Life at Low D

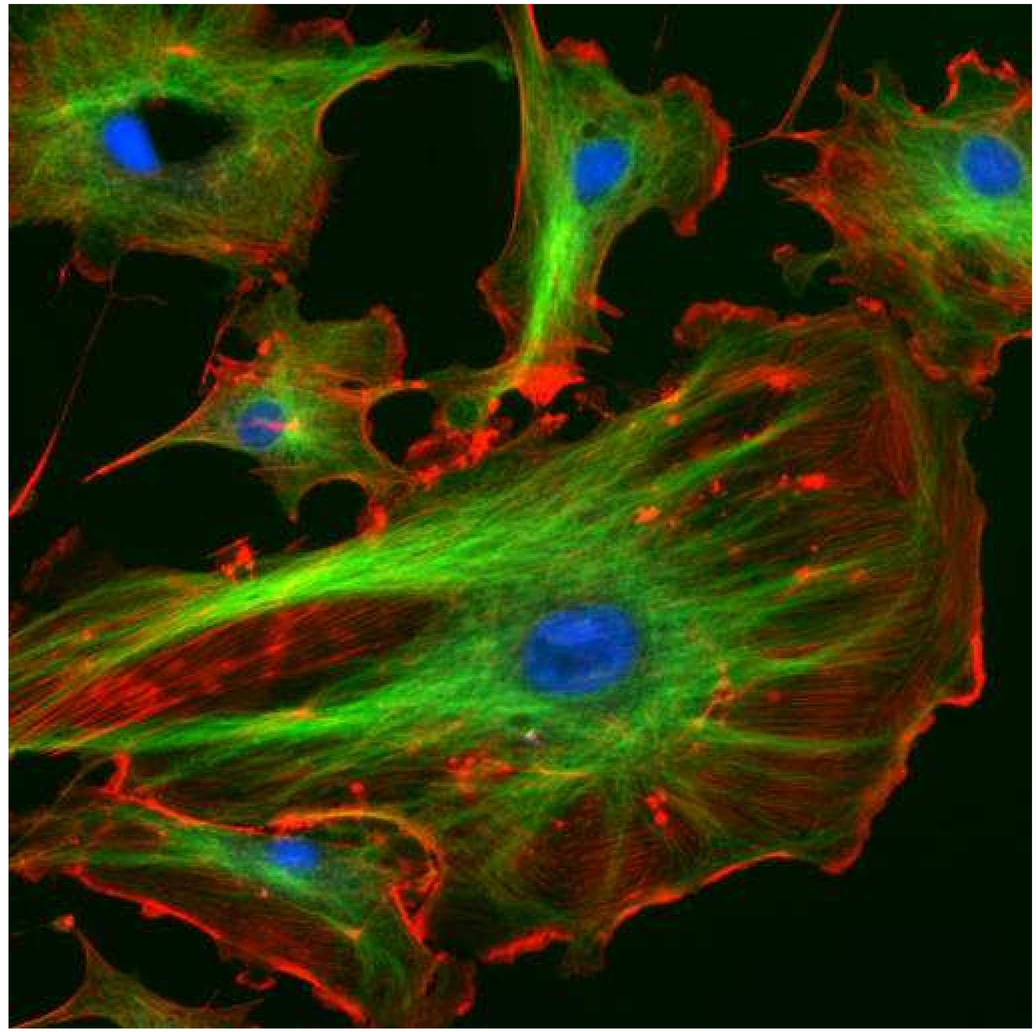
Lecture I : Biological Filaments (1D)

Cytoskeleton

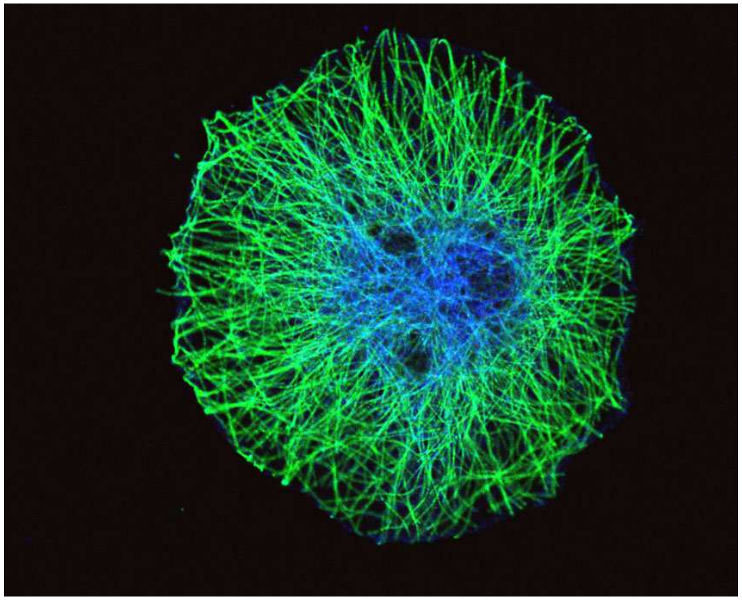
The "scaffolding" or "skeleton" contained within the cytoplasm

Cytoskeleton comprised of 3 types of protein filaments:

a) **Actin**
b) **Microtubules**
c) **Intermediate Filaments**

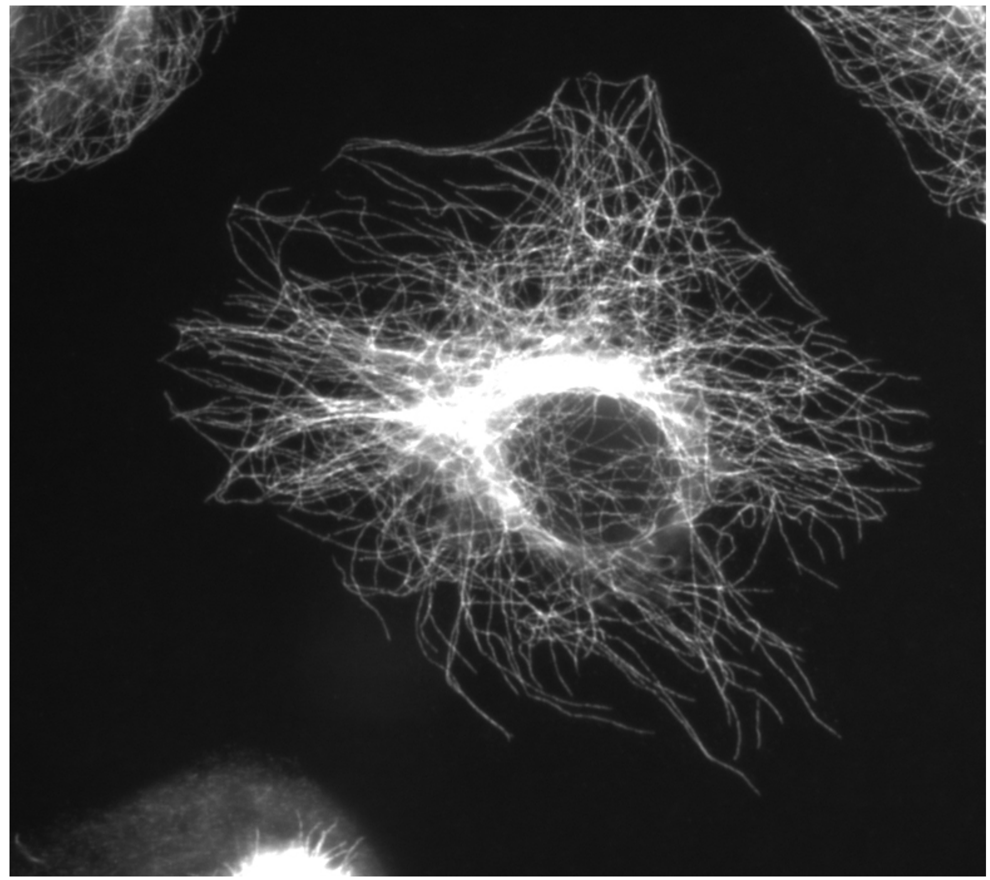


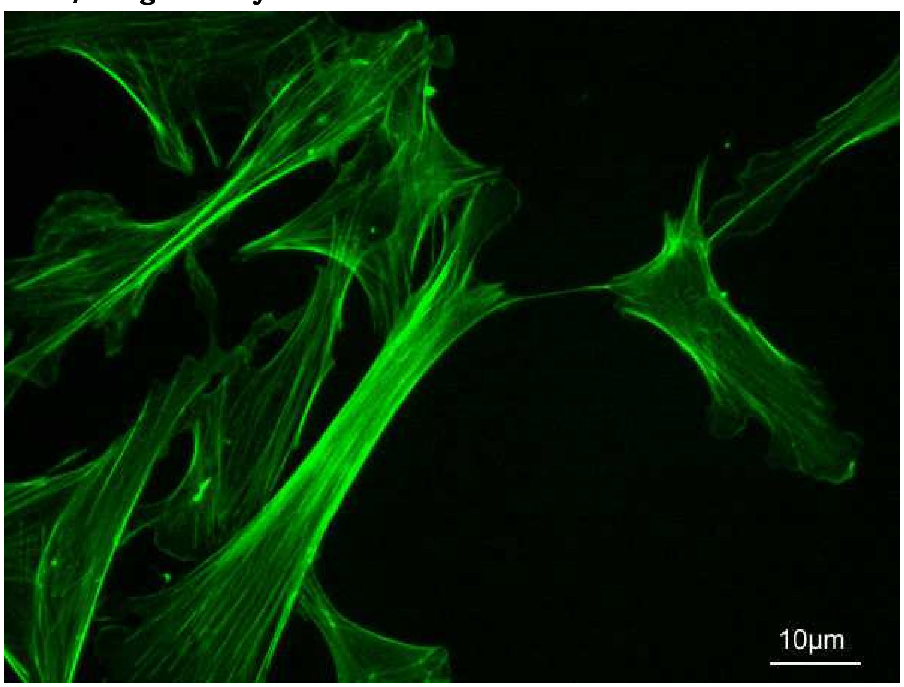
Endothelial cells. **Nuclei** are stained blue (DAPI), **microtubles** green (antibody bound to FITC), and **actin** filaments are red (phalloidin bound to TRITC). Bovine pulmonary arthery endothelial cells



A Drosophila tissue culture cell labeled for:
microtubules in green and **DNA** in blue

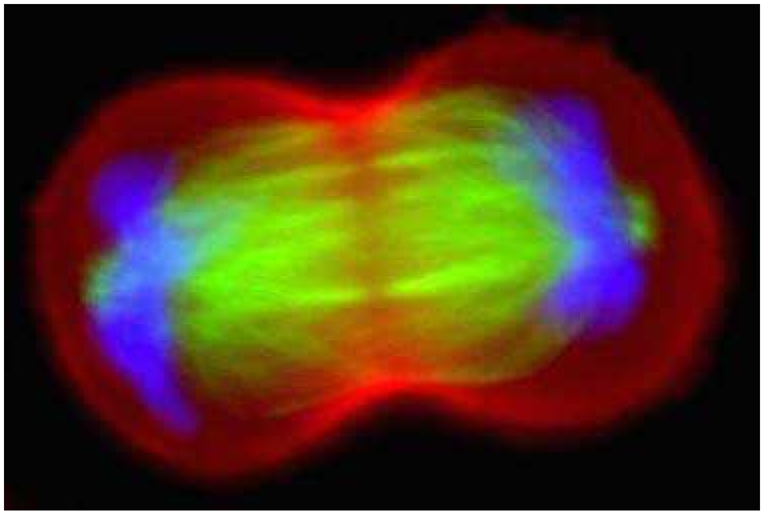
http://www.pdn.cam.ac.uk/groups/roperlab/Roper Lab/ImageGallery.html



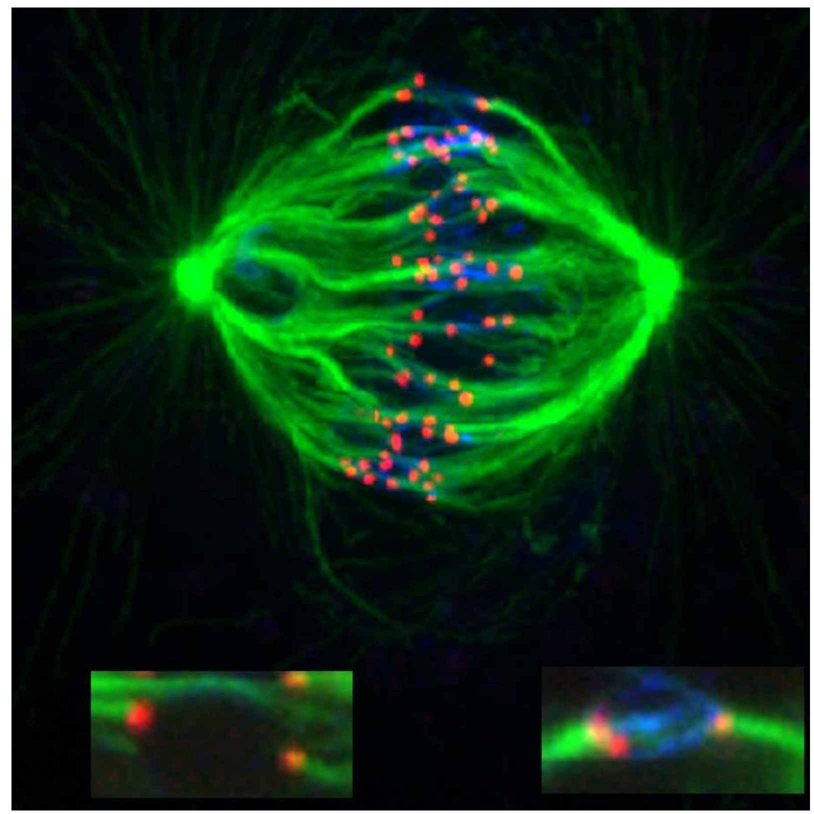


Actin filaments of mouse embryo fibroblasts, stained with FITC-phalloidin

Mitosis



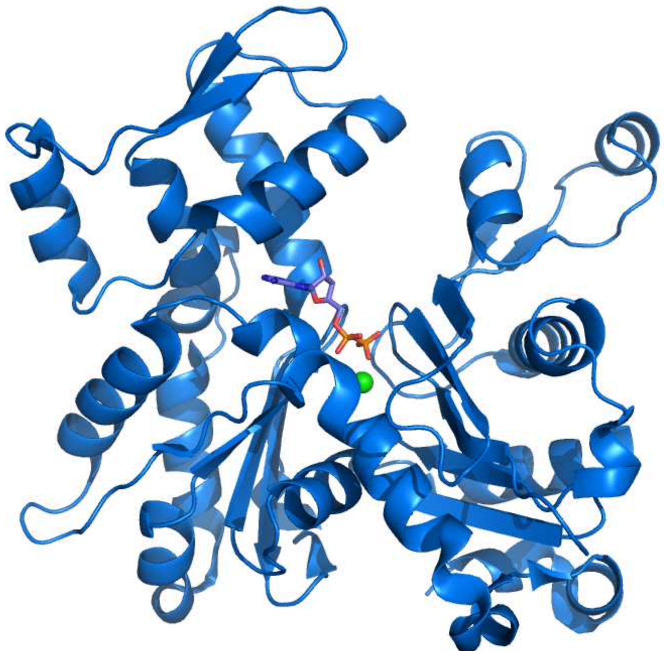
A Chinese hamster ovary cell (CHO) cell in anaphase. **Actin** (red), **tubulin** (green) and **DNA** (blue) are labeled



people.virginia.edu/~djb6t/LabWeb/anne.htm

A mitotic spindle with **kinetochores** (motors that bend microtubules, thereby destabilizing them and promoting depolymerization) in red and **microtubules** stained green

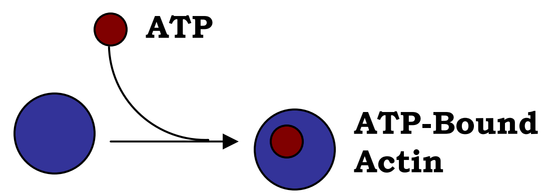
Biological Filaments Actin

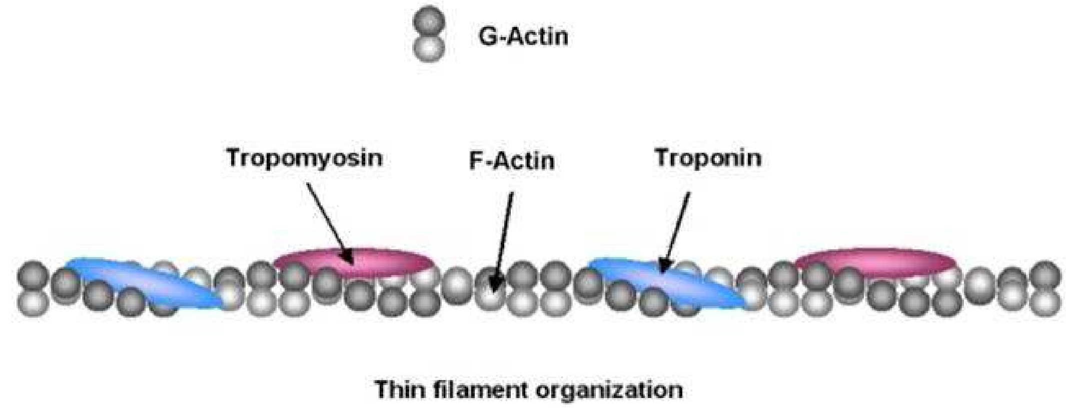


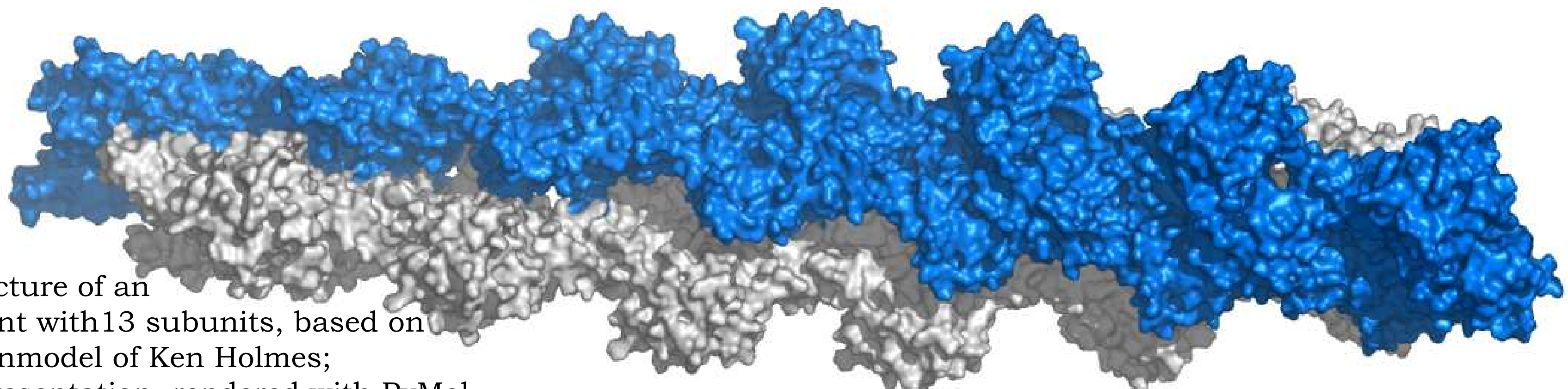
$$43kDa \sim 43nm^3$$

G-Actin

ADP and the divalent cation highlighted







atomic structure of an
actin filament with13 subunits, based on
actin filamenmodel of Ken Holmes;
surface representation, rendered with PyMol

Biological Filaments Microtubules

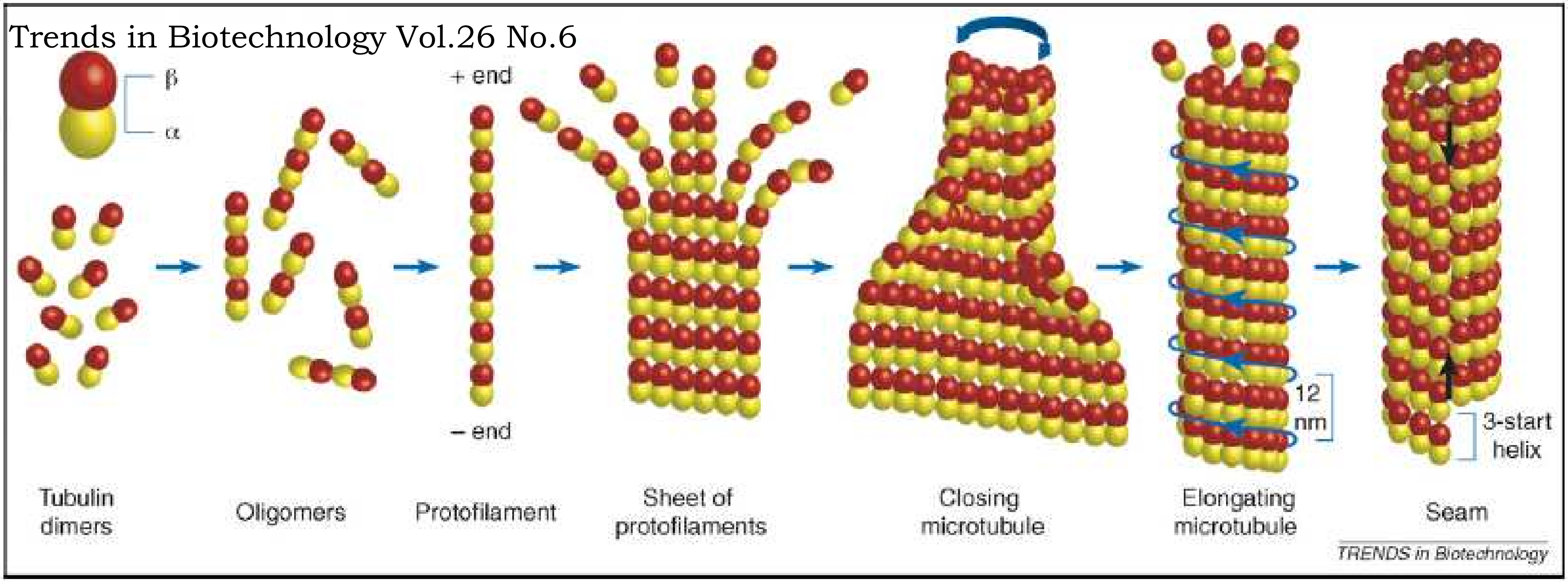
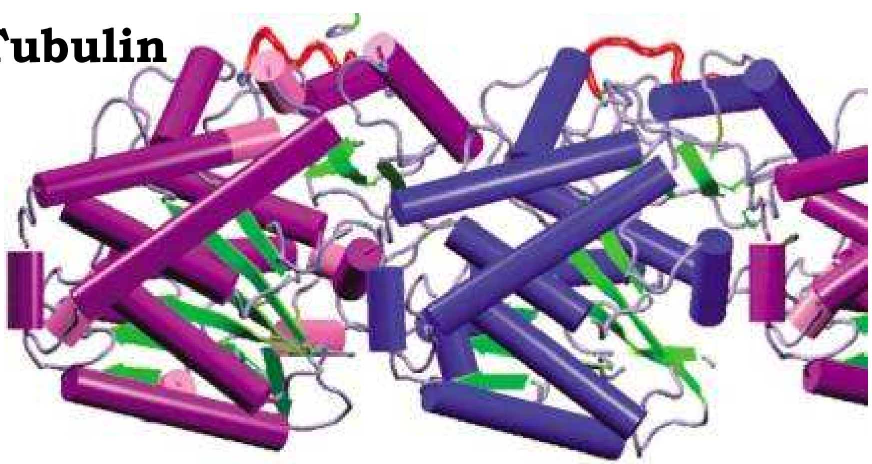
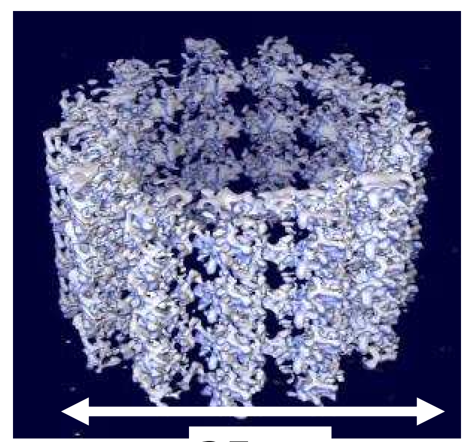


Figure 1. Polymerization of microtubules. Tubulin dimers assemble 'head-to-tail', forming oligomers that elongate into protofilaments. As the protofilaments reach an estimated critical length of 12 ± 2 dimers [65] they start to interact laterally, forming sheets with a characteristic intrinsic inward curvature. At a typical number of 13 protofilaments, the tubulin sheet closes into a tube, forming a microtubule. The tubulin lattice has a left-handed helical symmetry. The microtubule closes at the seam (black arrows), where there is a discontinuity point in the helical lattice.

Tubulin

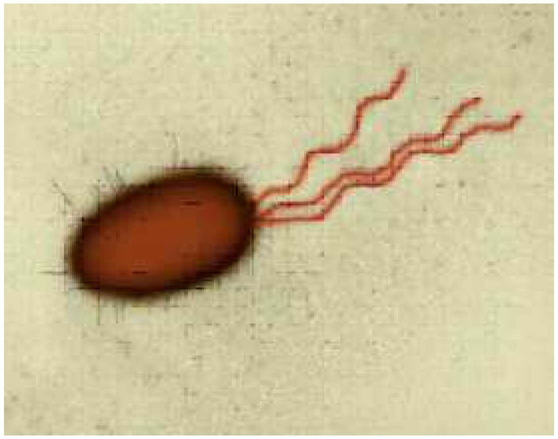


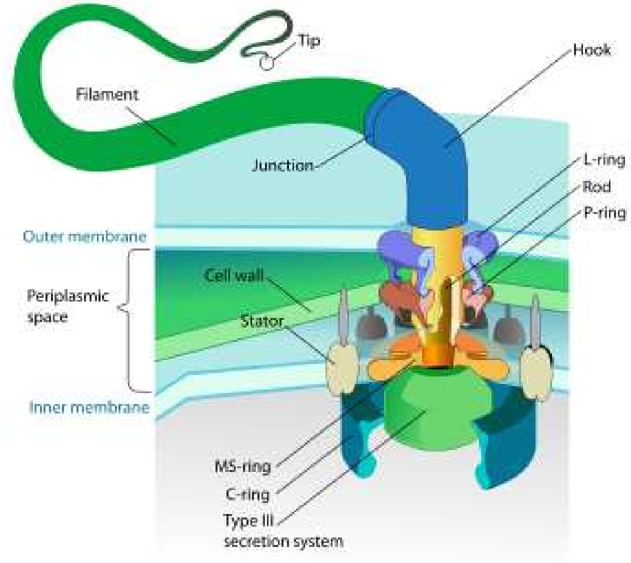


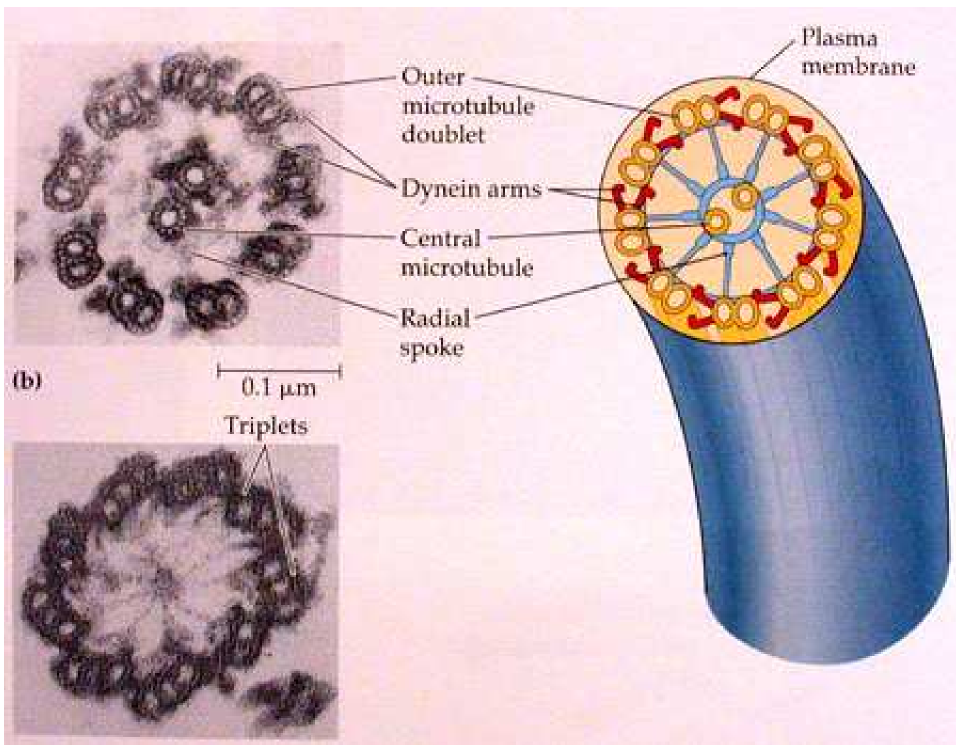
$25nm$

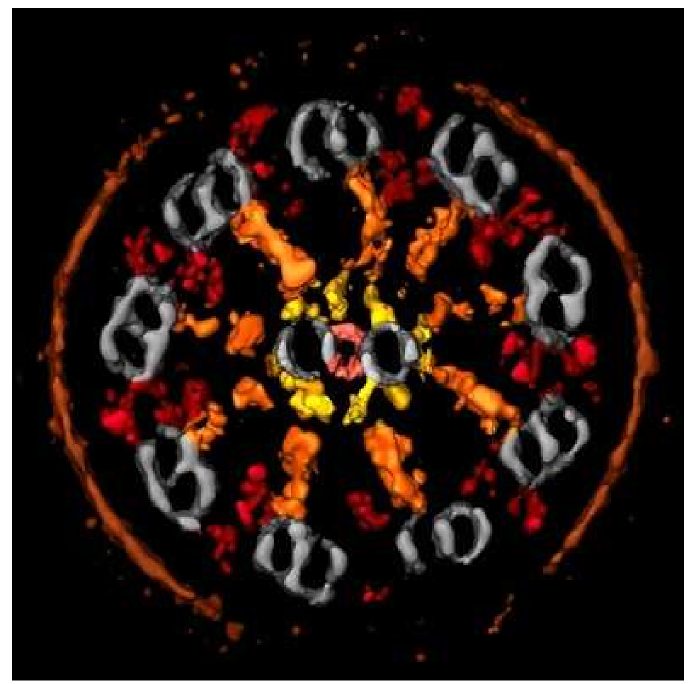
Biological Filaments “from” Actin and Microtubules

Flagella



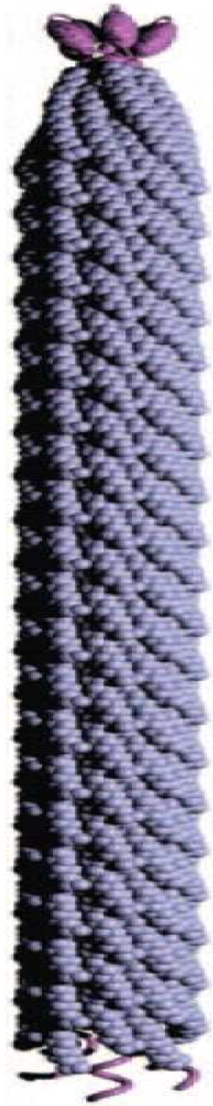


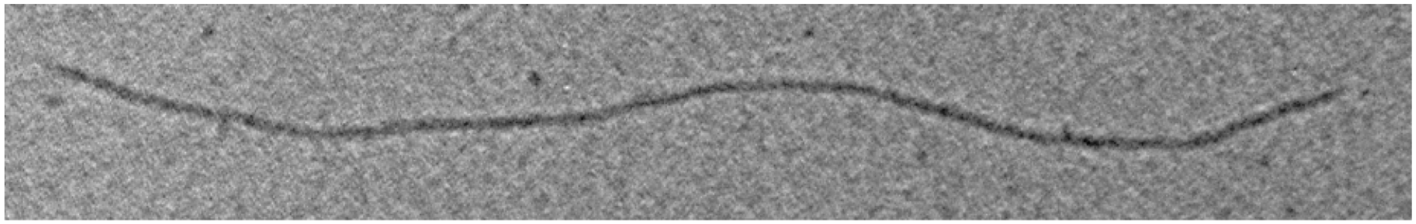




http://academic.brooklyn.cuny.edu/biology/bio4fv/page/flagella1068.JPG

Biological Filaments Filamentous Bacteriophages (M13)

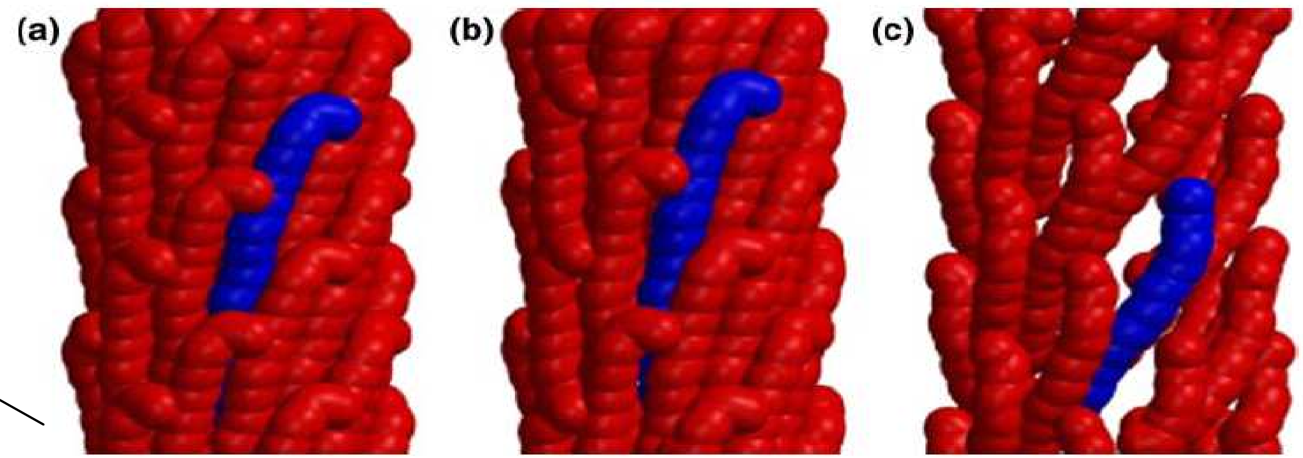




$L \sim 1\mu m$

geneIII and geneVI proteins
infective tip

geneVIII protein ~50aa monomer, 2700 copies



6nm

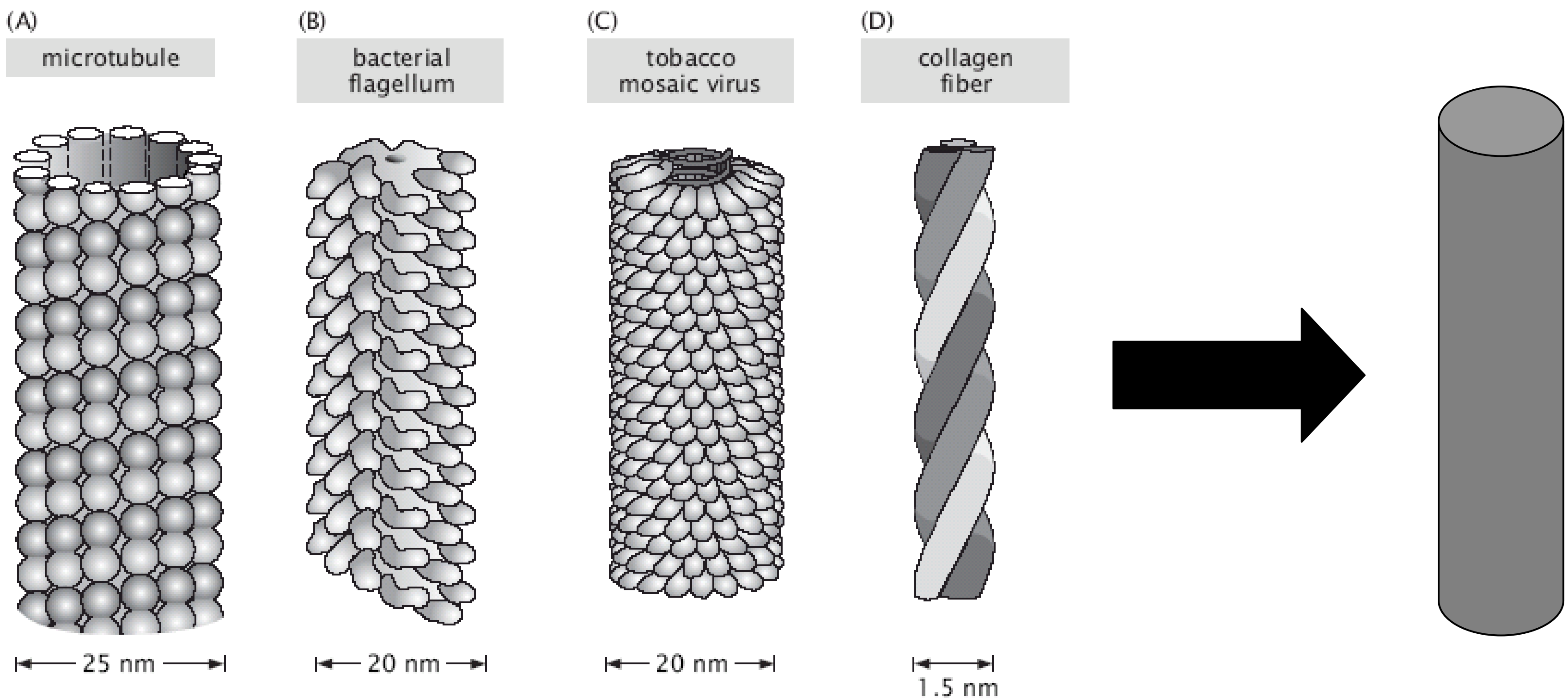
EurBiophysJ **37**. p.521(2008)

geneVII and geneIX proteins
remote tip

6nm

ssDNA 7259bp = 2177nm

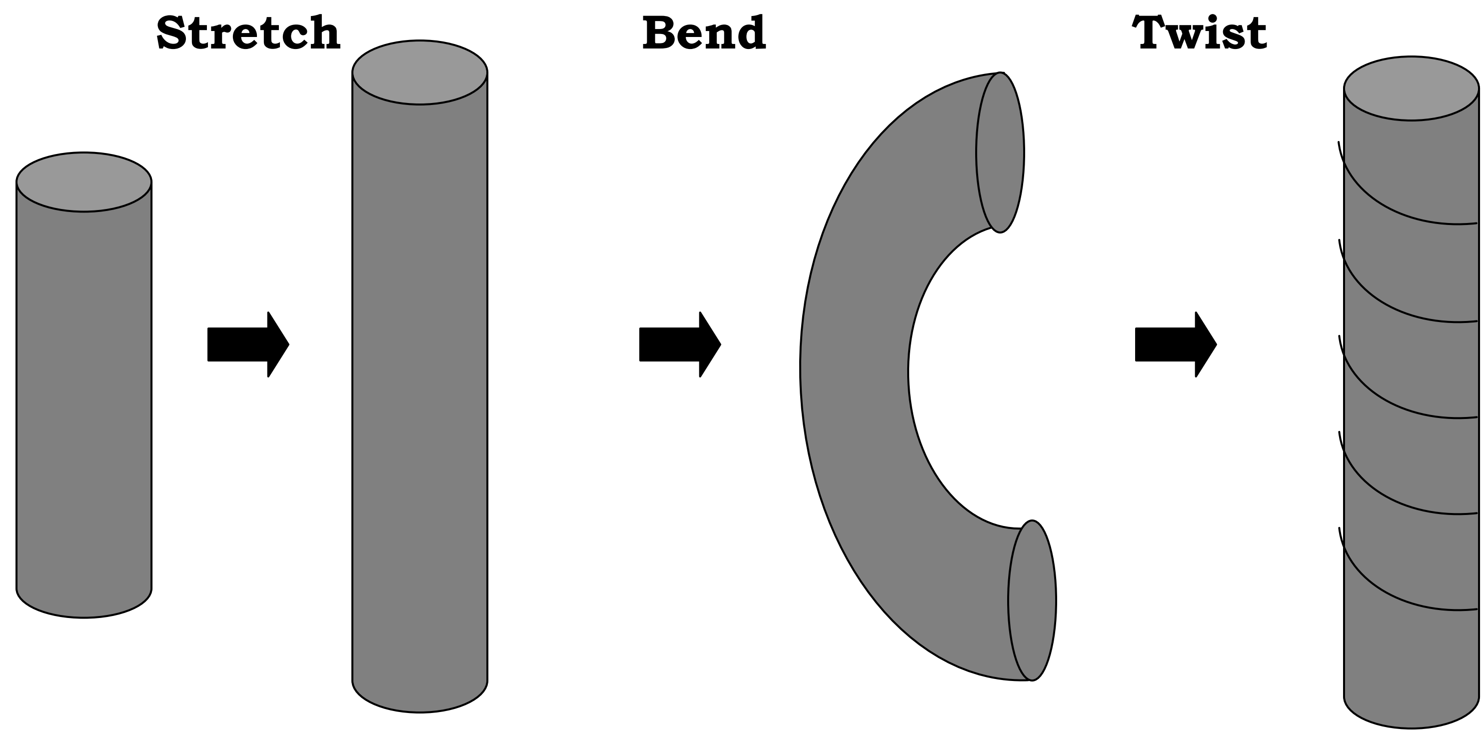
Biological Filaments



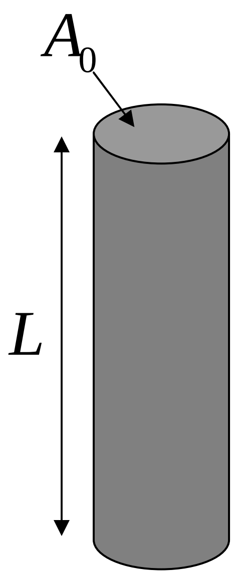
Phys. Biol. Of the Cell

Biological Filaments

What are the types of deformations encountered in a filament’s lifetime?



Stretching (Stress and Strain)



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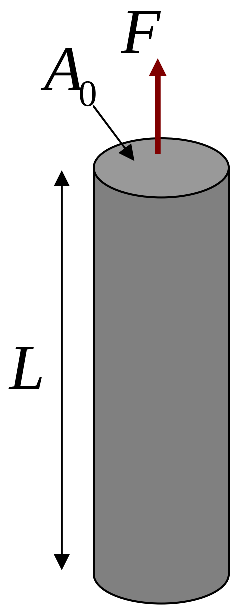
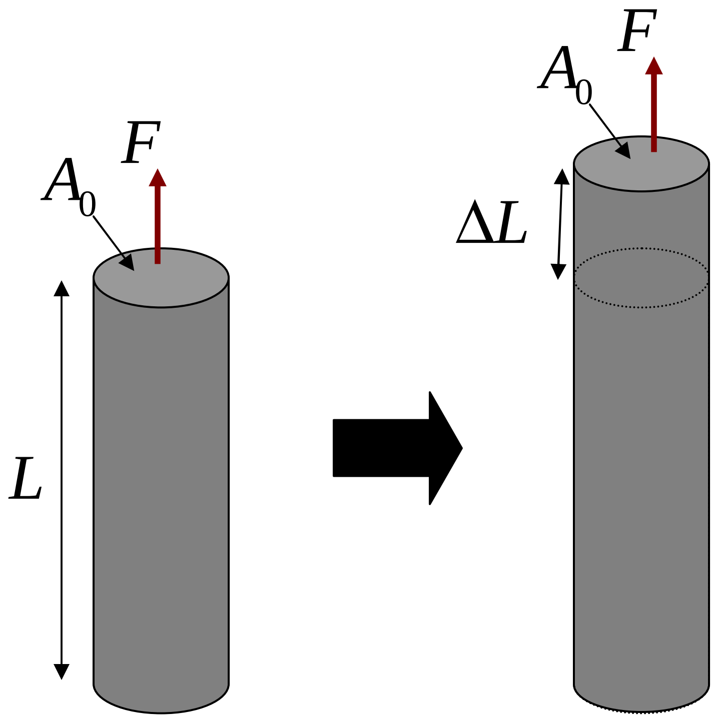


Stress

$$\sigma = \frac{F}{A_0}$$

Stretching (Stress and Strain)



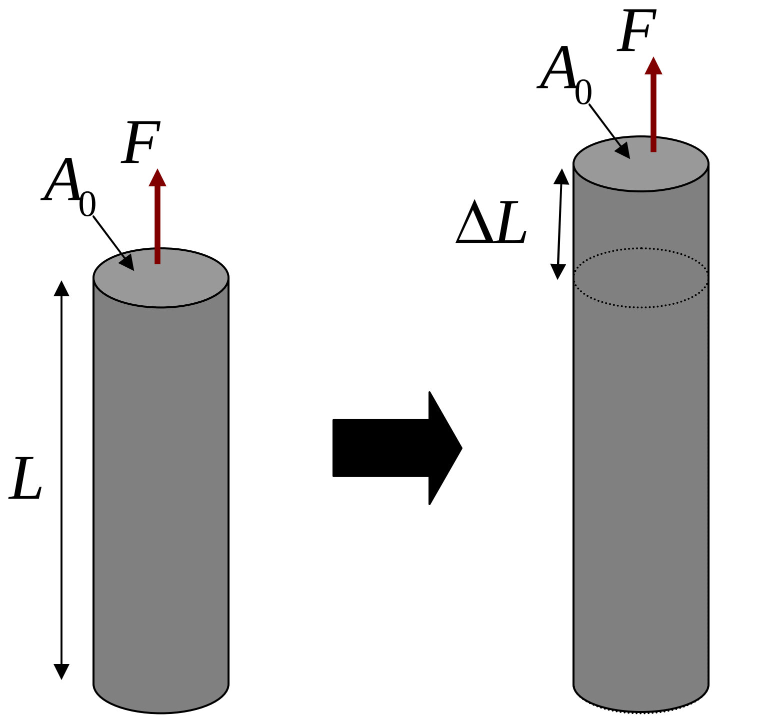
Stress

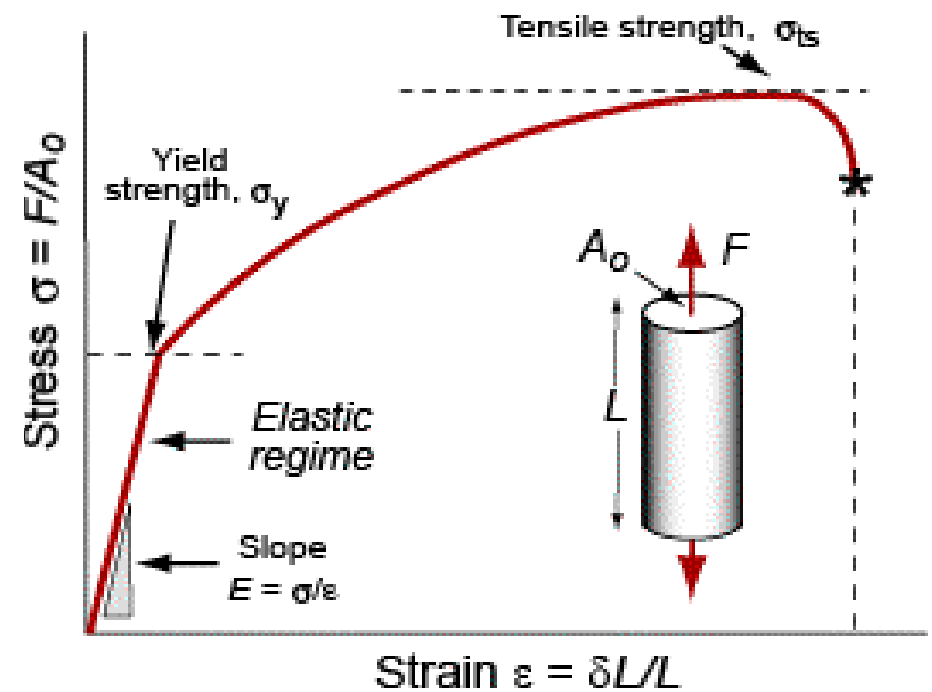
$$\sigma = \frac{F}{A_0}$$

Strain

$$\varepsilon = \frac{\Delta L}{L}$$

Stretching (Stress and Strain)





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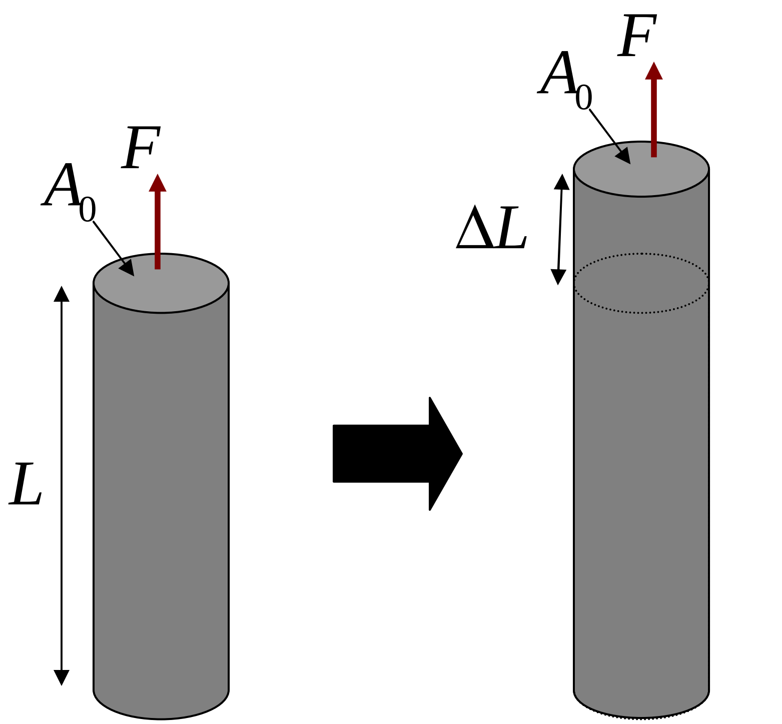
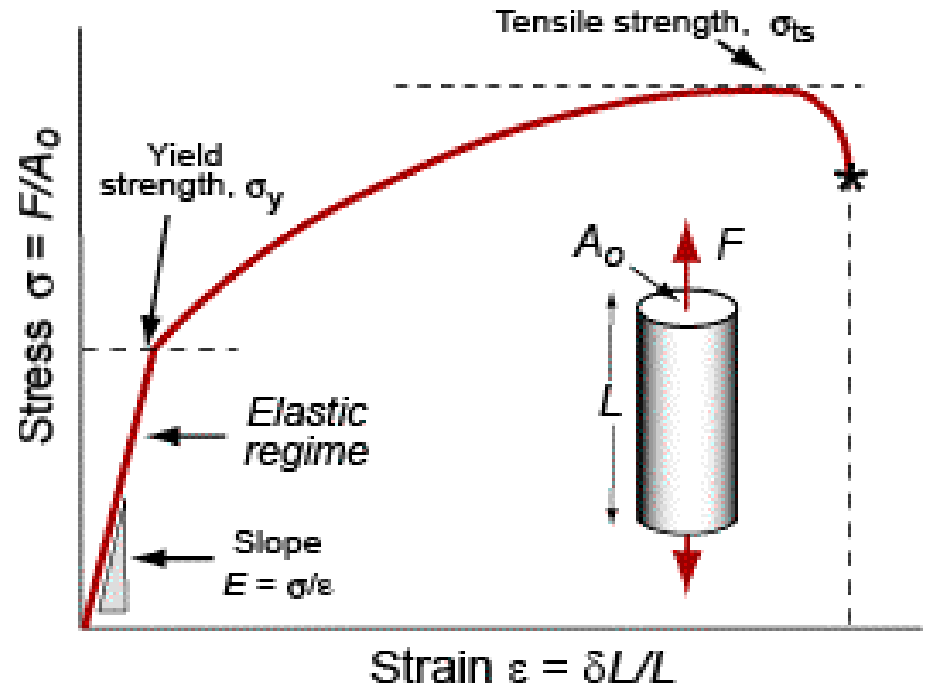




Stress

$$\sigma = \frac{F}{A_0}$$

Strain

$$\varepsilon = \frac{\Delta L}{L}$$

$$\boxed{\sigma = E\varepsilon}$$

$$E = \left[\frac{Force}{Area}\right] = \left[\frac{Energy}{Volume}\right]$$

Peruvian Example Rubber Band

Stress

$$A_0 = 0.1cm^2$$

$$1lbs \approx 0.5kg$$

$$F \sim (2.5kg)(9.8\, {}^{m}/_{s^2}) \approx 25N$$

$$\sigma = {}^{F}/_{A_0} \approx 250\, {}^{N}/_{cm^2}$$

Strain

$$\varepsilon = \frac{\Delta L}{L} \approx 0.75$$

$$\boxed{\sigma = E\varepsilon}$$

$$E \approx 300\, {}^{N}/_{cm^2} = 0.003GPa$$

Young's Modulus

Stress

$$\sigma = \frac{F}{A_0}$$

Strain

$$\varepsilon = \frac{\Delta L}{L}$$

$$\boxed{\sigma = E\varepsilon}$$

E = Young's Modulus

UNITS!!!

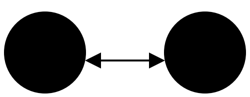
$$E = \left[\frac{Force}{Area}\right] = \left[\frac{Energy}{Volume}\right]$$

MATERIAL	E [G Pa]
Diamond	1200
Steel	211
Glass	100
Wood	10
Plastic	2.4
Rubber	0.02

$$1Pa = 1N/m^2 = 1J/m^3$$

Estimates

Metals

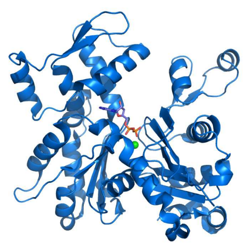


$154\,pm$

$$U_{\text{int}} \sim eV$$

$$E \approx 200\,GPa$$

G-Actin

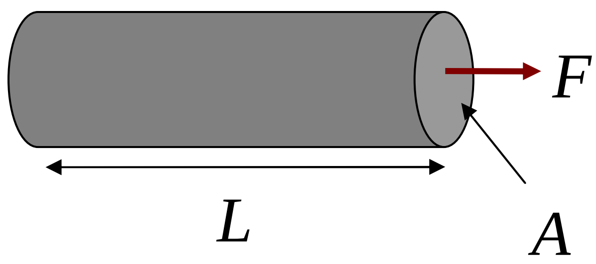


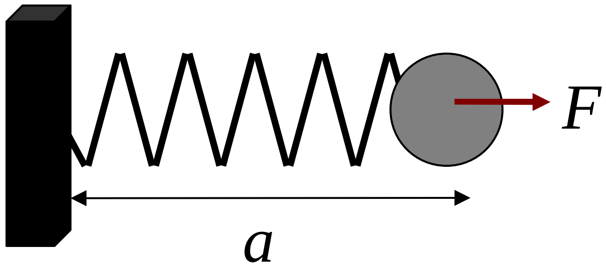
$$V \sim 43nm^3$$

$$U_{\text{int}} \sim 10k_BT$$

$$E \approx 2\,GPa$$

Spring Analogy

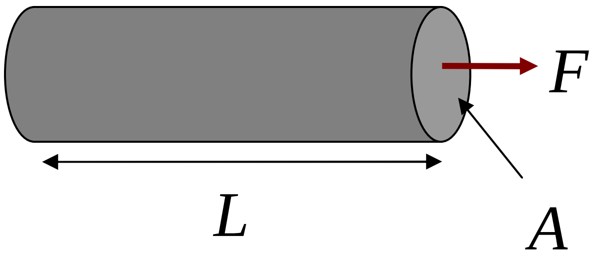


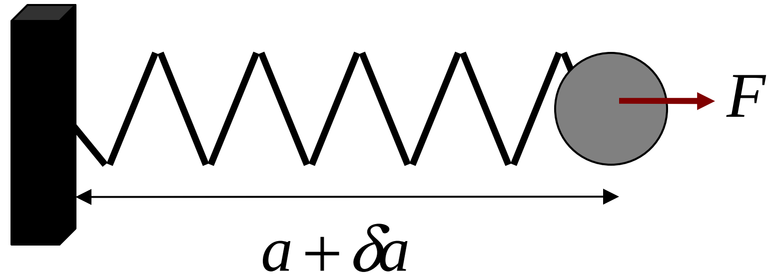


$$F = kx$$

$$U = \frac{1}{2}kx^2$$

Spring Analogy

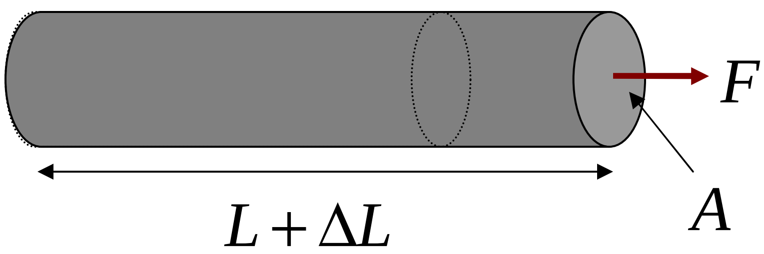


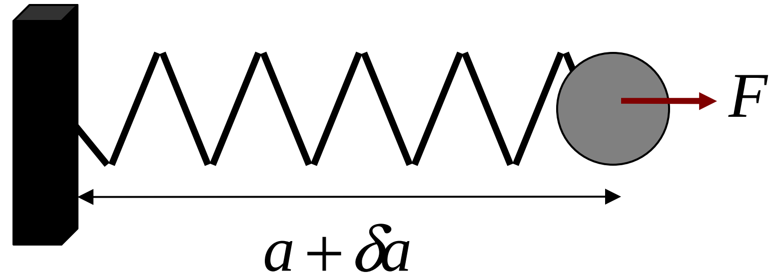


$$F = k(\delta a)$$

$$U = \frac{1}{2}k(\delta a)^2$$

Spring Analogy





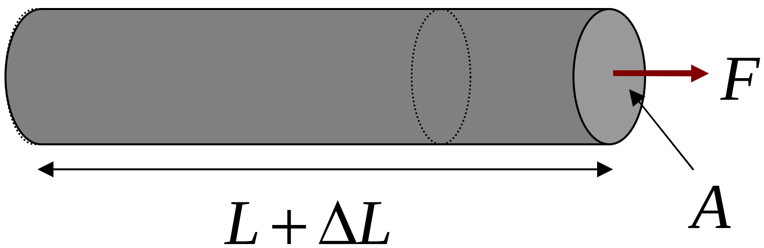
$$\frac{F}{A}=E\frac{\Delta L}{L} \Rightarrow F=\frac{EA}{L}(\Delta L)$$

$$F=k(\delta a)$$

$$U=\frac{1}{2}\frac{EA}{L}(\Delta L)^2$$

$$U=\frac{1}{2}k(\delta a)^2$$

Strain Energy

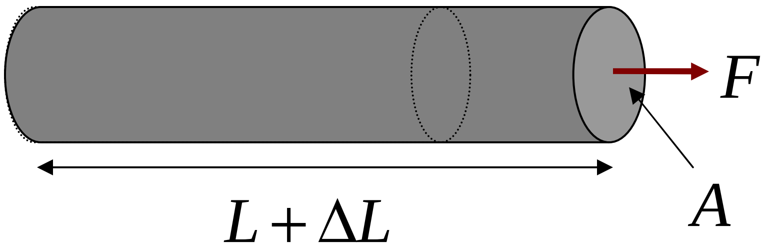


$$L + \Delta L$$

$$\frac{F}{A} = E\frac{\Delta L}{L}$$

$$U_{Strain} = \frac{1}{2}\frac{EA}{L}(\Delta L)^2$$

Strain Energy

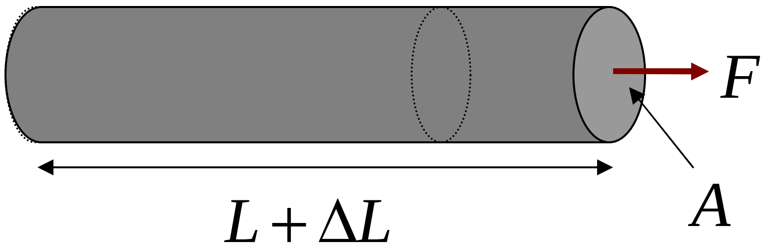


$$\frac{F}{A} = E\frac{\Delta L}{L}$$

$$U_{Strain} = \frac{1}{2}\frac{EA}{L}(\Delta L)^2$$

$$U_{Strain} = \frac{1}{2}E\left(\frac{\Delta L}{L_0}\right)^2 AL_0 = \frac{1}{2}E\varepsilon^2 V$$

Strain Energy



$$\frac{F}{A} = E\frac{\Delta L}{L}$$

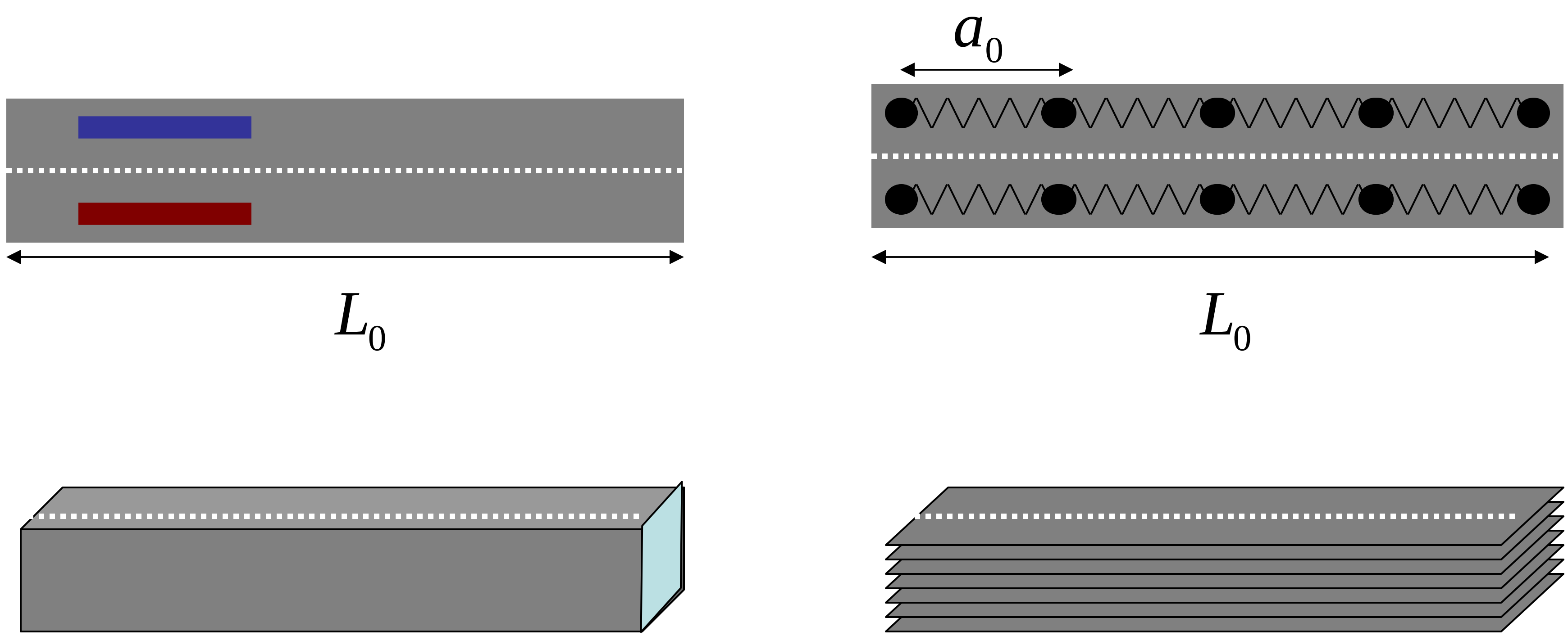
$$U_{Strain} = \frac{1}{2}\frac{EA}{L}(\Delta L)^2$$

$$U_{Strain} = \frac{1}{2}E\left(\frac{\Delta L}{L_0}\right)^2 AL_0 = \frac{1}{2}E\varepsilon^2 V$$

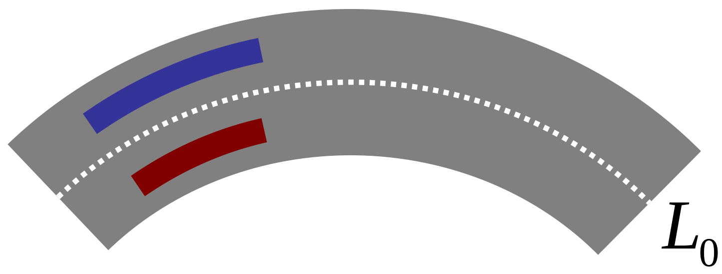
$$\frac{U_{Strain}}{Volume} = \frac{1}{2}E\varepsilon^2$$

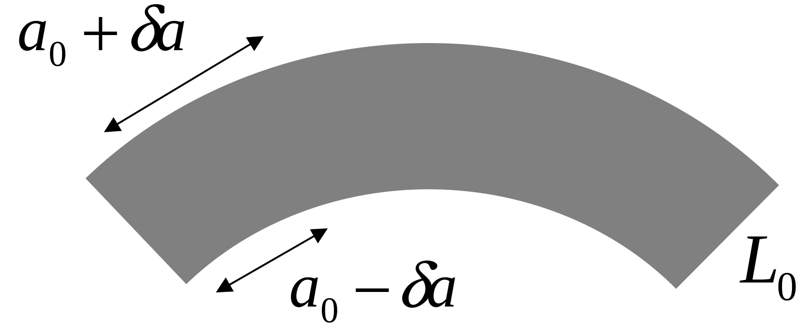
$$U_{Strain} = \frac{E}{2}\int \varepsilon^2(x,y,z)\,dxdydz$$

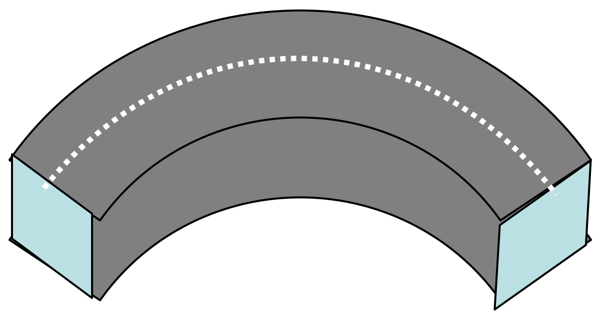
Bending

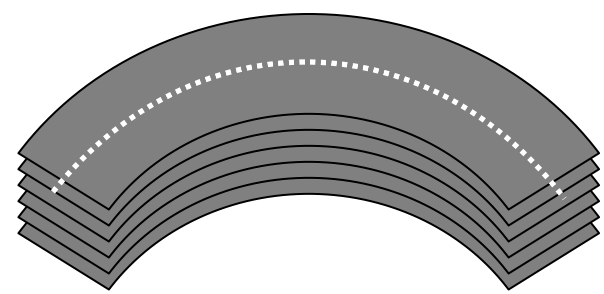


Bending

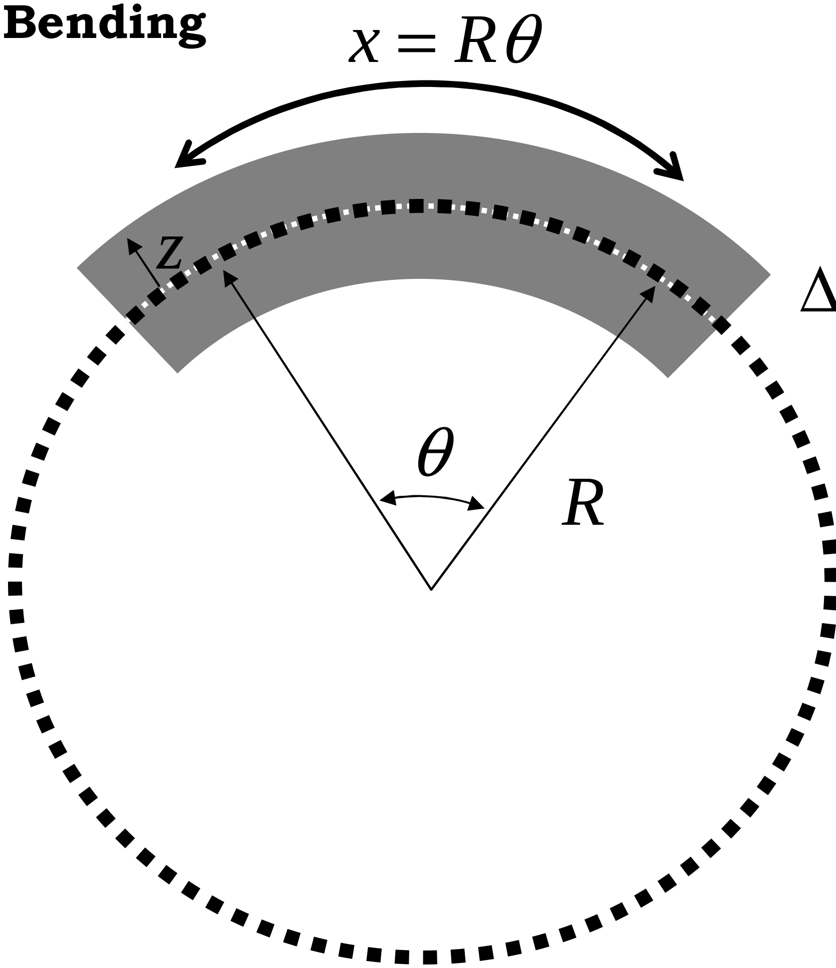








Bending



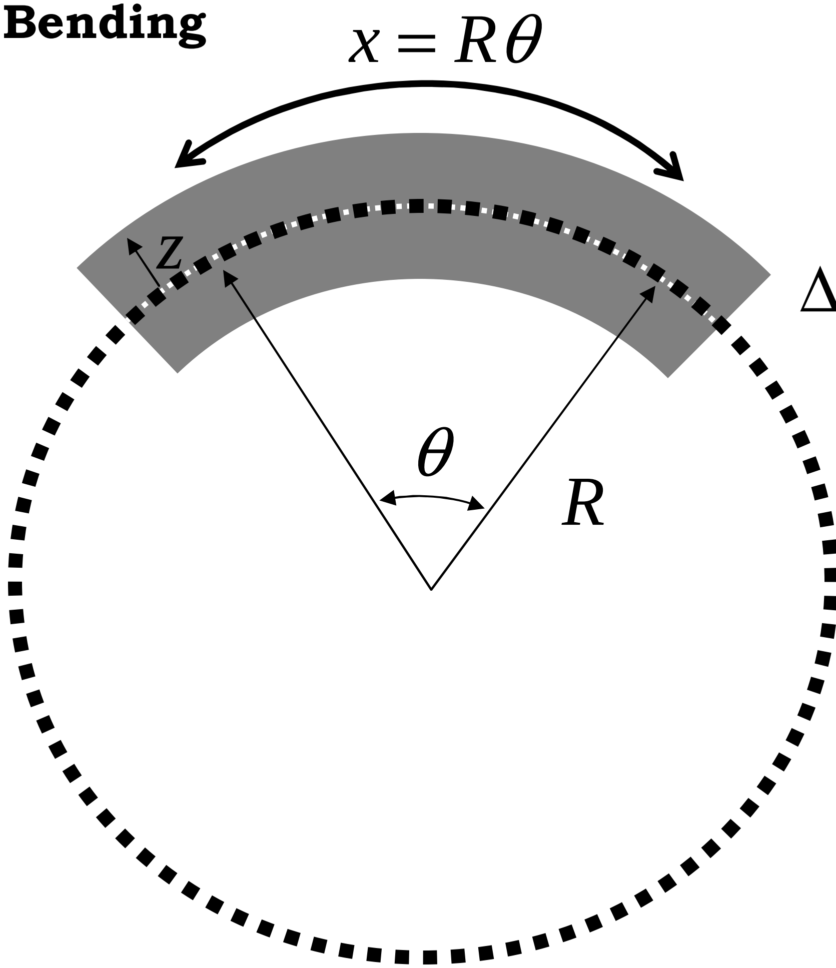
$$L = L(z)$$

$$L(0) = L_0$$

$$\Delta L(z) = L(z) - L_0 = (R + z)\frac{L_0}{R} - L_0 = \frac{zL_0}{R}$$

$$\varepsilon = \frac{\Delta L(z)}{L_0} = \frac{z}{R}$$

Bending



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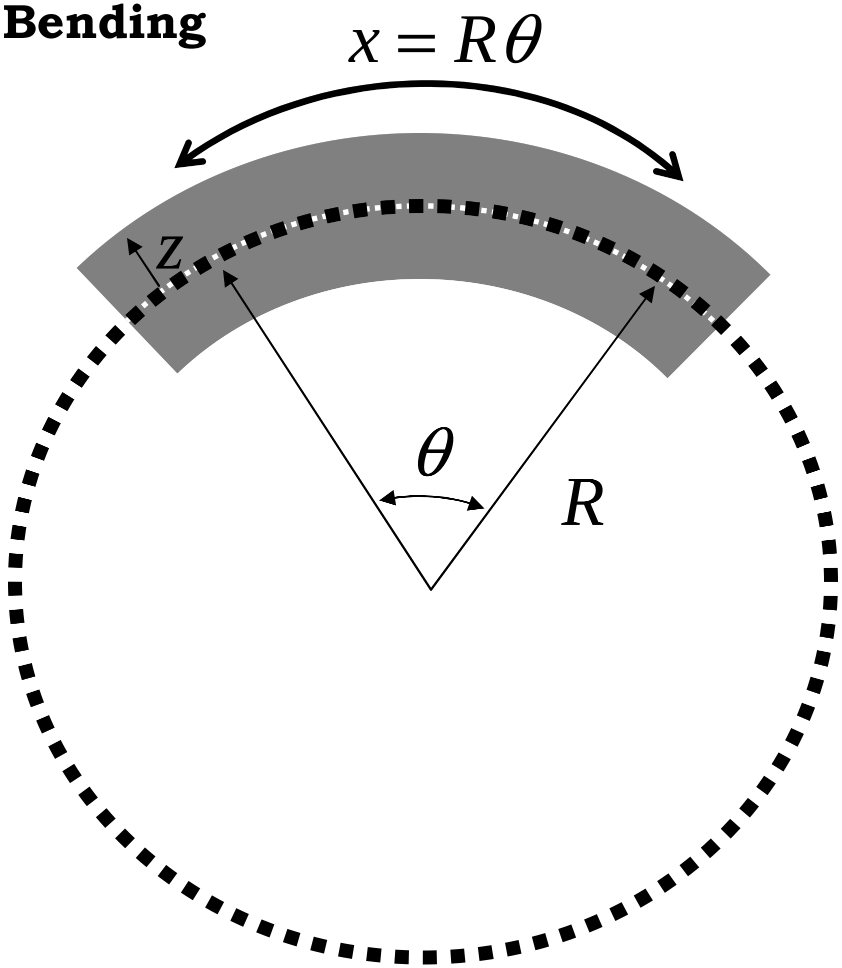
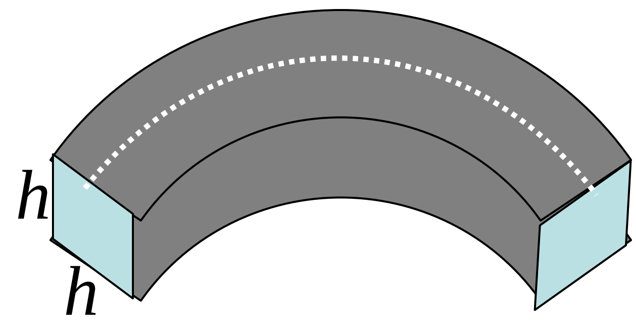
$$\varepsilon = \frac{\Delta L(z)}{L_0} = \frac{z}{R}$$

$$U_{Strain} = \frac{E}{2}\int \varepsilon^2(x,y,z)\,dxdydz$$

Bending





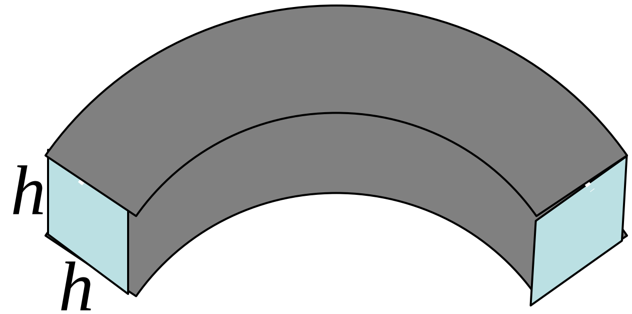
$$\varepsilon = \frac{\Delta L(z)}{L_0} = \frac{z}{R}$$

$$U_{Strain} = \frac{E}{2}\int \varepsilon^2(x,y,z)\,dxdydz$$

$$U_{Bend} = \frac{E}{2}\int_0^L dx \int_{-h/2}^{h/2} dy \int_{-h/2}^{h/2} \left(\frac{z}{R}\right)^2 dz = \frac{E}{2}\frac{1}{R^2} L \cdot h \cdot \frac{h^3}{8 \cdot 3} \cdot 2 = \frac{E}{2}\left(\frac{h^4}{12}\right)\frac{L}{R^2}$$

Bend Energy

$$U_{Bend} = \frac{E}{2}\left(\frac{h^4}{12}\right)\frac{L}{R^2}$$



Bend Energy

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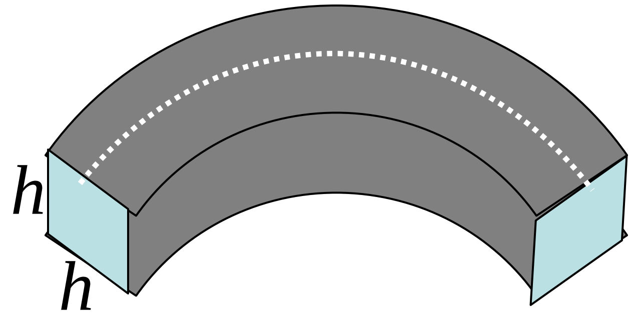
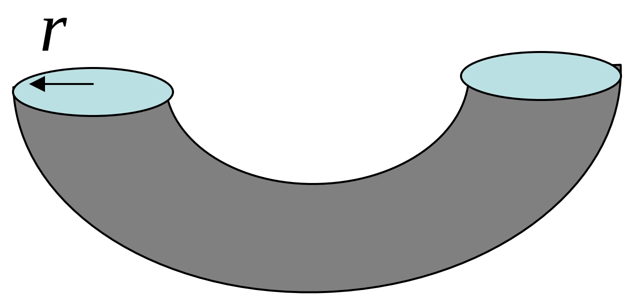
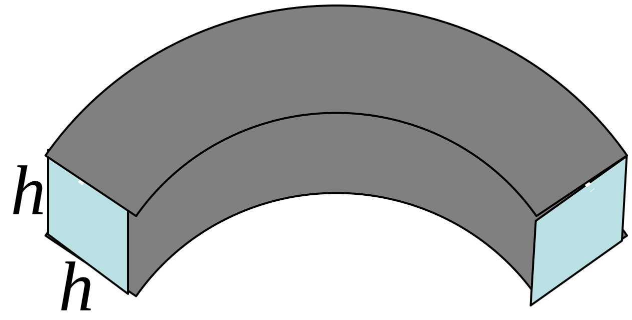


$$U_{Bend} = \frac{E}{2}\left(\frac{\pi\, r^4}{4}\right)\frac{L}{R^2}$$



Bend Energy

$$U_{Bend} = \frac{E}{2}\left(\frac{h^4}{12}\right)\frac{L}{R^2}$$



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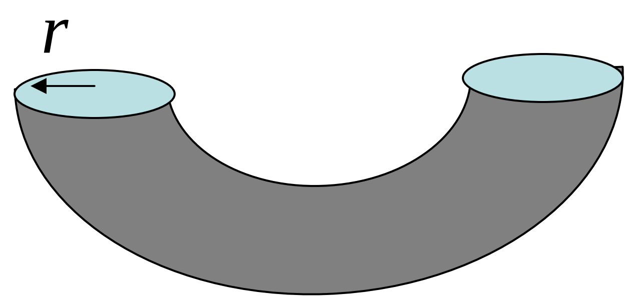
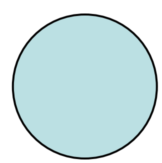
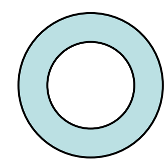


Areal Moment of Inertia

$$I = \frac{wh^3}{12}$$



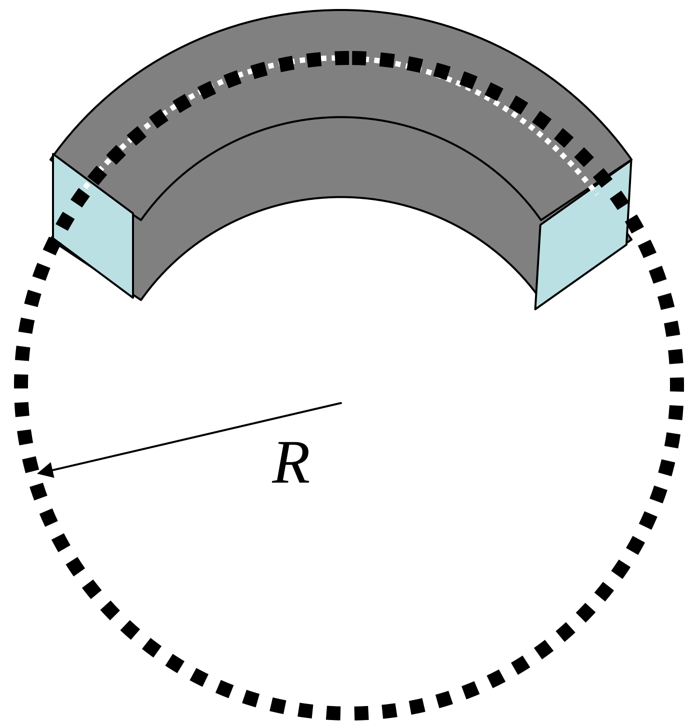
$$I = \frac{\pi\, r^4}{4}$$



$$I = \frac{\pi\,(r^4 - r_0^4)}{4}$$

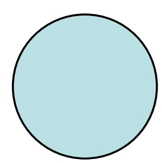
Bend Energy

$$U_{Bend} = \frac{EI}{2}\frac{L}{R^2}$$

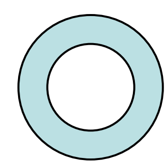


Areal Moment of Inertia

$$I = \frac{wh^3}{12}$$



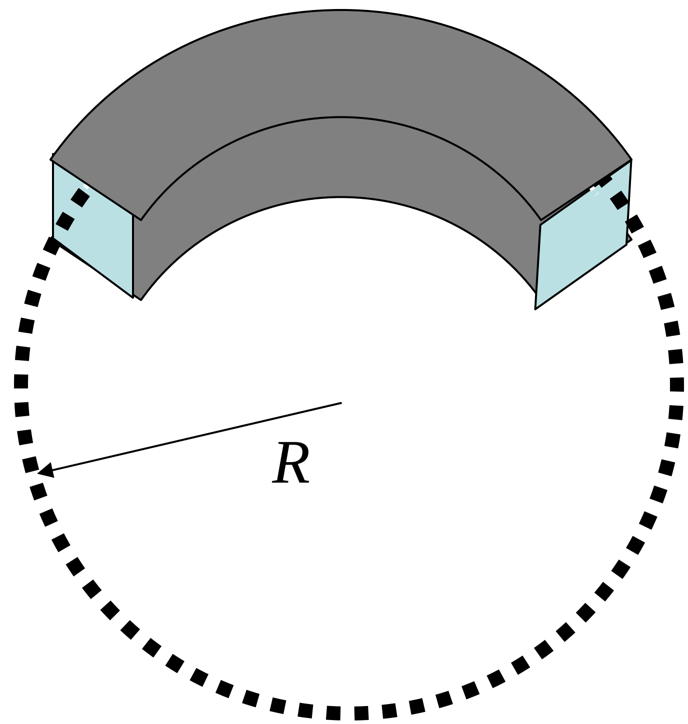
$$I = \frac{\pi r^4}{4}$$



$$I = \frac{\pi (r^4 - r_0^4)}{4}$$

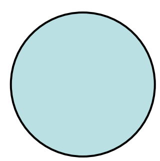
Bend Energy

$$U_{Bend} = \frac{EI}{2}\frac{L}{R^2}$$

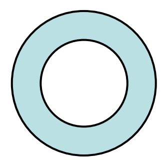


Areal Moment of Inertia

$$I = \frac{wh^3}{12}$$



$$I = \frac{\pi r^4}{4}$$

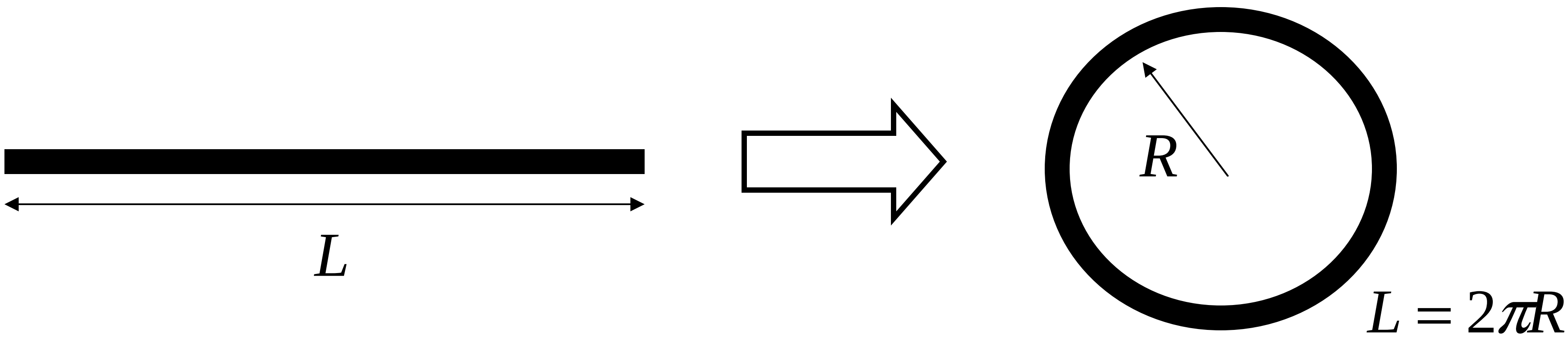


$$I = \frac{\pi (r^4 - r_0^4)}{4}$$

Bend Energy

$$U_{Bend} = \frac{EI}{2}\frac{L}{R^2}$$

Energy required to bend filament into circular arc of Length, L, and radius, R

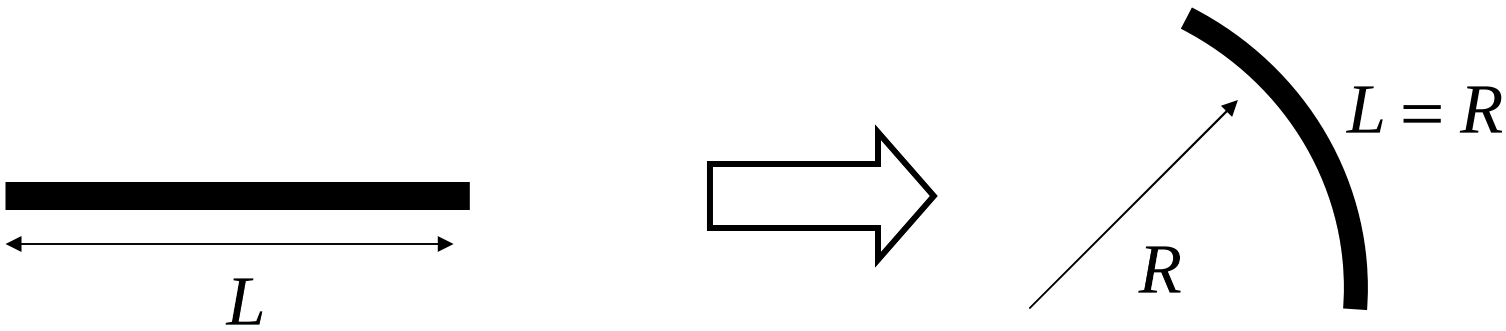


$$U_{Loop} = \frac{YI(2\pi R)}{2R^2} = \frac{\pi EI}{R}$$

Bend Energy

$$U_{Bend} = \frac{EI}{2}\frac{L}{R^2}$$

Energy required to bend filament into circular arc of Radius, R, and Length L=R

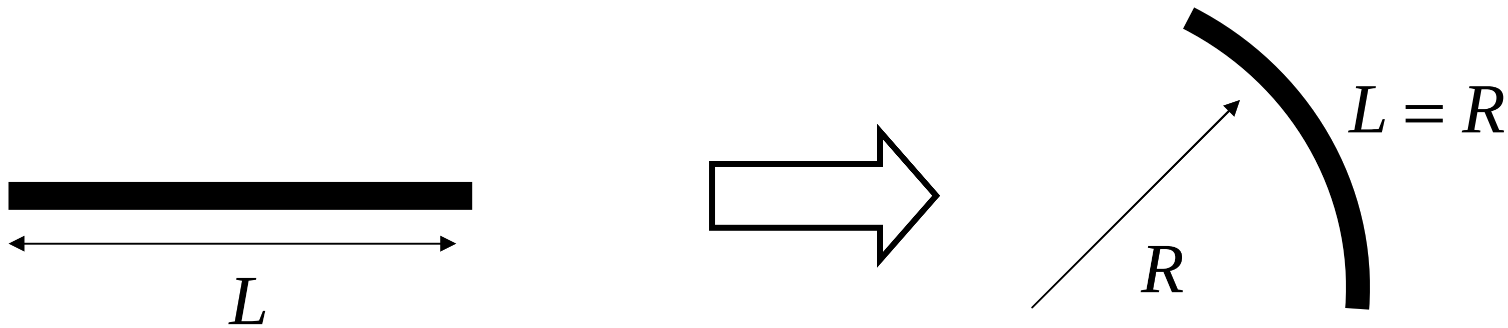


$$U_{Rad} = \frac{YI(R)}{2R^2} = \frac{EI}{2R} = \frac{EI}{2L}$$

Bend Energy

$$U_{Bend} = \frac{EI}{2}\frac{L}{R^2}$$

Energy required to bend filament into circular arc of Radius, R, and Length L=R

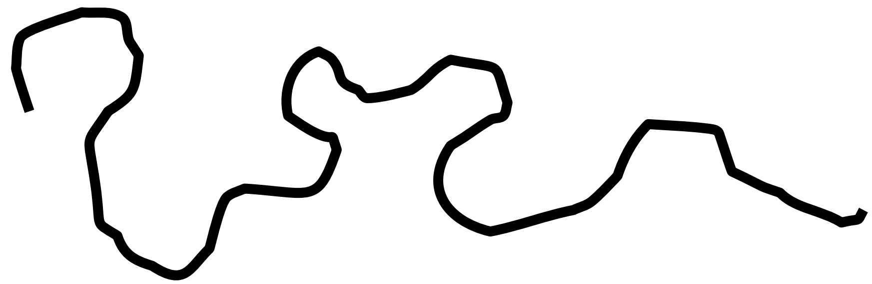


$$U_{Rad} = \frac{YI(R)}{2R^2} = \frac{EI}{2R} = \frac{EI}{2L}$$

$$\frac{1}{2}k_BT = \frac{EI}{2\xi} \Rightarrow \xi = \frac{EI}{k_BT}$$

Persistence Length

$$\xi = \frac{EI}{k_B T}$$



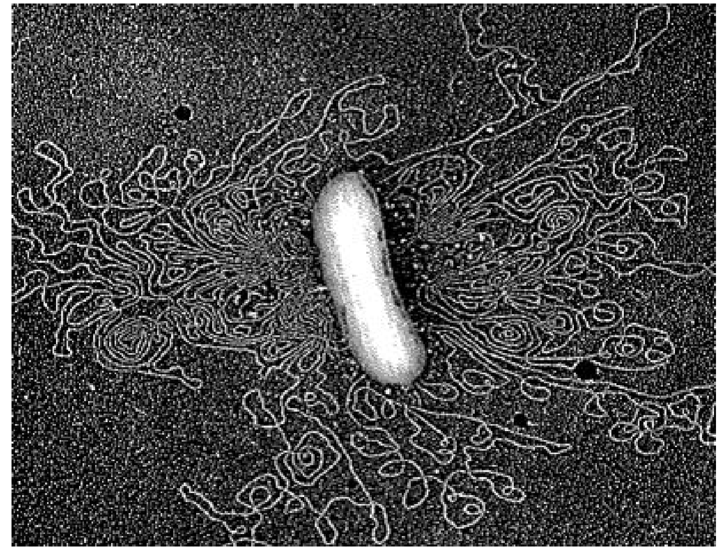
Correlation of Tangent Angles

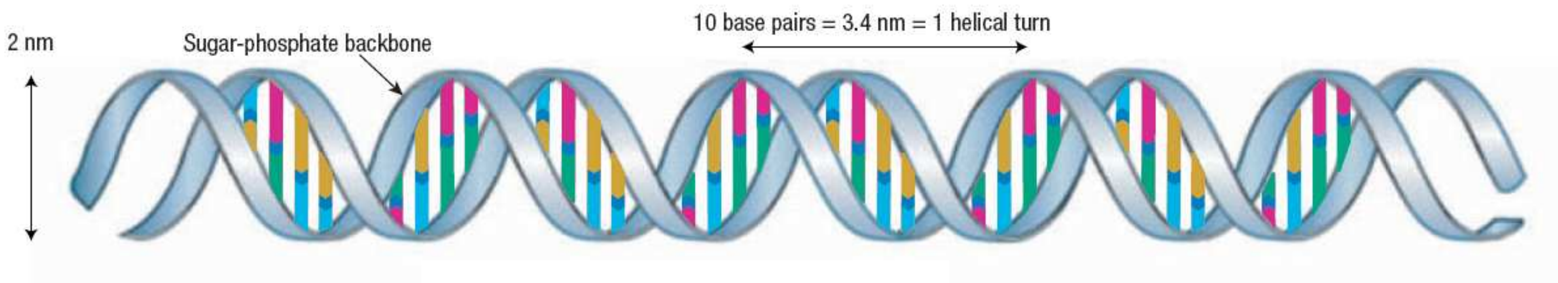
Oooooh what fun…!

Just wait until Lecture II

Persistence Length DNA

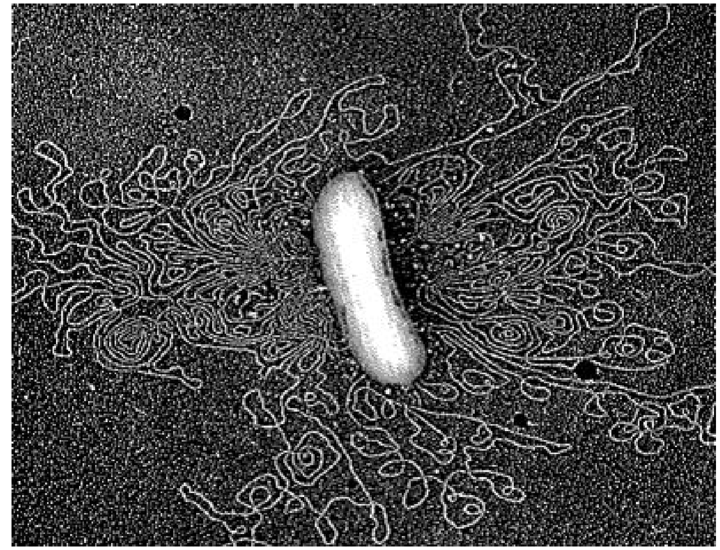
$$\xi = \frac{EI}{k_B T}$$

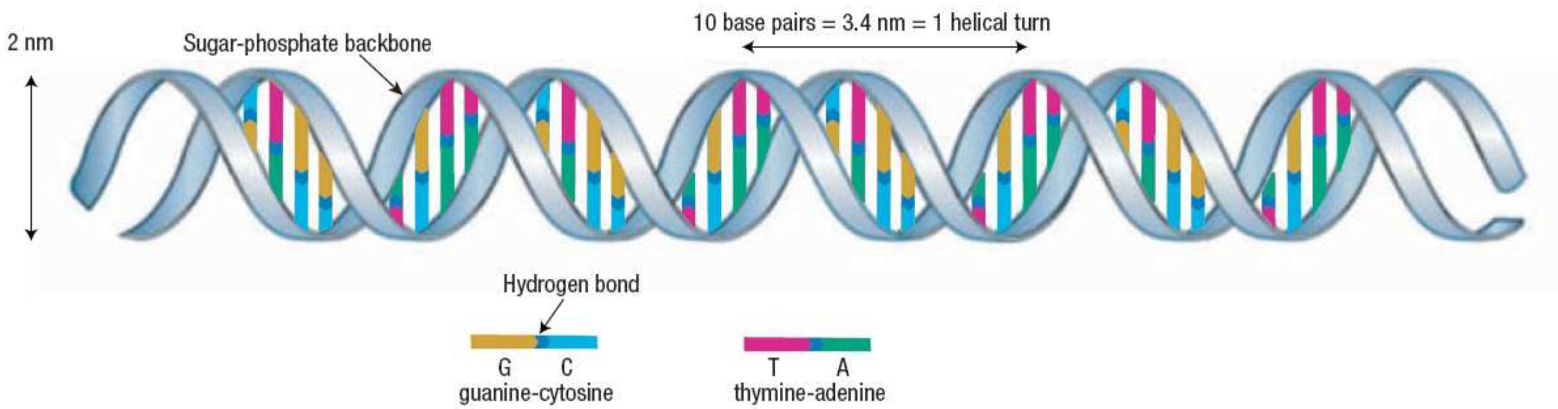




Persistence Length DNA

$$\xi = \frac{EI}{k_B T}$$



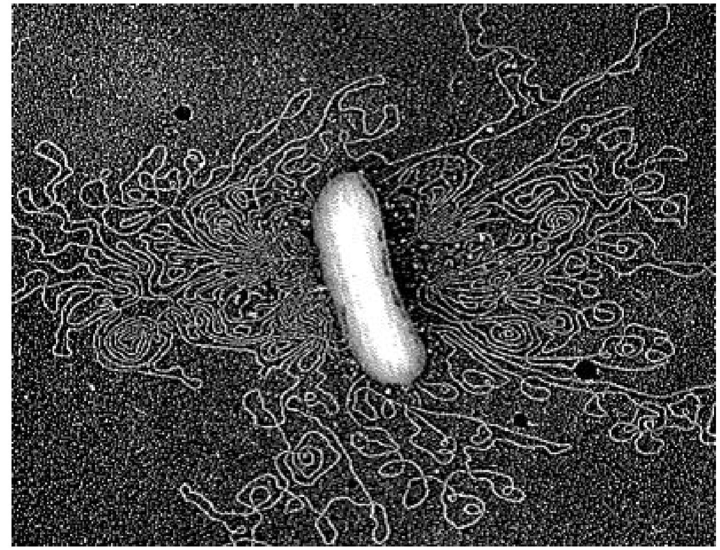


4

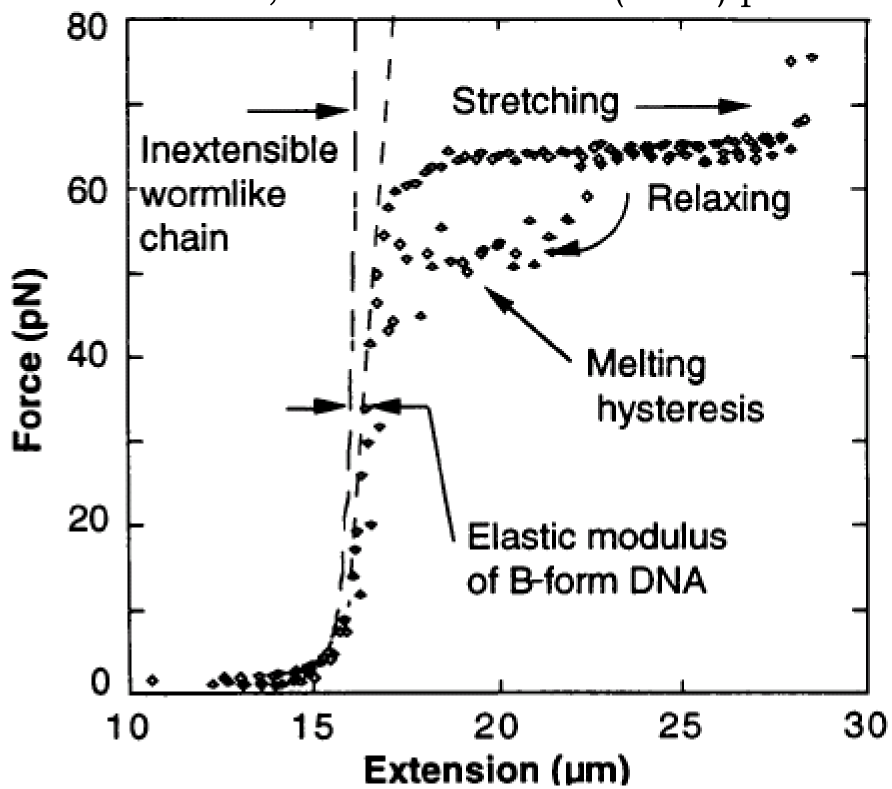
$$\xi \approx 340nm$$

Persistence Length DNA

$$\xi = \frac{EI}{k_B T}$$



Smith, et.al *Science* **271** (1996) p.795



$$\sigma = E\varepsilon$$

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi r^2} \qquad \varepsilon = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}$$

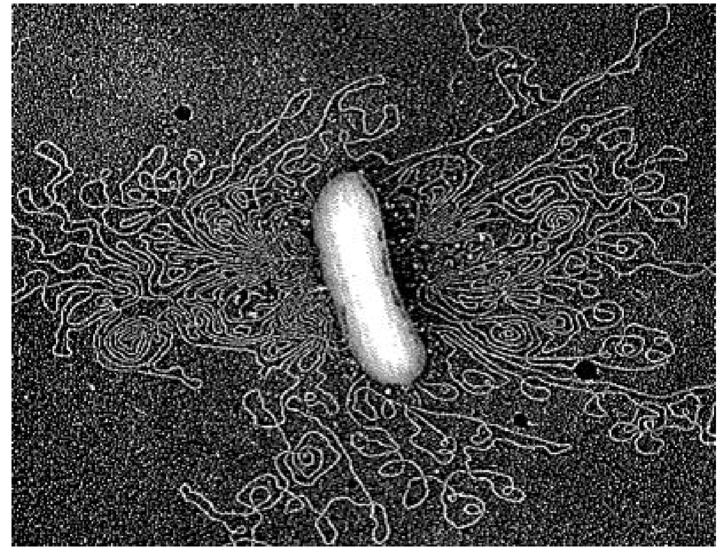
$$r = 10\,\mathring{A}$$

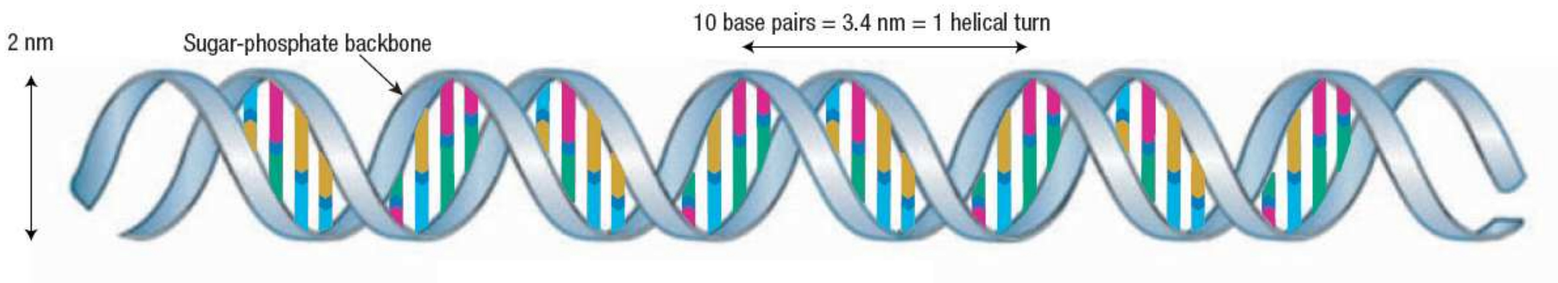
$$L_0 = 48{,}500bp = 16.4\mu m$$

$$E = \frac{F}{\pi r^2}\frac{L_0}{L - L_0} = 0.34GPa$$

Persistence Length DNA

$$\xi = \frac{EI}{k_B T}$$



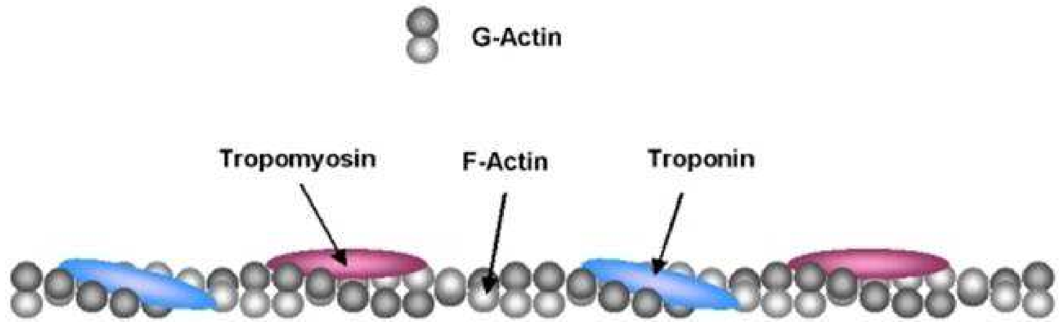


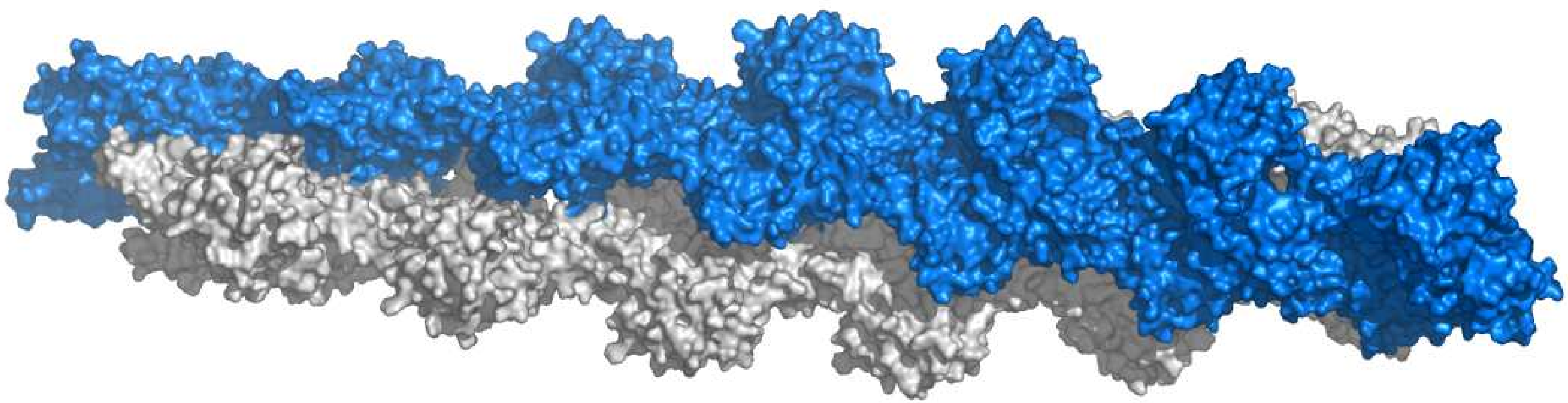
$$I = \frac{\pi R^4}{4} \approx 0.78 nm^4 \qquad E \approx 0.34 GPa$$

$$\xi \approx 66 nm$$

Persistence Length Actin

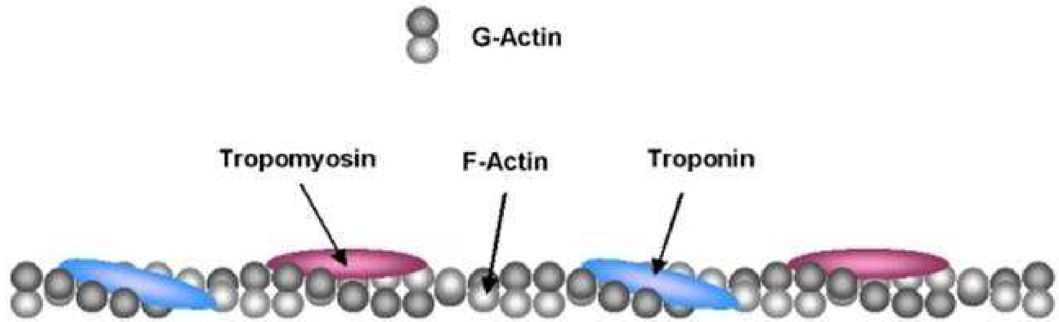
$$\xi = \frac{EI}{k_B T}$$



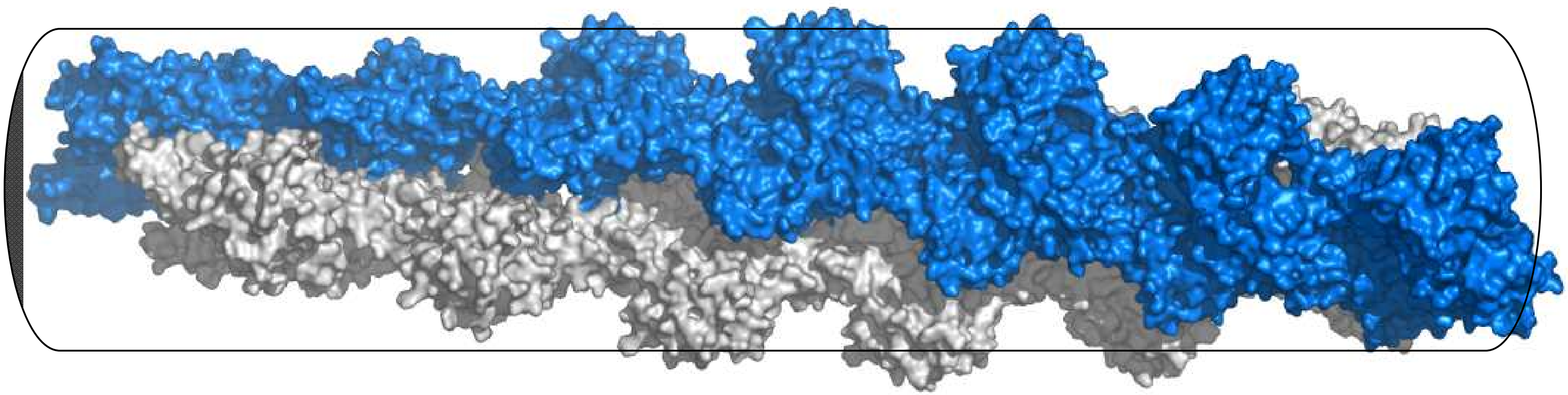


Persistence Length Actin

$$\xi = \frac{EI}{k_B T}$$



$R \sim 4nm$

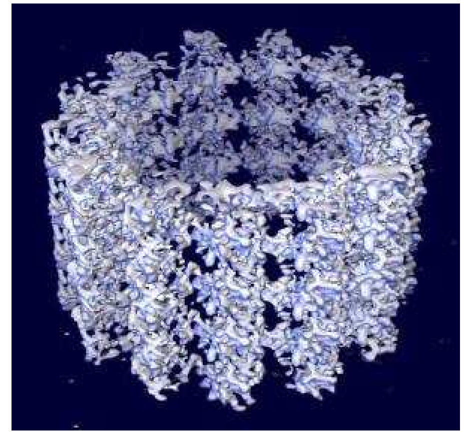


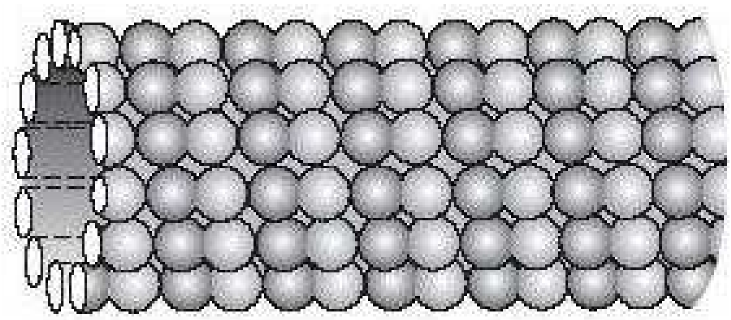
$$I = \frac{\pi R^4}{4} \approx 0.78 nm^4 \qquad E \approx 1.6 GPa$$

$$\xi \approx 47 \mu m$$

Persistence Length Microtubules

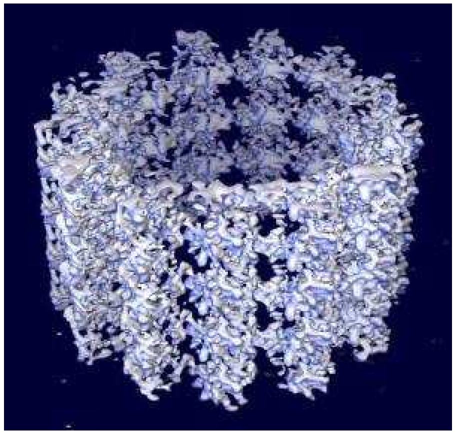
$$\xi = \frac{EI}{k_B T}$$



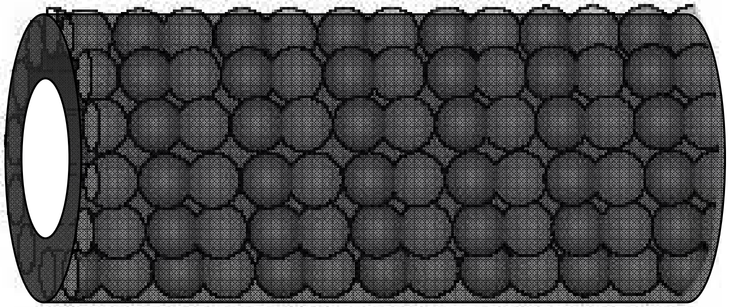


Persistence Length Microtubules

$$\xi = \frac{EI}{k_B T}$$



$$R \approx 12.5nm$$
$$R_0 \approx 10nm$$



$$I = \frac{\pi (R^4 - R_0^4)}{4} \approx 0.78nm^4 \qquad E \approx 1.6GPa$$

$$\xi \approx 4.5mm$$

Persistence Length A length dependence?

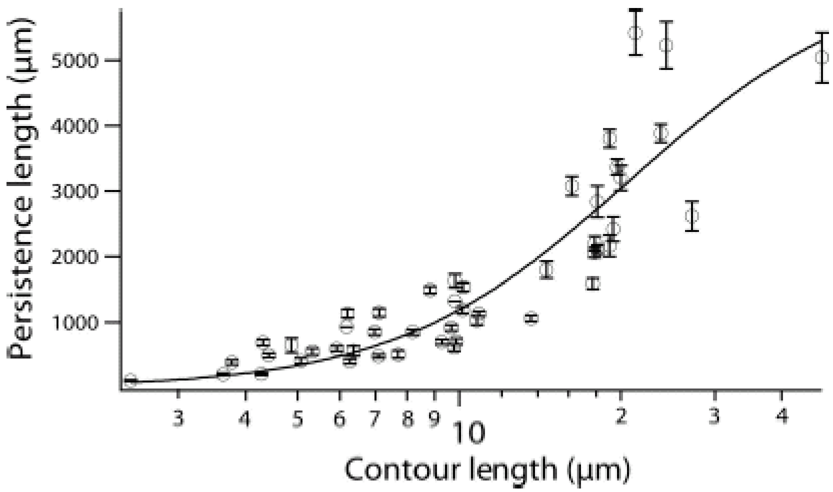


Fig. 3. The persistence length of MTs as a function of their contour lengths. The contour length ranges from 2.6 to 47.5 μm. The fit with Eq. 4 is superimposed on the experimental points.

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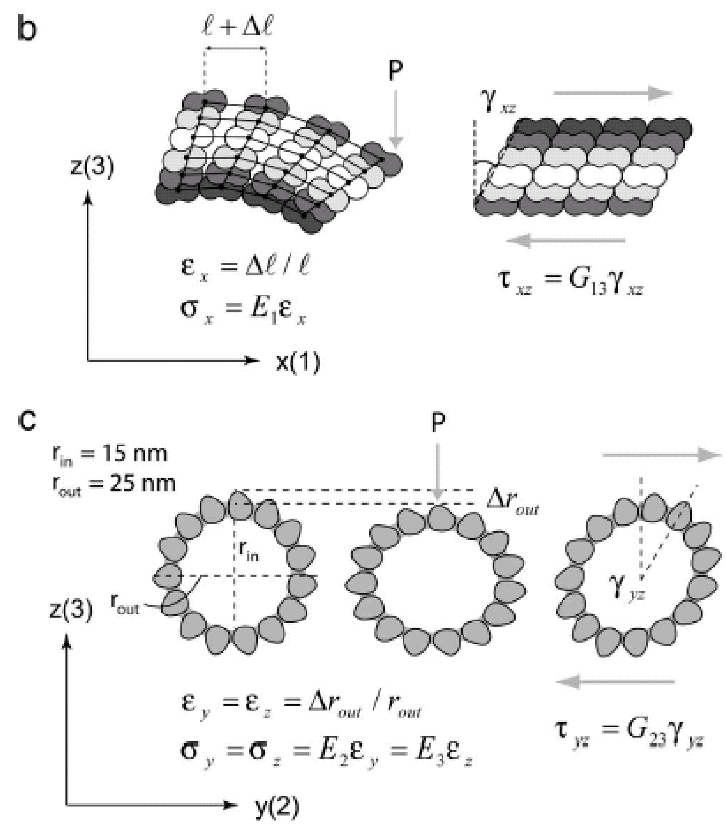
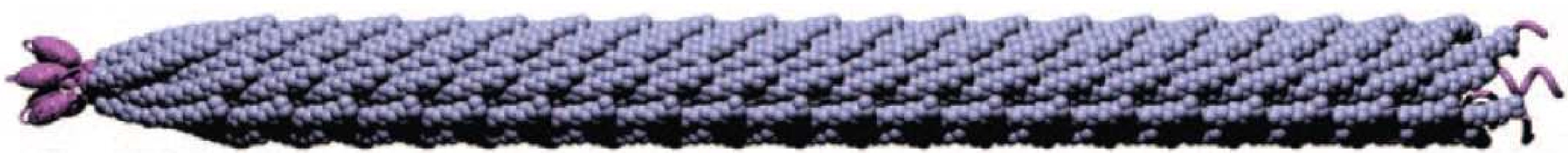
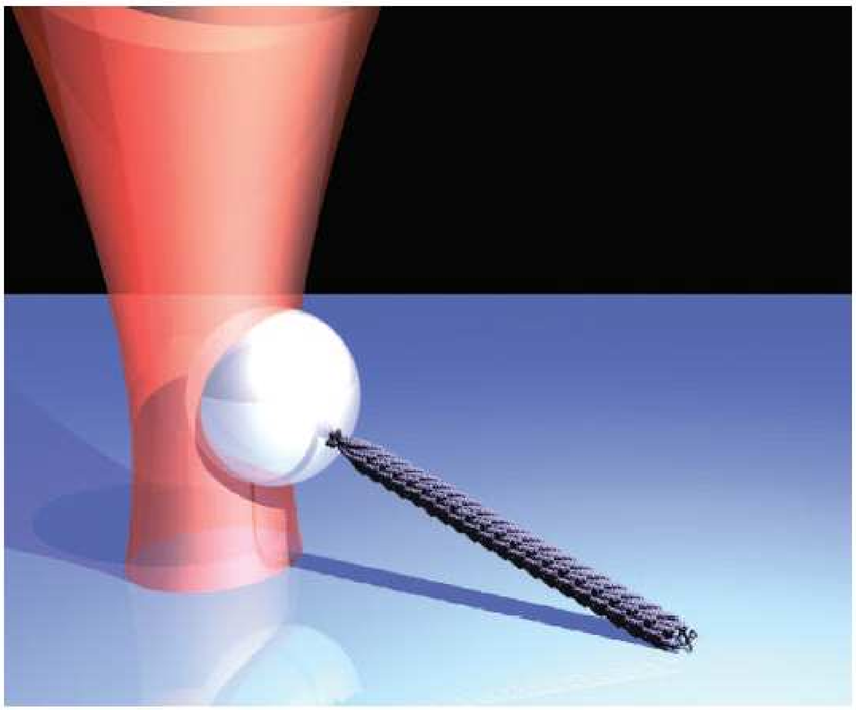
$$U_{Strain} = \frac{E}{2}\int \varepsilon^2 dxdydz$$

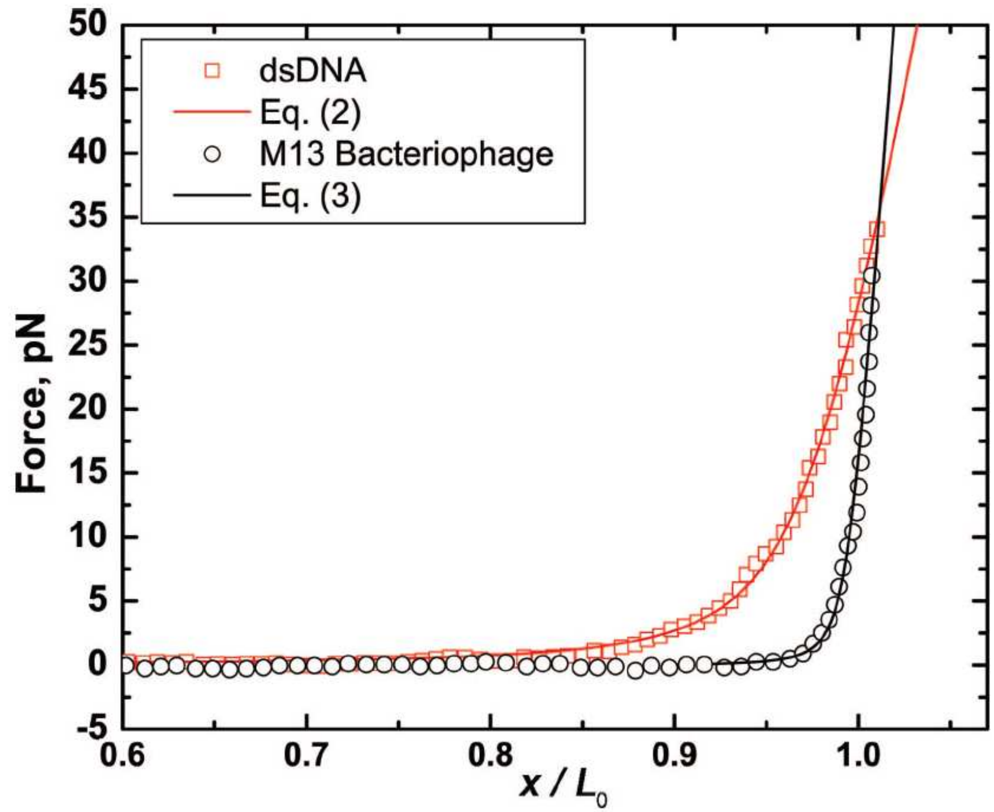


Stretching Experiments M13 virus





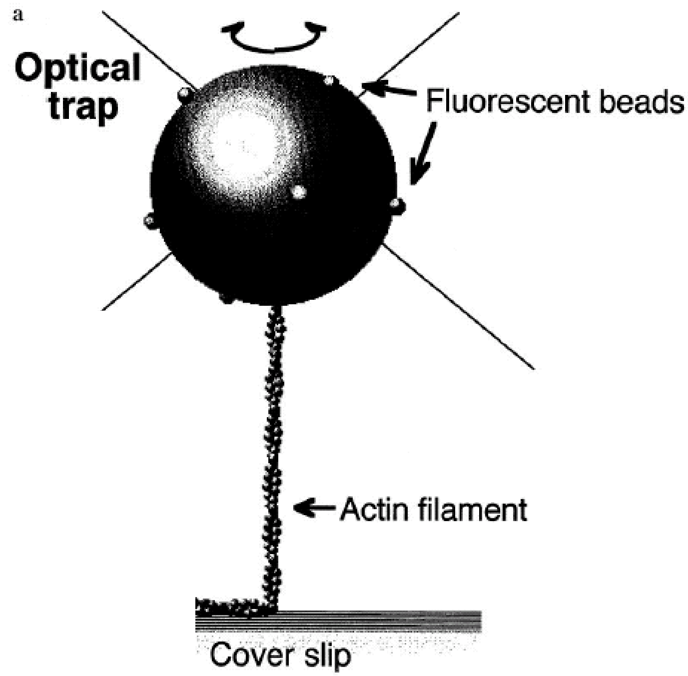
400nm Polystyrene Bead

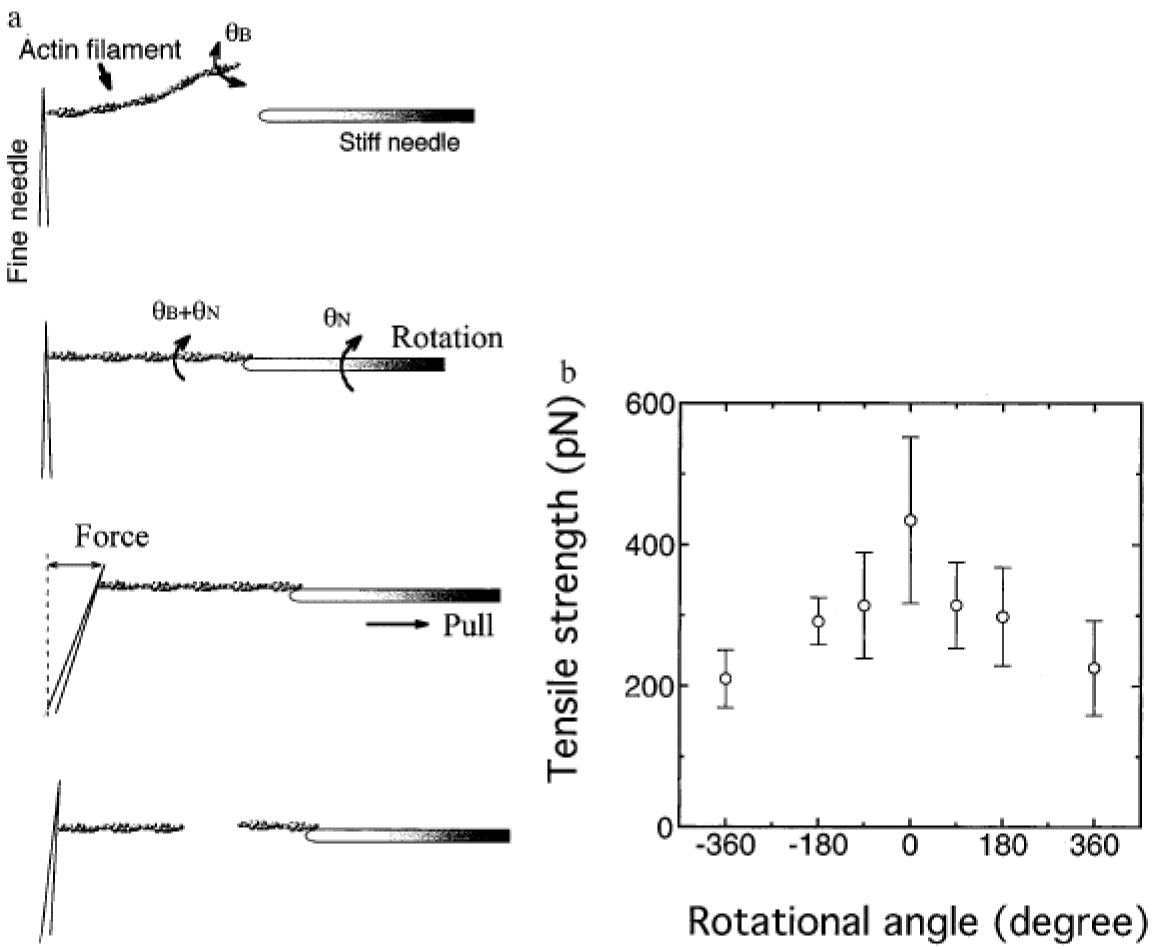


Belcher, A. et al. *PNAS* **104** (2007) p.4892

Stretching Experiments Actin

Tsuda *et al. PNAS* ***93*** *(1996)*





Life at Low D

END Lecture I

Thank you.

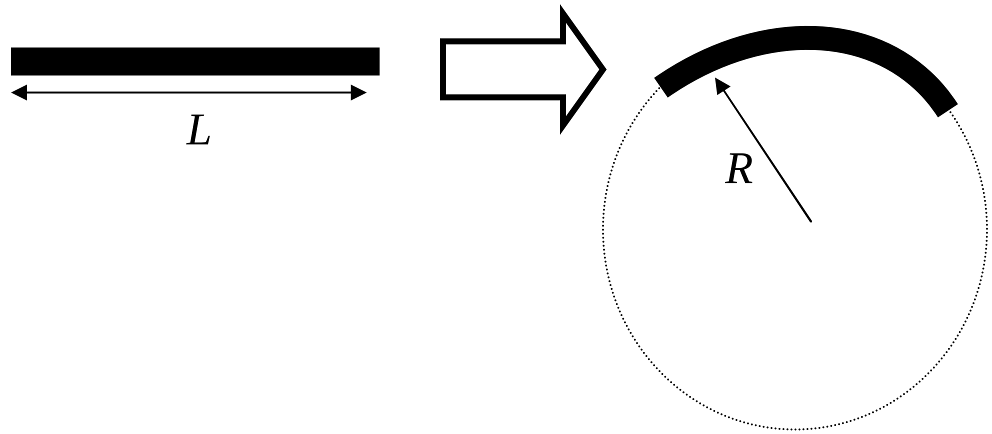
Life at Low D

Lecture II: Vitrual Lab

Bend Energy

$$U_{Bend} = \frac{EI}{2}\frac{L}{R^2}$$

Energy required to bend filament of Length L into circular arc of Radius, R



Bend Energy

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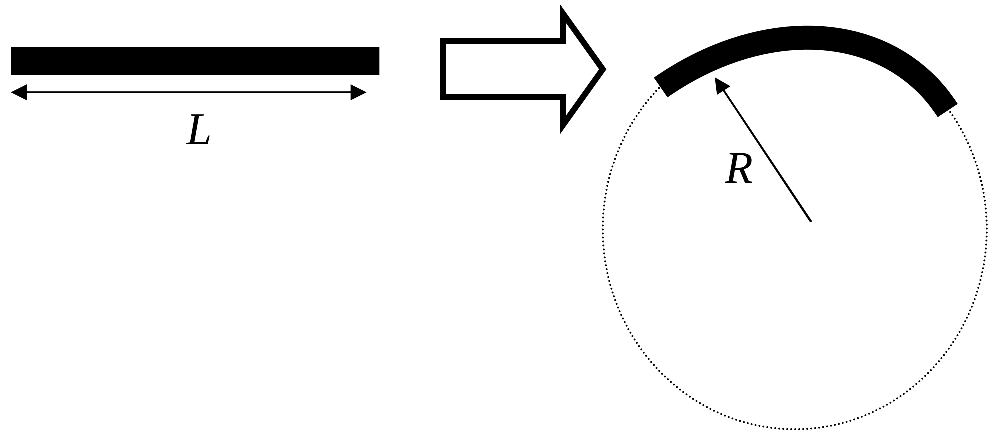
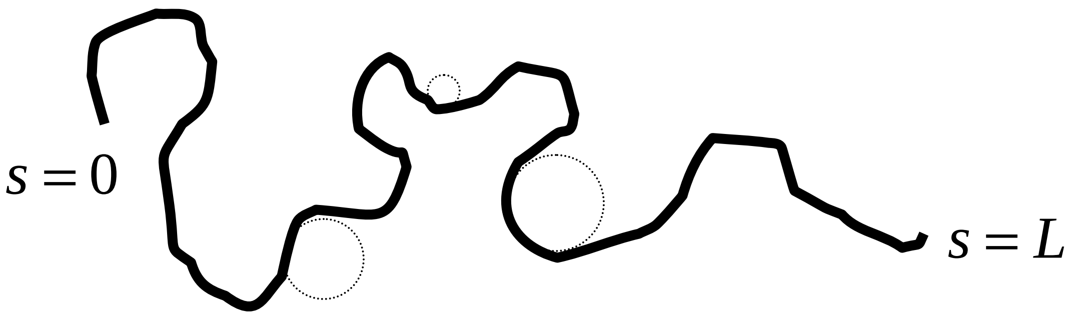
Energy required to bend filament of Length L into circular arc of Radius, R

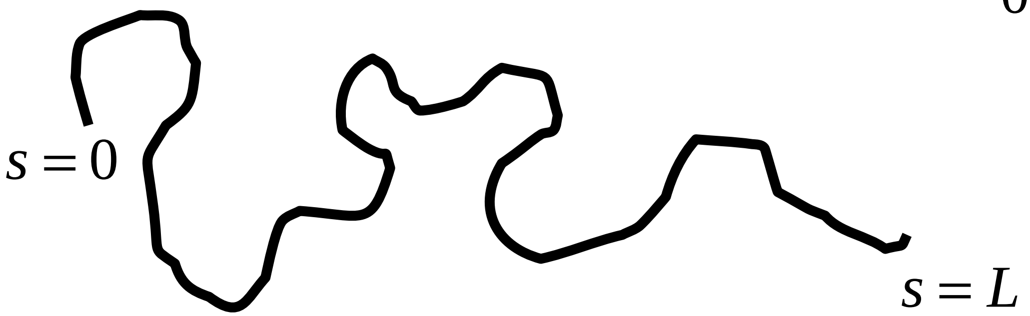


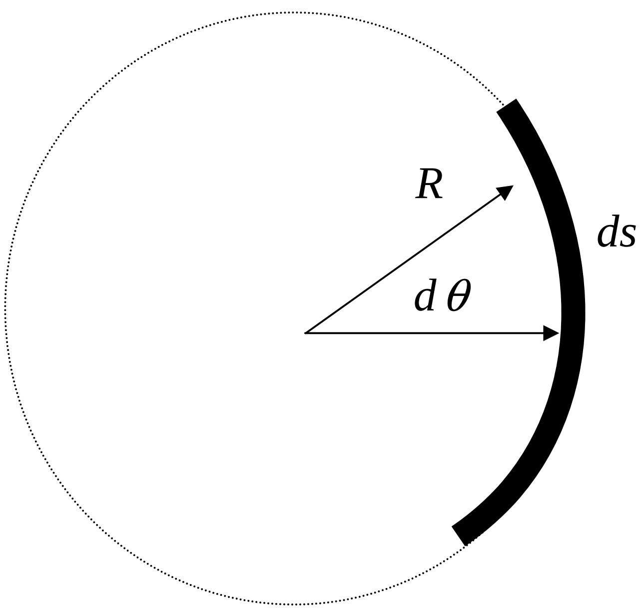


$$U_{Bend} = \sum_{all\ arcs} U^{o}_{Bend} = \frac{EIL}{2}\int_0^L \frac{1}{R^2(s)}ds$$

Bend Energy

$$U_{Bend} = \frac{EIL}{2}\int_0^L \frac{1}{R^2(s)}ds$$



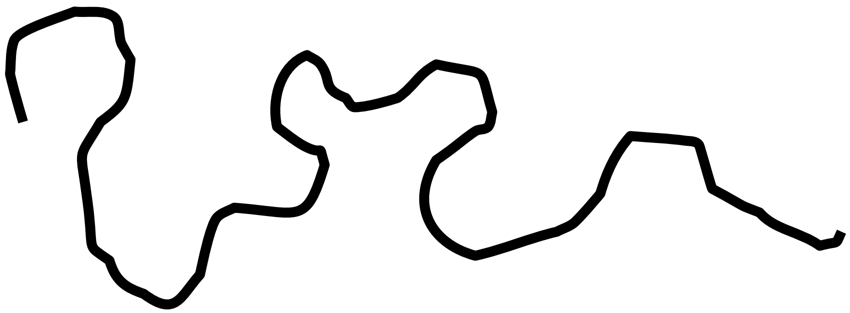


$$ds = Rd\theta \quad \Rightarrow \quad \frac{1}{R} = \frac{d\theta}{ds}$$

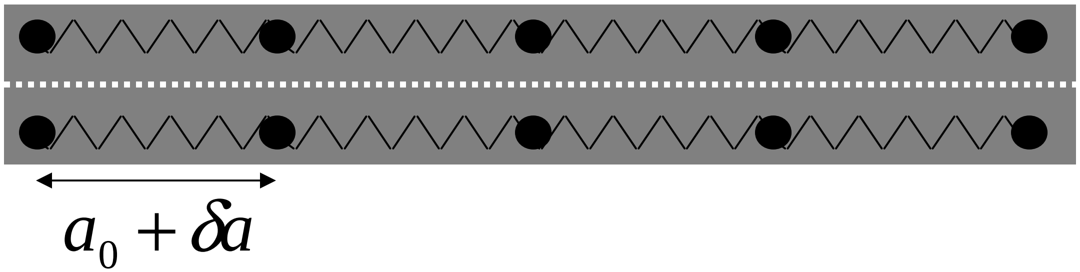
$$U_{Bend} = \frac{EIL}{2}\int_0^L \left(\frac{d\theta}{ds}\right)^2 ds$$

Bend Energy

$$U_{Bend} = \frac{EIL}{2}\int_0^L \left(\frac{d\theta}{ds}\right)^2 ds$$



"Molecular" Model - Estimates from Molecular Interactions



$$F = k_S(\delta a)$$

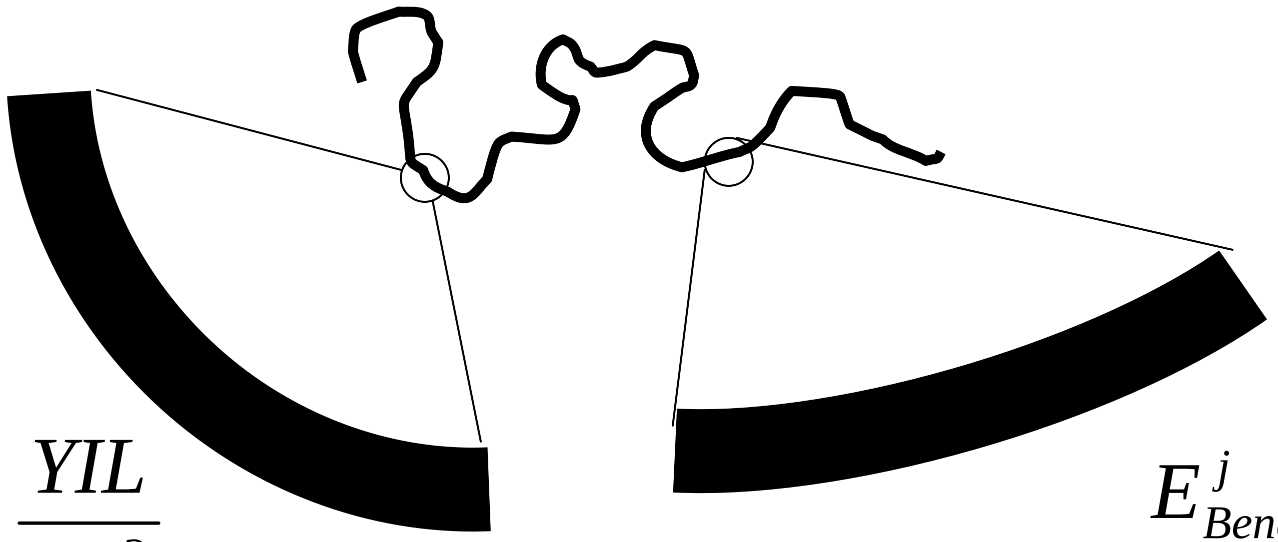
$$\varepsilon = \frac{\delta a}{a_0} \qquad \sigma = \frac{F}{a_0^2} = \frac{k_S \delta a}{a_0^2} \Rightarrow \sigma = \frac{k_S}{a_0}\varepsilon$$

$$E = \frac{k_S}{a_0}$$

"Monomer"	K (N/m)	a (A)
G-Actin	2	

$$k_S^{actin} = 2\ {}^{N}\!/_{m} \qquad a_0^{actin} = 1\,\mathring{A}$$

Bending Deformations (as collection of stretched beams)



$$E^{i}_{Bend} = \frac{YIL}{2R_i^2}$$

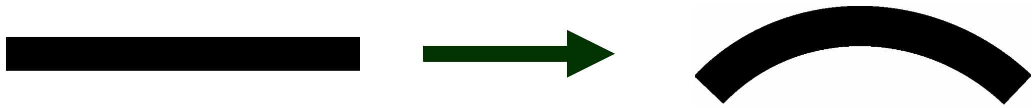
$$E^{j}_{Bend} = \frac{YIL}{2R_j^2}$$

$$E_{Bend} = \sum_{n} \frac{YIL}{2R_n^2}$$

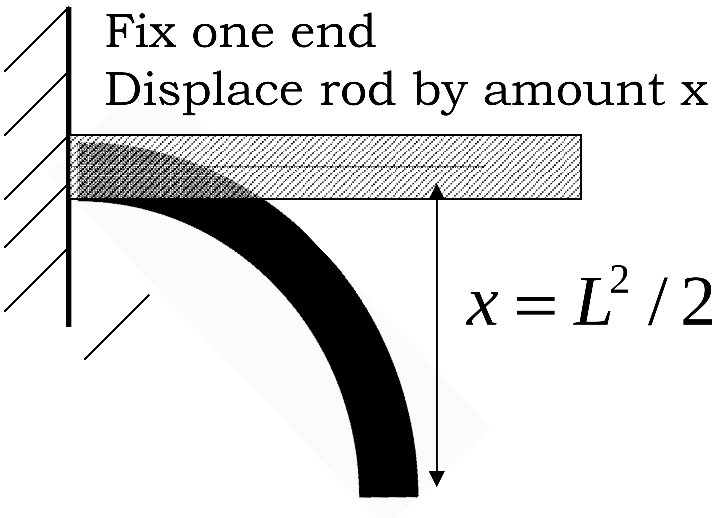
$$E_{Bend} = \frac{YIL}{2}\int_0^L \frac{1}{R^2(s)}ds$$

Bending Moment

Flexural Rigidity



$$E_{Bend} = \frac{YIL}{2R^2} = \frac{BL}{2R^2}$$



$$x = L^2 / 2R$$

$$E = \frac{2Bx^2}{L^3}$$

Harmonic Oscillator

$$E = \frac{k}{2}x^2$$

where $k = \frac{4B}{L^3}$

$$\langle x^2 \rangle = \frac{k_B T}{k} = \frac{L^3 k_B T}{4B}$$

Compare "deflections" for the filament models

Energies and Lengthscales

$$1k_BT \approx 4\times10^{-21}\,J \qquad\qquad 1J = 1N\cdot m$$

Gigajoule - 1 billion joules.

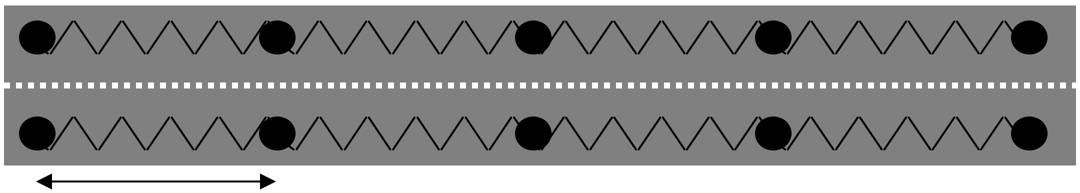
Six gigajoules is about the amount of chemical energy in a barrel of oil.

Terajoule - 1 trillion joules.

About 60 terajoules were released by the bomb that exploded over Hiroshima.

$$1k_BT = 4\,pN\cdot nm$$

“Molecular” Model



$a_0 + \delta a$

$$F = k_S(\delta a)$$

$$\varepsilon = \frac{\delta a}{a_0} \qquad \sigma = \frac{F}{a_0^2}$$

$$\sigma = \frac{k_S}{a_0}\varepsilon$$

$$E = \frac{k_S}{a_0}$$

$$k_S^{actin} = 2\ {}^{N}\!/_{m} \qquad a_0^{actin} = 1\,\mathring{A}$$

“Monomer”	K (N/m)	a (A)
G-Actin	2	

http://www.uic.edu/classes/phys/phys450/MARKO/N014.html