Linear Representations of Finite Groups

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I had the pleasure of leading a project for the spring 2026 Guided Reading Program at Brandeis University, organized by the mathematics department. My project was on Linear Representations of Finite Groups, a classical subject at the intersection of algebra, character theory, and geometry with various applications to the real world, such as computer science, machine learning, deep learning (LLMs), operational research and chemistry, and so on.

The project was centered on the basic question of how a finite group can act linearly on a vector space. Although this question is elementary to state, it quickly leads to a rich and powerful theory. One of the appealing features of the subject is that it gives a concrete way to study abstract groups: instead of working only with multiplication tables or presentations, one studies matrices, characters, invariant subspaces, and decompositions into irreducible representations. Because of its deep nature in pure mathematics as well as real life applications, this GRP of mine had a lot of participants. We had at least six undergraduates joining whose background ranged from pure math or applied math major to physics or economics major. I think this participation alone showed how useful this subject is and how attractive GRP is to undergraduate students.

We mainly used our department's own representation theorist Justin Campbell's online notes as our primary reference. The notes are by no means easy: Prof Campbell wrote down those notes while teaching a graduate course at Caltech. However, the notes are very well-written and the Brandeis students enjoyed reading them. Another nice thing here is that Prof Campbell was glad we used his notes and was willing to answer the students' questions when reading the notes. I think this also creates a better environment for the relationship between the department and students.

Our journey began with the definition of a representation of a finite group over the complex numbers, together with basic examples coming from permutation representations, one-

dimensional characters, and the regular representation. We then studied invariant subspaces, irreducible representations, and Schur's lemma, which is one of the first points where the theory becomes both elegant and powerful.

Toward the end of the project, we briefly discussed induction and restriction of representations. This gave the students a glimpse of how representations of a subgroup can be related to representations of the whole group, and vice versa. We also discussed how character theory provides efficient tools for decomposing representations without explicitly diagonalizing matrices or writing down all submodules by hand.

I believe this was a very successful Guided Reading Program project. We used a different approach to learning this time. Each week we had one student presenting part of the notes. Most of them presented. I think this is a great way for them to learn things better (by teaching others) and to learn how to present mathematics on the court. Additionally, I think it also created a better place for them to communicate and know each other, where they might in the future learn further math together.

Alan Hou is a PhD candidate who joined the department in 2022. He works with Professor Solomon Friedberg and Professor Omer Offen studying representation theory, in particular regarding the Langlands functoriality program and representations of reductive groups over local fields.