   Metric spaces, completeness, Baire Category Theorem
   Stone-Weierstrass Theorem
   Arzela-Ascoli Theorem, an application to Peano’s Existence Theorem

2) Banach spaces:
   Topological vector spaces; normed spaces
   Linear functionals, dual spaces, Hahn-Banach Theorem
   Banach spaces
   Contraction principle, applications to Picard’s Existence Theorem and Implicit Function
   Theorem
   Hilbert spaces (basic theory), Riesz Representation Theorem

3) Measure theory:
   Algebras and sigma-algebras of sets, measurable functions
   Measure spaces
   Integrable functions, integration and convergence theorems
   Extension of measures from algebras to sigma-algebras
   Lebesgue measurable sets, Lebesgue measure on \( \mathbb{R}^n \)
   Products measures, Fubini’s Theorem
   Signed/complex measures, Radon-Nikodym Theorem, Hahn and Jordan decompositions
   \( L^p \)-spaces
   Egorov’s Theorem, Lusin’s Theorem

Additional topics (if time permits):
   • Open mapping theorem, closed graph theorem (to be covered in Functional Analysis)
   • Functions of bounded variation, Lebesgue-Stieltjes integral
   • Convolution in \( L^1(\mathbb{R}^n) \)
   • Fourier transform, Fourier inversion
   • Fourier series, Poisson summation, Fejer’s Theorem
   • Probability theory. Basic ergodic theory.

Possible Texts:
   • Kolmogorov/Fomin: Introductory Real Analysis
Math 211b: Complex Analysis

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) Complex analytic functions
   Riemann sphere and rational functions
   Complex derivatives and Cauchy-Riemann equations
   Holomorphic functions in one variable (basic theory)

2) Integration
   Cauchy’s theorem, Cauchy’s integral formula
   Applications to: Fundamental Theorem of Algebra, Liouville’s theorem, Morera’s theorem, Gauss’ mean value theorem
   Maximum principle, Rouche’s theorem, argument principle
   Schwarz reflection principle, analytic continuation

3) Conformal maps
   Fractional-linear transformations
   Open mapping theorem
   Riemann mapping theorem
   Harmonic and subharmonic functions, Poisson's formula

4) Power series, partial fractions, special functions
   Taylor series
   Classification of singularities
   Laurent series
   Weierstrass theorem
   Mittag-Leffler theorem
   Infinite products and partial sums
   Elliptic functions, Weierstrass \( \wp \)-function

Additional topics (if time permits):
Math 221a: Topology I

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) CW-Complexes
   Definitions, direct limit topology

2) Covering Spaces and Fundamental Group
   Basic Definitions (homotopy, fundamental group)
   Existence and classification of covering spaces
   Correspondence between subgroups and covering spaces
   Van Kampen’s theorem

3) Homology Theory:
   Definitions of simplicial complexes and simplicial homology
   Definition of singular homology
   Long exact sequence of a pair, excision, Mayer-Vietoris sequence
   Homology of cell complexes and/or CW complexes
   Computing homology of basic spaces: eg., spheres, projective spaces

4) Applications of homology:
   Maps between spheres; degree of map
   Vector fields
   Fixed point theorems
   Separation theorems (Jordan Curve theorem)

Additional topics (if time permits):
   • Homology with coefficients

Possible Texts:
   • Hatcher: Algebraic Topology
   • Greenberg and Harper: Algebraic Topology: A First Course
   • Munkres: Elements of Algebraic Topology
Math 221b: Topology II

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) Cohomology theory
   - Definition of cohomology
   - Basic properties
   - Cup and cap products

2) Universal coefficients:
   - Tor and homology
   - Ext and cohomology
   - Kunneth theorems

3) Poincare duality
   - Poincare duality for manifolds with and without boundaries

Additional topics (if time permits):
   - Homotopy theory: Basic properties, Hurewicz theorem, path spaces, fibrations
   - Eilenberg-MacLane spaces

Possible Texts:
   - Hatcher: Algebraic Topology
   - Greenberg and Harper: Algebraic Topology: A First Course
   - Munkres: Elements of Algebraic Topology


Core topics (ALWAYS covered): Please use this checklist as you go through the course.

1) Numerical linear algebra:
   - Floating point arithmetic
   - Polynomial interpolation
   - Linear systems and LU factorization
   - Least squares and QR factorization
   - Singular Value Decomposition

2) Numerical differential equations:
   - Quadrature methods
   - Euler and Runge-Kutta methods
Accuracy and stability of timestepping schemes

Additional topics (if time permits):
• Optimization, eigenvalue problems, finite difference methods for PDE’s, Lax Equivalence Theorem

Possible Texts:
• Heath: Scientific Computing: An Introductory Survey
Trefethen and Bau: Numerical Linear Algebra