

Physics Placement Assessment

Instructions

Complete the following problems below without any reference materials whatsoever. You may use a calculator where you need numbers. Do as many problems as you can in three hours; it is *neither necessary nor expected* to get them all right. We just want to see what you can do in order to advise you about which class to take.

When you are finished download the answers and score your test, giving yourself one point for each part of each problem that you get right. Then follow the suggestions on the answer sheet to decide in which course to enroll.

Please don't guess – this is not an SAT. No one but you will know your score unless you wish to discuss your placement when you get to campus in August. If that is the case, make an appointment to see the Physics Advising Head.

Problems

1. Evaluate these derivatives, where x and y are variables and a , b , & c are constants:

$D_1 = \frac{d}{dx} \left[b \sin(a\sqrt{x^2 + c^2}) \right]$ (a) $b \cos(a\sqrt{x^2 + c^2})$ (b) $-ba \cos(a\sqrt{x^2 + c^2})$ (c) $\frac{3abx^2 \cos(a\sqrt{x^2 + c^2})}{2\sqrt{x^3 + x^2}}$ (d) $\frac{3ab \cos(a\sqrt{x^2 + c^2})}{2}$	$D_2 = \frac{\partial}{\partial x} \left(\frac{xy^2}{\sin cy} \right)$ (a) $\left(\frac{y^2}{\sin cy} \right)$ (b) $\left(\frac{y^2}{\sin c} \right)$ (c) $\left(\frac{2xy}{\sin cy} \right)$ (d) $\left(\frac{cy^3}{2} \right)$
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2. Evaluate the following integrals:

$I_1 = \int \tan x dx$ (a) $\ln \cos x$ (b) $-\ln \cos x$ (c) $\sin^2 x$ (d) $\ln x$	$I_2 = \int x e^{-x} dx$ (a) $-x e^{-x}$ (b) $-(1 + x^2) e^{-x}$ (c) $-e^{-x}$ (d) $-(1 + x) e^{-x}$	$I_3 = \int_1^2 dx/x$ (a) $\ln 2$ (b) $\ln x$ (c) 1 (d) $3/2$
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3. Integrate the function $f(x, y) = x + y^2$ over the part of the x-y plane bounded by the positive x-axis, the curve $x = 2$, and the curve $y = x^2$.

- (a) 4
- (b) $2 + x^4$
- (c) 18
- (d) $212/21$

4. A mass m is suspended vertically in gravity of acceleration g by a spring of negligible mass with spring constant k . Let the y-axis be vertical and let the location of the end of the spring when no mass is attached be $y = 0$.

A) Find the equilibrium position y_0 of the mass.

- (a) mg/k
- (b) $-mg/k$
- (c) $mk > g$
- (d) 0

B) At time $t = 0$ the mass is released from rest at position $y(0) = 2y_0$. Find the subsequent motion of the mass, that is, $y(t > 0)$.

- (a) $y(t) = -(mg/k)(1 + \cos \omega t)$
- (b) $y(t) = -(mk/g)(1 + \cos \omega t)$
- (c) $y(t) = (mg/k) \cos \omega t$
- (d) $y(t) = (mg/k) \sin \omega t$

5. A uniform disk and a hoop of the same mass and radius start to roll down a ramp at the same time. Which will reach the bottom of the ramp first? Explain your answer.

- (a) the hoop because its moment of inertia is larger
- (b) the hoop because its moment of inertia is smaller
- (c) the disk because its moment of inertia is larger
- (d) the disk because its moment of inertia is smaller.

6. A planet of mass m is in circular orbit about a star of mass $M \gg m$.

A) Find the relationship among the period of the orbit P , the radius of the orbit a , and the masses m and/or M .

- (a) $P^2 = a^3$
- (b) $P^2 = a^3 \left(\frac{4\pi^2}{GM} \right)$
- (c) $P^3 = a^2 \left(\frac{4\pi^2}{GM} \right)$
- (d) $P^3 = a^2$

B) The radius of Earth's orbit is $a = 150 \times 10^6$ km. Find the mass of Sun in kilograms. Newton's constant of gravity is $G = 6.7 \times 10^{-11}$ in SI units, and there are 3.15×10^7 seconds in a year.

- (a) 2.0×10^{30} kg
- (b) 2.0×10^{30} g
- (c) 4.0×10^{30} kg
- (d) 4.0×10^{30} g

7. Determine which of the following are conservative forces, and if the answer is yes, find the corresponding potential energy function.

A) $\vec{F}_1 = A(3, z, y)$:

- (a) not conservative
- (b) conservative, $U = -3Ayz$
- (c) conservative, $U = -(-3x, yz, -yz)$
- (d) conservative, $U = -A(3x + yz)$

B) $\vec{F}_2 = Axyz(1, 1, 1)$:

- (a) not conservative
- (b) conservative, $U = (1, 1, 1)$
- (c) conservative, $U = -(x, y, z)$
- (d) conservative, $U = -(A/8)x^2y^2z^2$

8. Consider these two charges: $q_1 = 1$ nC, $r_1 = (1, 0, 0)$ cm and $q_2 = -2$ nC, $r_2 = (0, 1, 0)$ cm. Find the vector electric field at the origin. You may take $1/4\pi\epsilon_0 = 9.0 \times 10^9$ in SI units.

- (a) $\vec{E} = 9.0 \times 10^4$ N/C (+1, +2, 0)
- (b) $\vec{E} = 9.0 \times 10^4$ N/C (-1, -2, 0)
- (c) $\vec{E} = 9.0 \times 10^4$ N/C (-1, +2, 0)
- (d) $\vec{E} = 9.0 \times 10^4$ N/C (-2, +1, 0)

9. An electron with kinetic energy of 100 MeV moves in a plane perpendicular to a uniform magnetic field of 0.1 T.

A) What is the shape of the electron's path?

- (a) parabola
- (b) hyperbola
- (c) straight line
- (d) circle

B) Find the electron's orbital frequency. One electron volt is 1.6×10^{-19} J and the mass of an electron is 9.1×10^{-31} kg = 511 keV/c².

- (a) 14 MHz
- (b) 2.8 GHz
- (c) ∞
- (d) 0

10. An inductor $L = 100$ μ H is in series with a capacitor $C = 4$ μ F.

A) What is the oscillation frequency of this circuit in Hertz?

- (a) 5.0×10^4 Hz
- (b) 8.0×10^3 Hz
- (c) 3.1×10^5 Hz
- (d) no oscillations, dying exponential instead.

B) Suppose there were also a resistor in the circuit. Describe qualitatively how the behavior of the circuit would change.

- (a) no change
- (b) oscillations grow
- (c) oscillations die out
- (d) no oscillations ever possible.

11. A sinusoidal plane electromagnetic wave of frequency ν and amplitude E_0 in free space moves in the $+x$ direction and is polarized with the electric field along the y axis. Write an expression in SI units for the magnetic field of the wave as a function of position and time. You may write $k = 2\pi/\lambda$ and $\omega = 2\pi\nu$.

(a) $\vec{B}(x) = \hat{z} \frac{E_0}{c} \sin(kx + \omega t)$

(b) $\vec{B}(x) = \hat{z} \frac{E_0}{c} \sin(ky - \omega t)$

(c) $\vec{B}(x) = \hat{z} \frac{E_0}{c} \sin(ky + \omega t)$

(d) $\vec{B}(x) = \hat{z} \frac{E_0}{c} \sin(kx - \omega t)$

12. Find the charge density corresponding to the following electric fields:

A) $\vec{E} = (1, 2, 3)$ V/m:	B) $\vec{E} = \frac{1}{1cm}(x, y, z)$ V/m:	(C) $\vec{E} = \hat{r} \text{ 1C}/4\pi\epsilon_0 r^2$:
(a) 0	(a) 0	(a) 0
(b) $6\epsilon_0 \text{ Cm}^{-3}$	(b) $3\epsilon_0/1cm \text{ Cm}^{-3}$	(b) $1 \text{ C}/r^2$
(c) $-6/\epsilon_0 \text{ Cm}^{-3}$	(c) $-(3\epsilon_0/1cm) \text{ Cm}^{-3}$	(c) $-1 \text{ C at } (0, 0, 0)$
(d) $(x, 2y, 3z)\epsilon_0 \text{ Cm}^{-3}$	(d) $3/\epsilon_0 1cm \text{ Cm}^{-3}$	(d) $1 \text{ C at } (0, 0, 0)$

13. A long cylindrical wire of radius a is made of a metal of conductivity σ . It carries an AC current $i(t) = i_0 \cos \omega t$.

A) Find the displacement current in the wire.	B) Find the frequency at which the displacement current would be equal in magnitude to the conduction current.
(a) 0	(a) 0
(b) $i_0 \cos \omega t$	(b) $\omega/\epsilon_0 \sigma$
(c) $-\omega i_0 \sin \omega t$	(c) σ/ϵ_0
(d) $-\frac{\epsilon_0 \omega i_0}{\sigma} \sin \omega t$	(d) ∞
(C) Find that frequency if the wire is copper with conductivity $\sigma = 6 \times 10^7 \text{ } \Omega^{-1}\text{m}^{-1}$.	(D) Is such a frequency feasible in a copper wire?
(a) 0	(a) yes
(b) $1.1 \times 10^{18} \text{ Hz}$	(b) no
(c) $6.7 \times 10^{18} \text{ Hz}$	
(d) ∞	

14. A spacecraft is designed to travel 20 light years (as measured in the Earth frame) in 10 years of astronaut time. A) Find the speed $\beta = v/c$ and Lorentz factor γ required for the trip. Hint: The answer is not $\gamma = 2$.

- (a) $\beta = 5$ and $\gamma = \infty$
- (b) $\beta = 4/5$ and $\gamma = 5/3$
- (c) $\beta = 1/\sqrt{3}$ and $\gamma = \sqrt{3}/2$
- (d) $\beta = \sqrt{4}/5$ and $\gamma = \sqrt{5}$

B) Find the space-time interval between departure and arrival in the spacecraft frame, where y is a year and L is a light-year.

- (a) $10y$
- (b) $-100L^2$
- (c) 0
- (d) $20L$

(C) Find the the space-time interval between departure and arrival in the Earth frame.

- (a) $-100L^2$
- (b) $100L^2$
- (c) $10y$
- (d) 0