Self-organized buckling patterns underlie transition from macroscopic extension to contraction in active nonlinear elastic networks

Brandeis bioinspired MRSEC

Kanaya Malakar[1], John Berezney[1], Daniel Goldstein[2], Deshpreet Bedi[1] , and Bulbul Chakraborty[1]

[1]Brandeis University, Waltham, MA, USA 02453, [2]Tufts University, MA, USA

Brandeis UNIVERSITY

ABSTRACT

Many fundamental cellular processes require exquisitely orchestrated large-scale reorganization of structural filaments. One mechanism of reorganization is via internal forces generated by motor proteins. The transmission of these forces is mediated by a highly non-linear network of fiber-like filaments. To understand the role of buckling and failure in such networks, we examine a model nonlinear elastic network subjected to an internal force dipole. Such networks exhibit non-monotonic elastic deformation in response to the applied force. We observe a transition from linear and non-linear extensibility to global contractile behavior. We demonstrate this emergence of contractile behavior is associated with a large-scale transformation of the underlying lattice structure. These results recapitulate observations of active microtubule/actin gels which transition from extensile flows to global contraction [J. Berezney et. al., arXiv 2110.00166]. This work underscores the importance of cytoskeletal networks and metamaterials whose failure modes and nonlinear mechanics can be engineered to generate complex and adaptive large-scale phenomena.

EXPERIMENTAL MOTIVATION

Active composites are a mixture of active microtubules and passive actin. Experiments done with active composites show a wide range of dynamical organizations (as shown below).

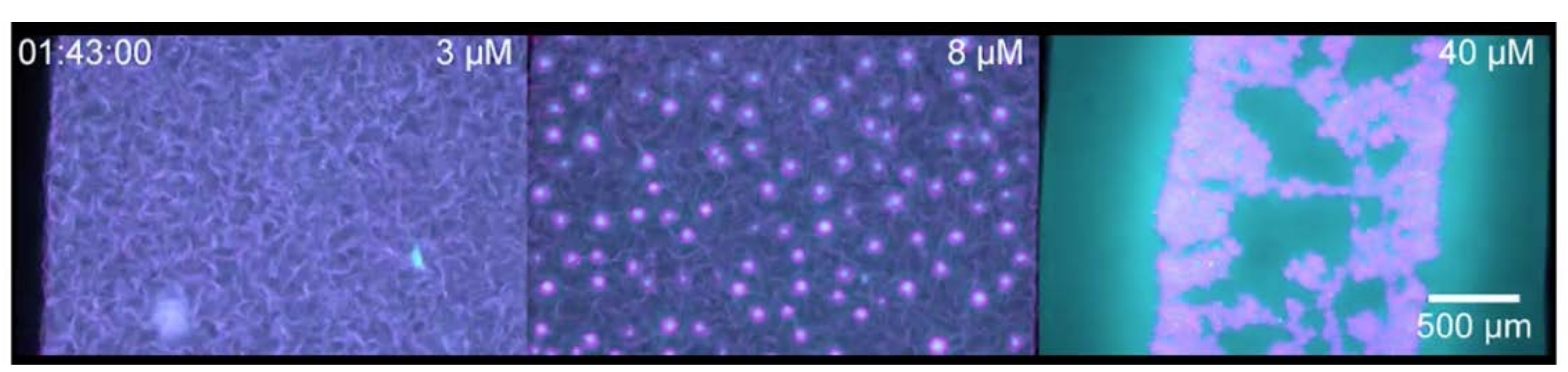


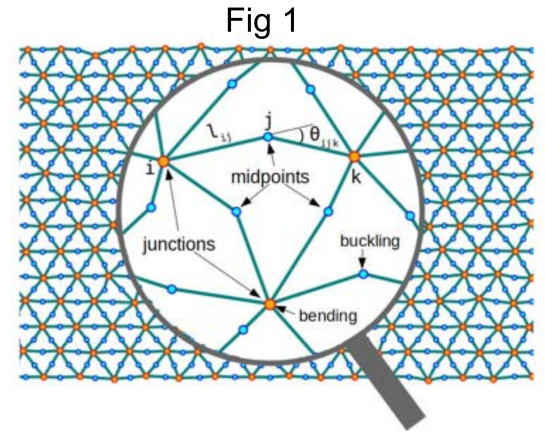
Image: John Berezney

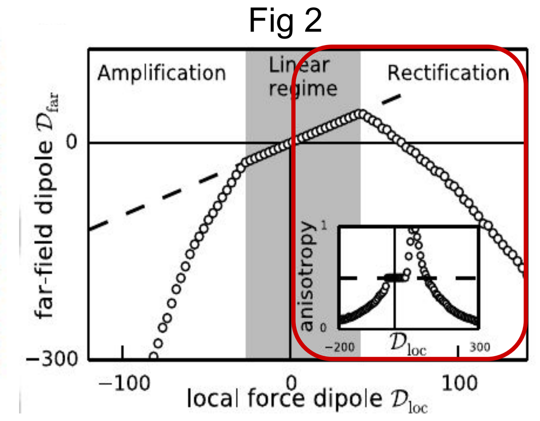
Left: Low actin concentration results in extensile flows. **Middle**: Intermediate actin concentration shows aster formation, and subsequent coexistence of extensile and contractile regions. **Right**: High actin concentration results in global contraction.

Here we attempt to understand the global contractile behavior using a network model of non-linear springs where the springs are allowed to buckle at their midpoints.

THEORETICAL MODEL

$$H = \underbrace{\sum_{bonds\{ij\}} \frac{\mu}{2}(l_{ij} - l_{eq})^2}_{\text{stretching}} + \underbrace{\sum_{angle\{ijk\}} \kappa \sin^2 \frac{\theta_{ijk}}{2}}_{\text{buckling}} - \underbrace{\sum_{forces\{i\}} \mathbf{F}_i . \mathbf{r}_i}_{\text{Active force dipole}}$$





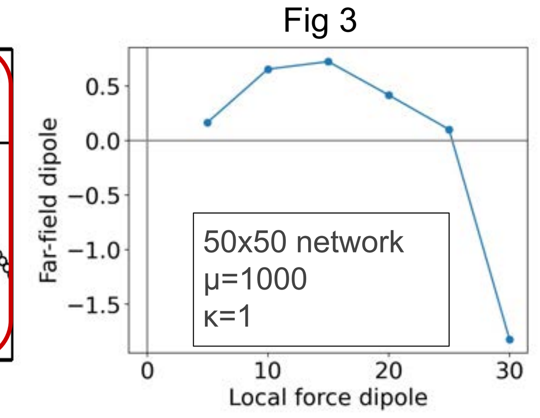
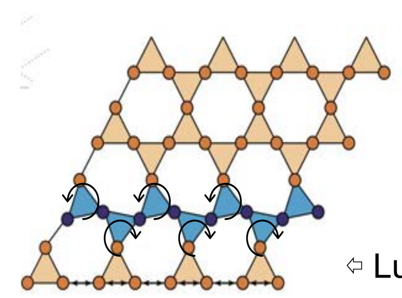


Fig.1: Schematic of non-linear spring network. **Fig.2**: (Ronceray et. al., PNAS, 2016) Far-field dipole response shows contractile behavior for local *extensile* dipole. **Fig.3**: Rectification observed in our simulation where far-field stress is measured using the following formula.

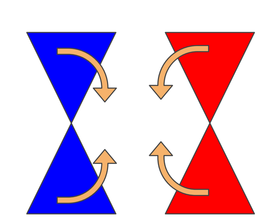
$$\sigma_i = \sum_{bonds\{ij\}} \sigma_{ij} = \sum_j r_{ij} \otimes F_{ij}$$

SELF-ORGANIZED BUCKLING PATTERNS

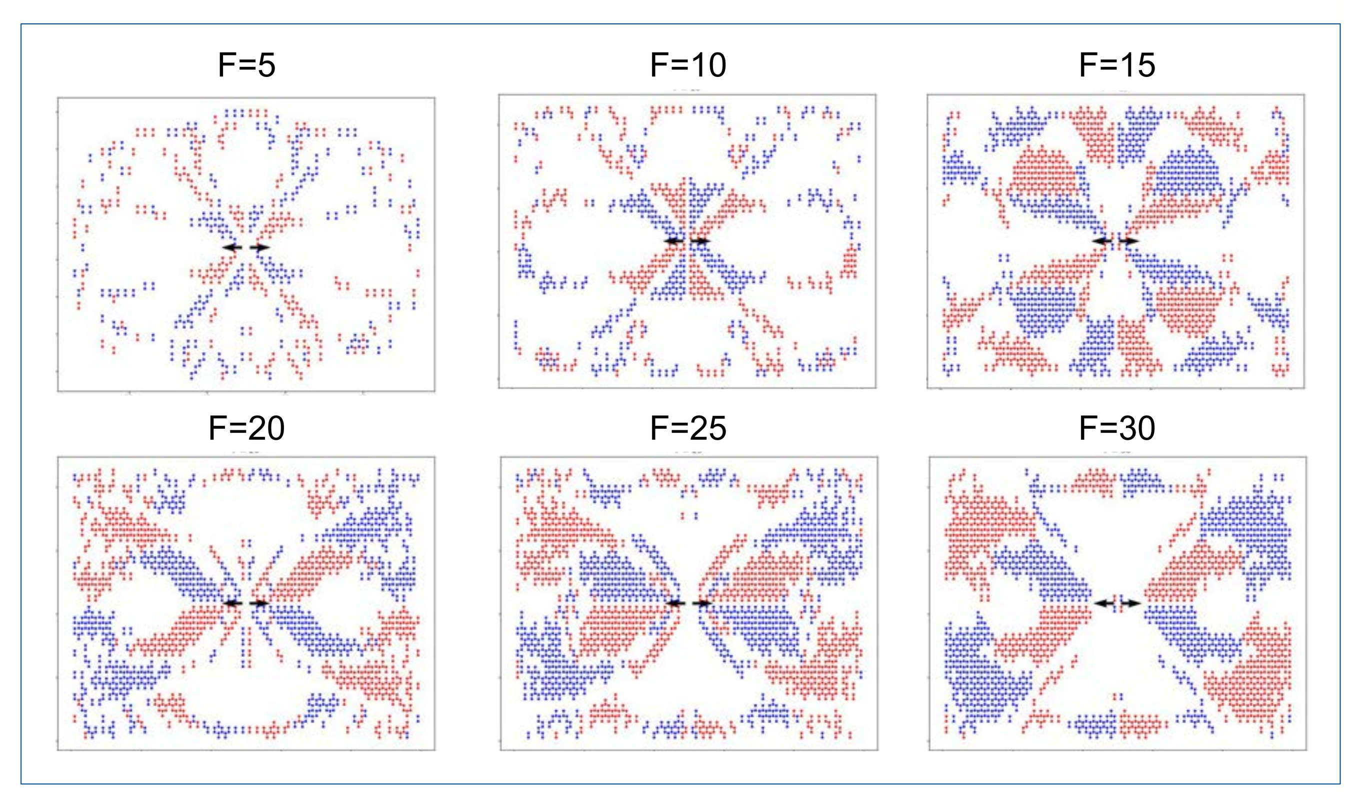


Midpoints of a triangular network form a Kagome network that has zero modes where the lattice can deform without energy cost.

⇦ Lubensky et. al., Rep. Prog. Phys. **78** (2015)

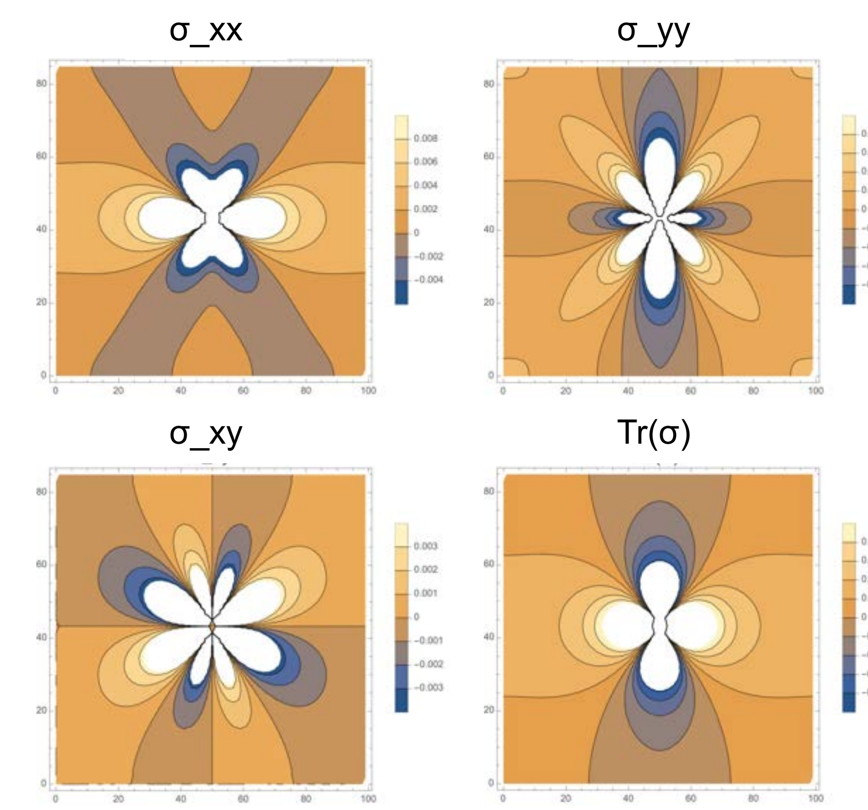


⇧ Pairs of adjacent triangles rotating in opposite directions form an unit of twisted Kagome lattice.



⇧ Spatial distribution of Kagome domains of two types of chirality. (Network size = 50x50, μ=1000, κ=1)

SPATIAL STRESS PATTERN



$$\partial_i \sigma_{ij} = -f_j^{ext}$$

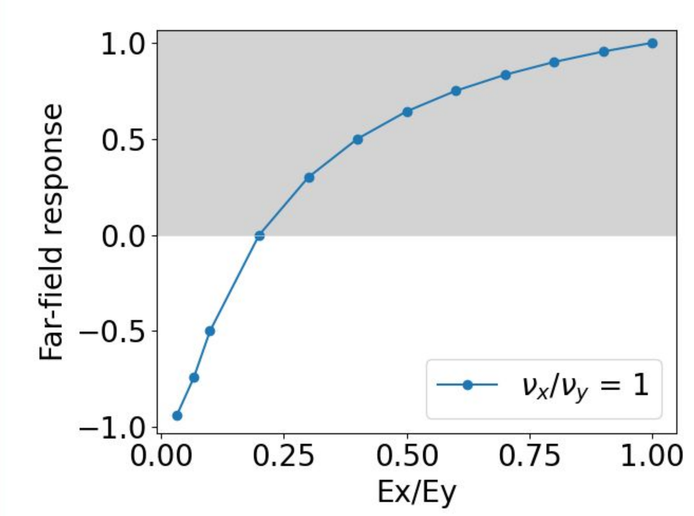
$$\sigma_{ij}(x, y) = \mathcal{G}_{ijx}(x, y) f_x$$

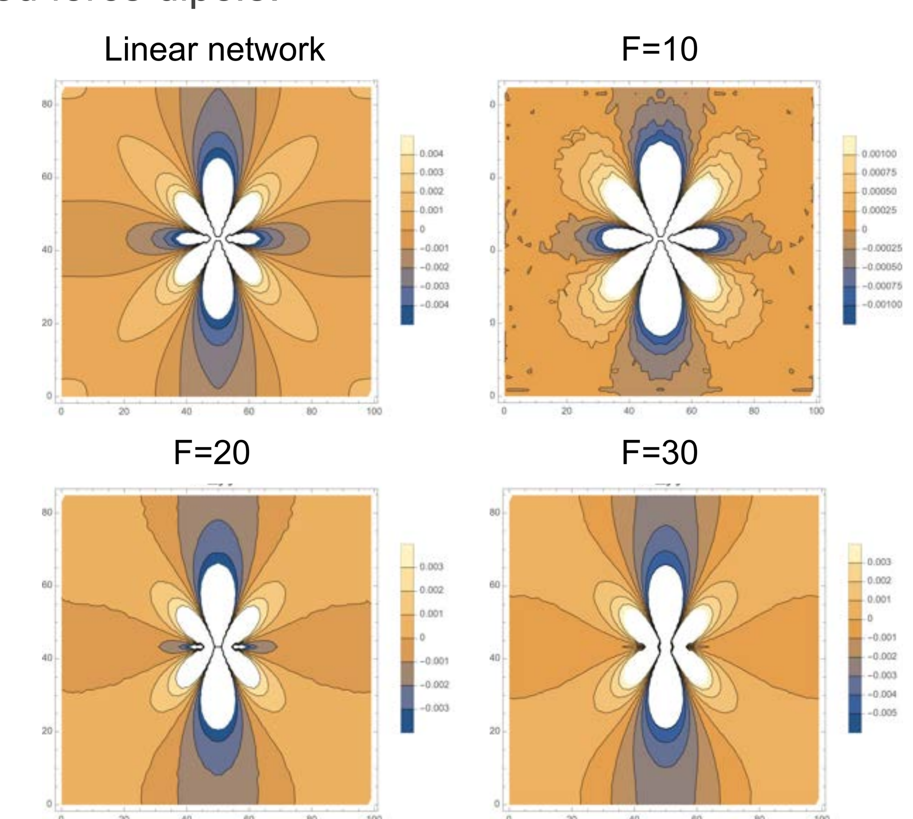
⇦ Spatial pattern of the three independent components and trace of stress tensor in a linear-spring network (no buckling).

A linear network is characterized by isotropic elastic modulus described completely by two Lamé parameters.

$$\lambda = G = \frac{\sqrt{3}\mu}{8}$$

Component of stress perpendicular to the direction of applied force dipole for a non-linear network (κ/μ=1e-3) shows significant deviation from that of a linear network for large magnitudes of applied force dipole. ⇩





⇧ Buckling renormalizes the elastic moduli creating a highly anisotropic one that leads to rectification.

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