Syllabi for Required Courses

Math 131a: Algebra I

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) **Group theory:**
- Quick review of the basic theory (subgroups, homomorphisms, etc.).
- Group actions
- Sylow theorems.
- Solvable and nilpotent groups.
- Free groups, presentations.

2) **Category theory**
- Basic notions of categories and functors
- Example of categories, basic constructions (products), universal objects
- Use of Category language when treating the different part of the course
- Natural transformations

3) **Rings and Modules:**
- Review of basic theory (subrings, ideals, fields, homomorphisms, etc.)
- PID's, UFD's, Polynomial rings.
- Modules (over a commutative ring)
- Tensor products, exterior and symmetric powers, determinants.
- Finitely generated modules over a PID and applications.

4) **Field theory:**
- Field extensions, splitting fields, finite fields.
- Separable and inseparable extensions, algebraic closure.
- Fundamental theorem of Galois theory, solvability by radicals.

Additional topics (if time permits):
- Field theory (trace and norm, transcendental extensions, purely inseparable extensions, infinite Galois extensions, Kummer theory).
- Category theory (adjoint functors, Yoneda's lemma, limits).

Possible Texts:
- Lang: Algebra
- Jacobson: Basic Algebra

Math 131b: Algebra II

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) **Homological algebra:**
- <u>Exact sequences</u>
- <u>Complexes and homology</u>
- Projective and injective modules
- Ext and Tor

2) **Commutative algebra:**
- <u>Chain conditions</u>
- <u>Hilbert basis theorem</u>
- <u>Localization.</u>
- Nullstellensatz

3) **Representation theory (of finite groups):**
- <u>Maschke's theorem</u>
- <u>Schur's Lemma</u>
- <u>Fundamental isomorphism theorem for the group algebra</u>
- <u>Characters.</u>
- Frobenius reciprocity

Additional topics (if time permits):
- Non-commutative algebra (Semisimple rings, Wedderburn's theorem).
- Additional representation theory (representations of Sn, Brauer's theorem, representations in finite characteristic, representations of Lie algebras and Lie groups).
- Commutative algebra/number theory (integrality, completion, DVR's, Dedekind domains).
- Commutative algebra/algebraic geometry (dimension theory, Noether normalization, the ideal-variety correspondence, primary decomposition).

Possible Texts:
- Lang: Algebra
- Jacobson: Basic Algebra

140a: Geometric Analysis

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) **Manifolds:**
- Change of coordinates
- Differential structure

2) **Tangent vectors**
- Tangent bundle
- Derivations
- Vector fields
- Lie bracket
- Tensors

3) **Vector bundles**
- Basics of vector bundles
- Normal bundles
- Pullback construction

4) **Differential topology**
- Inverse and implicit function theorems—as assigned reading
- Transversality
- Sard's theorem—discussion without proof

5) **Differential equations and systems**
- Frobenius Theorem
- Existence and uniqueness theorems for ODE's—discussion without proof

6) **Differential forms:**
- Closed and exact
- Poincaré Lemma

7) **Integration**
- Basics of Integration
- Stokes Theorem
- Orientations and volume elements

Additional topics (if time permits).
- Basic Lie Groups: Lie algebra, one parameter subgroups, structural equations, left and right invariant vector fields.
- Principal bundles; connections on vector bundles
- Frobenius Theorem in differential form version
- de Rham cohomology and theorem

Possible Texts:

- Lee: Introduction to Smooth Manifolds
- Hitchin's Oxford Notes: Differentiable Manifolds (http://people.maths.ox.ac.uk/hitchin/files/LectureNotes/Differentiable_manifolds/manifolds2014.pdf)
- Spivak: A Comprehensive Introduction to Differential Geometry, vol. I

Additional References:

- Warner: Foundations of Differentiable Manifolds and Lie groups
- Milnor: Topology from the Differentiable Viewpoint
- Bott and Tu: Differential Forms in Algebraic Topology

Math 141a: Real Analysis

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) **General topology**

- Basic axioms of topology, continuous maps
- Compact spaces
- Metric spaces, completeness, Baire Category Theorem
- Stone-Weierstrass Theorem
- Arzela-Ascoli Theorem, an application to Peano's Existence Theorem

2) **Banach spaces:**

- Topological vector spaces; normed spaces
- Linear functionals, dual spaces, Hahn-Banach Theorem
- Banach spaces
- Contraction principle, applications to Picard's Existence Theorem and Implicit Function Theorem
- Hilbert spaces (basic theory), Riesz Representation Theorem

3) **Measure theory:**

- Algebras and sigma-algebras of sets, measurable functions
- Measure spaces
- Integrable functions, integration and convergence theorems
- Extension of measures from algebras to sigma-algebras
- Lebesgue measurable sets, Lebesgue measure on R^n
- Products measures, Fubini's Theorem
- Signed/complex measures, Radon-Nikodym Theorem, Hahn and Jordan decompositions
- L^p-spaces
- Egorov's Theorem, Lusin's Theorem

Additional topics (if time permits):

- Open mapping theorem, closed graph theorem (to be covered in Functional Analysis)
- Functions of bounded variation, Lebesgue-Stieltjes integral
- Convolution in L^1(R^n)
- Fourier transform, Fourier inversion
- Fourier series, Poisson summation, Fejer's Theorem
- Probability theory. Basic ergodic theory.

Possible Texts:

- Kolmogorov/Fomin: Introductory Real Analysis
- Lang: Real and Functional Analysis
- Loomis: Abstract Harmonic Analysis
- Royden: Real Analysis
- Rudin: Real and Complex Analysis
- Stein/Shakarchi: Complex Analysis

Math 141b: Complex Analysis

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) **Complex analytic functions**
- <u>Riemann sphere and rational functions</u>
- <u>Complex derivatives and Cauchy-Riemann equations</u>
- <u>Holomorphic functions in one variable (basic theory)</u>

2) **Integration**
- <u>Cauchy’s theorem, Cauchy’s integral formula</u>
- <u>Applications to: Fundamental Theorem of Algebra, Liouville’s theorem, Morera’s theorem, Gauss’ mean value theorem</u>
- <u>Maximum principle, Rouche’s theorem, argument principle</u>
- <u>Schwarz reflection principle, analytic continuation</u>

3) **Conformal maps**
- <u>Fractional-linear transformations</u>
- Open mapping theorem
- Riemann mapping theorem
- Harmonic and subharmonic functions, Poisson's formula

4) **Power series, partial fractions, special functions**
- <u>Taylor series</u>
- <u>Classification of singularities</u>
- <u>Laurent series</u>
- <u>Weierstrass theorem</u>
- Mittag-Leffler theorem
- Infinite products and partial sums
- Elliptic functions, Weierstrass $\wp$-function

Additional topics (if time permits):
- Introduction to Riemann surfaces. Connections with the theory of covering spaces and cohomology. Gamma and zeta functions. Picard’s theorem. Runge’s theorem. Inhomogeneous Cauchy-Riemann equation. Several complex variables (Hartog’s theorem). Phragmen-Lindelof theorem. Vitali-Montel Theorem. Jensen’s Formula.

Possible Texts:
- Ahlfors: Complex Analysis
- Conway: Functions of One Complex Variable
- Narasimhan/Nievergelt: Complex Analysis in One Variable
- Gameiln: Comlpex Analysis
- Stein/Shakarchi: Real Analysis

Math 151a: Topology I

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) **CW-Complexes**
- Definitions, direct limit topology

2) **Covering Spaces and Fundamental Group**
- Basic Definitions (homotopy, fundamental group)
- Existence and classification of covering spaces
- Correspondence between subgroups and covering spaces
- Van Kampen's theorem

3) **Homology Theory:**
- Definitions of simplicial complexes and simplicial homology
- Definition of singular homology
- Long exact sequence of a pair, excision, Mayer-Vietoris sequence
- Homology of cell complexes and/or CW complexes
- Computing homology of basic spaces: eg., spheres, projective spaces

4) **Applications of homology:**
- Maps between spheres; degree of map
- Vector fields
- Fixed point theorems
- Separation theorems (Jordan Curve theorem)

Additional topics (if time permits):
- Homology with coefficients

Possible Texts:
- Hatcher: Algebraic Topology
- Greenberg and Harper: Algebraic Topology: A First Course
- Munkres: Elements of Algebraic Topology

Math 151b: Topology II

Core topics: the topics underlined below should be ALWAYS covered; the rest should be mentioned and discussed, and the students should be directed to appropriate literature.

1) **Cohomology theory**
- Definition of cohomology
- Basic properties
- Cup and cap products

2) **Universal coefficients:**
- Tor and homology
- Ext and cohomology
- Kunneth theorems

3) **Poincare duality**
- Poincare duality for manifolds with and without boundaries

Additional topics (if time permits):
- Homotopy theory: Basic properties, Hurewicz theorem, path spaces, fibrations
- Eilenberg-MacLane spaces

Possible Texts:
- Hatcher: Algebraic Topology
- Greenberg and Harper: Algebraic Topology: A First Course
- Munkres: Elements of Algebraic Topology

Math 162: Numerical Methods for Scientific Computing

Core topics (ALWAYS covered): Please use this checklist as you go through the course.

1) Numerical linear algebra:

- ☐ Floating point arithmetic
- ☐ Polynomial interpolation
- ☐ Linear systems and LU factorization
- ☐ Least squares and QR factorization
- ☐ Singular Value Decomposition

2) Numerical differential equations:

- ☐ Quadrature methods
- ☐ Euler and Runge-Kutta methods
- ☐ Accuracy and stability of timestepping schemes

Additional topics (if time permits):

- Optimization, eigenvalue problems, finite difference methods for PDE's, Lax Equivalence Theorem

Possible Texts:

- Heath: Scientific Computing: An Introductory Survey

Trefethen and Bau: Numerical Linear Algebra