Syllabi for Required Courses

Math 201a: Algebra I

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) **Group theory:**
Quick review of the basic theory (subgroups, homomorphisms, etc.).
Group actions
Sylow theorems.
Solvable and nilpotent groups.
Free groups, presentations.

2) **Category theory**
Basic notions of categories and functors
Example of categories, basic constructions (products), universal objects
Use of Category language when treating the different part of the course
Natural transformations

3) **Rings and Modules:**
Review of basic theory (subrings, ideals, fields, homomorphisms, etc.)
PID's, UFD's, Polynomial rings.
Modules (over a commutative ring)
Tensor products, exterior and symmetric powers, determinants.
Finitely generated modules over a PID and applications.

4) **Field theory:**
Field extensions, splitting fields, finite fields.
Separable and inseparable extensions, algebraic closure.
Fundamental theorem of Galois theory, solvability by radicals.

Additional topics (if time permits):
- Field theory (trace and norm, transcendental extensions, purely inseparable extensions, infinite Galois extensions, Kummer theory).
- Category theory (adjoint functors, Yoneda's lemma, limits).

Possible Texts:
- Lang: Algebra
- Jacobson: Basic Algebra

Math 201b: Algebra II

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) **Homological algebra:**
Exact sequences
Complexes and homology
Projective and injective modules
Ext and Tor

2) **Commutative algebra:**
Chain conditions
Hilbert basis theorem
Localization.
Nullstellensatz

3) **Representation theory (of finite groups):**
Maschke's theorem
Schur's Lemma
Fundamental isomorphism theorem for the group algebra
Characters.
Frobenius reciprocity

Additional topics (if time permits):

• Non-commutative algebra (Semisimple rings, Wedderburn's theorem). • Additional representation theory (representations of Sn, Brauer's theorem, representations in finite characteristic, representations of Lie algebras and Lie groups).
• Commutative algebra/number theory (integrality, completion, DVR's, Dedekind domains). • Commutative algebra/algebraic geometry (dimension theory, Noether normalization, the ideal-variety correspondence, primary decomposition).

Possible Texts:

• Lang: Algebra
• Jacobson: Basic Algebra

225a: Geometry of Manifolds

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) Manifolds:

Change of coordinates
Differential structure

2) Tangent vectors
Tangent bundle
Derivations
Vector fields
Lie bracket
Tensors

3) Vector bundles
Basics of vector bundles
Normal bundles
Pullback construction

4) Differential topology
Inverse and implicit function theorems—as assigned reading
Transversality
Sard's theorem—discussion without proof

5) Differential equations and systems
Frobenius Theorem
Existence and uniqueness theorems for ODE's—discussion without proof

6) Differential forms:
Closed and exact
Poincaré Lemma

7) Integration
Basics of Integration
Stokes Theorem
Orientations and volume elements

Additional topics (if time permits).

- Basic Lie Groups: Lie algebra, one parameter subgroups, structural equations, left and right invariant vector fields.
- Principal bundles; connections on vector bundles
- Frobenius Theorem in differential form version
- de Rham cohomology and theorem

Possible Texts:

- Lee: Introduction to Smooth Manifolds
- Hitchin's Oxford Notes: Differentiable Manifolds (http://people.maths.ox.ac.uk/hitchin/files/LectureNotes/Differentiable_manifolds/manifolds2014. pdf)
- Spivak: A Comprehensive Introduction to Differential Geometry, vol. I

Additional References:

- Warner: Foundations of Differentiable Manifolds and Lie groups
- Milnor: Topology from the Differentiable Viewpoint
- Bott and Tu: Differential Forms in Algebraic Topology

Math 211a: Real Analysis

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) General topology

Basic axioms of topology, continuous maps
Compact spaces
Metric spaces, completeness, Baire Category Theorem
Stone-Weierstrass Theorem
Arzela-Ascoli Theorem, an application to Peano's Existence Theorem

2) Banach spaces:

Topological vector spaces; normed spaces
Linear functionals, dual spaces, Hahn-Banach Theorem
Banach spaces
Contraction principle, applications to Picard's Existence Theorem and Implicit Function Theorem
Hilbert spaces (basic theory), Riesz Representation Theorem

3) Measure theory:

Algebras and sigma-algebras of sets, measurable functions
Measure spaces
Integrable functions, integration and convergence theorems
Extension of measures from algebras to sigma-algebras
Lebesgue measurable sets, Lebesgue measure on R^n
Products measures, Fubini's Theorem
Signed/complex measures, Radon-Nikodym Theorem, Hahn and Jordan decompositions
L^p-spaces
Egorov's Theorem, Lusin's Theorem

Additional topics (if time permits):

- Open mapping theorem, closed graph theorem (to be covered in Functional Analysis)
- Functions of bounded variation, Lebesgue-Stieltjes integral
- Convolution in L^1(R^n)
- Fourier transform, Fourier inversion
- Fourier series, Poisson summation, Fejer's Theorem
- Probability theory. Basic ergodic theory.

Possible Texts:

- Kolmogorov/Fomin: Introductory Real Analysis

- Lang: Real and Functional Analysis
- Loomis: Abstract Harmonic Analysis
- Royden: Real Analysis
- Rudin: Real and Complex Analysis
- Stein/Shakarchi: Complex Analysis

Math 211b: Complex Analysis

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) Complex analytic functions

Riemann sphere and rational functions
Complex derivatives and Cauchy-Riemann equations
Holomorphic functions in one variable (basic theory)

2) Integration

Cauchy's theorem, Cauchy's integral formula
Applications to: Fundamental Theorem of Algebra, Liouville's theorem, Morera's theorem, Gauss' mean value theorem
Maximum principle, Rouche's theorem, argument principle
Schwarz reflection principle, analytic continuation

3) Conformal maps

Fractional-linear transformations
Open mapping theorem
Riemann mapping theorem
Harmonic and subharmonic functions, Poisson's formula

4) Power series, partial fractions, special functions

Taylor series
Classification of singularities
Laurent series
Weierstrass theorem
Mittag-Leffler theorem
Infinite products and partial sums
Elliptic functions, Weierstrass $\wp$-function

Additional topics (if time permits):

- Introduction to Riemann surfaces. Connections with the theory of covering spaces and cohomology. Gamma and zeta functions. Picard's theorem. Runge's theorem. Inhomogeneous Cauchy-Riemann equation. Several complex variables (Hartog's theorem). Phragmen-Lindelof theorem. Vitali-Montel Theorem. Jensen's Formula.

Possible Texts:

- Ahlfors: Complex Analysis
- Conway: Functions of One Complex Variable
- Narasimhan/Nievergelt: Complex Analysis in One Variable
- Gameiln: Comlpex Analysis
- Stein/Shakarchi: Real Analysis

Math 221a: Topology I

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) CW-Complexes
Definitions, direct limit topology

2) **Covering Spaces and Fundamental Group**
Basic Definitions (homotopy, fundamental group)
Existence and classification of covering spaces
Correspondence between subgroups and covering spaces
Van Kampen's theorem

3) Homology Theory:
Definitions of simplicial complexes and simplicial homology
Definition of singular homology
Long exact sequence of a pair, excision, Mayer-Vietoris sequence
Homology of cell complexes and/or CW complexes
Computing homology of basic spaces: eg., spheres, projective spaces

4) Applications of homology:
Maps between spheres; degree of map
Vector fields
Fixed point theorems
Separation theorems (Jordan Curve theorem)

Additional topics (if time permits):

- Homology with coefficients

Possible Texts:

- Hatcher: Algebraic Topology
- Greenberg and Harper: Algebraic Topology: A First Course
- Munkres: Elements of Algebraic Topology

Math 221b: Topology II

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) **Cohomology theory**
<u>Definition of cohomology</u>
<u>Basic properties</u>
<u>Cup and cap products</u>

2) Universal coefficients:
<u>Tor and homology</u>
<u>Ext and cohomology</u>
<u>Kunneth theorems</u>

3) Poincare duality
<u>Poincare duality for manifolds with and without boundaries</u>

Additional topics (if time permits):
- Homotopy theory: Basic properties, Hurewicz theorem, path spaces, fibrations
- Eilenberg-MacLane spaces

Possible Texts:
- Hatcher: Algebraic Topology
- Greenberg and Harper: Algebraic Topology: A First Course
- Munkres: Elements of Algebraic Topology

Math 232a: Numerical Methods for Scientific Computing

Core topics (ALWAYS covered): Please use this checklist as you go through the course.

1) Numerical linear algebra:
Floating point arithmetic
Polynomial interpolation
Linear systems and LU factorization
Least squares and QR factorization
Singular Value Decomposition

2) Numerical differential equations:
Quadrature methods
Euler and Runge-Kutta methods

Accuracy and stability of timestepping schemes

Additional topics (if time permits):

- Optimization, eigenvalue problems, finite difference methods for PDE's, Lax Equivalence Theorem

Possible Texts:

- Heath: Scientific Computing: An Introductory Survey

Trefethen and Bau: Numerical Linear Algebra