Linear Algebra/Multivariable Calculus Placement Exam

Instructions: This exam should be taken by students trying to decide between Math 15a or 20a, on the one hand, and Math 22a on the other. (For descriptions of these courses, see the Bulletin.) When you are done taking the exam, enter your answers into the answer sheet, which may be found at `http://www.brandeis.edu/departments/mathematics/undergraduate/math22placement.html`. You will need to login using your UNet id/password, which you should have already obtained. When you have completed the exam, you should then request an override on Workdays to enroll in Math 22a. Your placement test will be graded by the mathematics department. If your test results place you in 22a, an override would be provided. If your test results place you in Math 15a, you will be asked to enroll in Math 15s instead.
Most of the problems are multiple choice; for these you can just enter the correct letter choice for your answer. For the other problems, simply type in your answer. The answer sheet also asks for some additional information about your background in mathematics that will be helpful in determining your placement

1. If $|x| \geq 3$ and $|y| \leq 1$, which of the following is necessarily true?
(a) $x - y \geq 2$
(b) $|x + y| \leq 3$
(c) $|x + 2y| \geq 1$
(d) None of the above.

2. The unit sphere $x^2 + y^2 + z^2 = 1$ intersects the plane $2x + y - z = 0$ in 3-space in how many points?
(a) No points.
(b) Exactly 1 point.
(c) Exactly 2 points.
(d) More than 2 points.

3. Consider the line L in 3-space given by equations
$$\frac{x-1}{3} = \frac{y+2}{2} = z - 1.$$
Find the equation of a plane P through the origin, in the form $ax + by + cz = 0$, with the property that L and P do not intersect.

4. If $0 \leq x \leq \pi$, $0 \leq y \leq \pi$, and $x < y$, which of the following is necessarily true?
(a) $\sin(x) < \sin(y)$
(b) $\cos(x) < \cos(y)$
(c) $\sin(\frac{x}{2}) < \sin(\frac{y}{2})$
(d) $\sin(2x) < \sin(2y)$
(e) None of the above.

5. Let A and B be subsets of the real numbers $\mathbb{R}$. For any subset X of the real numbers, let X^c denote its complement (ie all real numbers **not** in X). The symbols $\cup$ and $\cap$ represent union and intersection, respectively. Which of the following sets is the same as $(A \cap B^c)^c$.
(a) $A^c \cup B$.
(b) $A^c \cap B$.
(c) $A \cup B^c$.
(d) None of the above.

6. Which of the following statements is logically equivalent to the statement:

Every polynomial function is differentiable.

(a) Every differentiable function is a polynomial.
(b) If a function is not differentiable, then it is not a polynomial.
(c) If a function is not a polynomial, then it is not differentiable.
(d) All of (a), (b), and (c).
(e) None of (a), (b), or (c).

7. Suppose S is a set of whole numbers with the property that every even number in S is divisible by 5. Which of the following must be true? List all correct answers.
(a) 2 is not in S.
(b) 5 is not in S.
(c) S contains all multiples of 10.
(d) Every even number in S is divisible by 10.
(e) S contains no odd numbers.

8. Find two numbers a and b such that the function $f(x) = ae^x + be^{-x}$ satisfies $f(0) = 5$ and $f'(0) = -1$.

9. Which of the following statements are true in Euclidean 3-space? List all correct answers.
(a) There is a unique line passing through any two distinct points.
(b) There is a unique plane passing through any three distinct points, unless they are co-linear.
(c) There is at least one plane passing through any three distinct points.
(d) There is at most one plane passing through any three distinct points.
(e) None of the above.

10. Consider Euclidean 3-space with x, y, z-axes, and a line L passing through the origin. Let α and β be the respective angles between L and the x-, y-axes. Which of the following statements are true (hint: consider a point (x, y, z) on L such that $x^2 + y^2 + z^2 = 1$):
(a) $\cos(\alpha)^2 + \cos(\beta)^2 \leq 1$.
(b) $\cos(\alpha)^2 + \cos(\beta)^2 = 1$.
(c) $\cos(\alpha)^2 + \sin(\beta)^2 \leq 1$.
(d) $\cos(\alpha)^2 + \sin(\beta)^2 = 1$.
(e) None of the above.