Life at Low D

Lecture I : Biological Filaments (1D)
**Cytoskeleton**

The "scaffolding" or "skeleton" contained within the cytoplasm

Cytoskeleton comprised of 3 types of protein filaments:

a) **Actin**
b) **Microtubules**
c) **Intermediate Filaments**

**Endothelial cells.** **Nuclei** are stained blue (DAPI), **microtubules** green (antibody bound to FITC), and **actin** filaments are red (phalloidin bound to TRITC). Bovine pulmonary arthery endothelial cells
A Drosophila tissue culture cell labeled for: **microtubules** in green and **DNA** in blue

Actin filaments of mouse embryo fibroblasts, stained with FITC-phalloidin
A Chinese hamster ovary cell (CHO) cell in anaphase. **Actin** (red), **tubulin** (green) and **DNA** (blue) are labeled.

A mitotic spindle with **kinetochores** (motors that bend microtubules, thereby destabilizing them and promoting depolymerization) in red and **microtubules** stained green.
**Biological Filaments** Actin

atomic structure of an actin filament with 13 subunits, based on the actin filament model of Ken Holmes; surface representation, rendered with PyMol

G-Actin
ADP and the divalent cation highlighted

ATP
ATP-Bound Actin

G-Actin

Thin filament organization

43kDa $\sim$ 43nm$^3$
**Biological Filaments** Microtubules

Figure 1. Polymerization of microtubules. Tubulin dimers assemble 'head-to-tail', forming oligomers that elongate into protofilaments. As the protofilaments reach an estimated critical length of 12 ± 2 dimers [65] they start to interact laterally, forming sheets with a characteristic intrinsic inward curvature. At a typical number of 13 protofilaments, the tubulin sheet closes into a tube, forming a microtubule. The tubulin lattice has a left-handed helical symmetry. The microtubule closes at the seam (black arrows), where there is a discontinuity point in the helical lattice.

**Tubulin**
**Biological Filaments** “from” Actin and Microtubules

**Flagella**

http://academic.brooklyn.cuny.edu/biology/bio4fv/page/flagella1068.JPG
**Biological Filaments** Filamentous Bacteriophages (M13)

**geneIII and geneVI proteins**
infective tip

**geneVIII protein** ~50aa monomer, 2700 copies

**geneVII and geneIX proteins**
remote tip

**ssDNA** 7259bp = 2177nm

*L ~ 1μm*

*EurBiophysJ 37. p.521(2008)*
Biological Filaments

(A) Microtubule
(B) Bacterial flagellum
(C) Tobacco mosaic virus
(D) Collagen fiber

Phys. Biol. Of the Cell
Biological Filaments
What are the types of deformations encountered in a filament’s lifetime?

- **Stretch**
- **Bend**
- **Twist**
Stretching (Stress and Strain)
Stretching (Stress and Strain)

Stress
\[ \sigma = \frac{F}{A_0} \]
Stretching (Stress and Strain)

\[ \varepsilon = \frac{\Delta L}{L} \]

\[ \sigma = \frac{F}{A_0} \]
Stretching (Stress and Strain)

Stress
\[ \sigma = \frac{F}{A_0} \]

Strain
\[ \varepsilon = \frac{\Delta L}{L} \]
Stretching (Stress and Strain)

\[ \sigma = \frac{F}{A_0} \]  
\[ \varepsilon = \frac{\Delta L}{L} \]  
\[ \sigma = E \varepsilon \]  
\[ E = \frac{\text{Force}}{\text{Area}} = \frac{\text{Energy}}{\text{Volume}} \]
**Peruvian Example** Rubber Band

\[ A_0 = 0.1 \text{cm}^2 \]

\[ 1 \text{lbs} \approx 0.5 \text{kg} \]

\[ F \sim (2.5 \text{kg})(9.8 \text{ m/s}^2) \approx 25 \text{N} \]

\[ \sigma = \frac{F}{A_0} \approx 250 \frac{\text{N}}{\text{cm}^2} \]

\[ \varepsilon = \frac{\Delta L}{L} \approx 0.75 \]

\[ \sigma = E\varepsilon \]

\[ E \approx 300 \frac{\text{N}}{\text{cm}^2} = 0.003 \text{ GPa} \]
Young’s Modulus

\[
\begin{align*}
\text{Stress} & \quad \sigma = \frac{F}{A_0} \\
\text{Strain} & \quad \varepsilon = \frac{\Delta L}{L}
\end{align*}
\]

\[
\sigma = E \varepsilon
\]

E = Young’s Modulus
UNITS!!!

\[
E = \left[ \frac{\text{Force}}{\text{Area}} \right] = \left[ \frac{\text{Energy}}{\text{Volume}} \right]
\]

Estimates

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>E [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamond</td>
<td>1200</td>
</tr>
<tr>
<td>Steel</td>
<td>211</td>
</tr>
<tr>
<td>Glass</td>
<td>100</td>
</tr>
<tr>
<td>Wood</td>
<td>10</td>
</tr>
<tr>
<td>Plastic</td>
<td>2.4</td>
</tr>
<tr>
<td>Rubber</td>
<td>0.02</td>
</tr>
</tbody>
</table>

1 Pa = 1 N/m² = 1 J/m³

Metals

\[
\begin{align*}
154 \text{ pm} & \\
U_{\text{int}} & \sim eV \quad \Rightarrow \quad E \approx 200 \text{ GPa}
\end{align*}
\]

G-Actin

\[
\begin{align*}
V & \sim 43 \text{nm}^3 \\
U_{\text{int}} & \sim 10k_B T \quad \Rightarrow \quad E \approx 2 \text{ GPa}
\end{align*}
\]
Spring Analogy

\[ F = kx \]

\[ U = \frac{1}{2} kx^2 \]
Spring Analogy

\[ F = k(\delta a) \]

\[ U = \frac{1}{2} k(\delta a)^2 \]
Spring Analogy

\[ F = E \frac{\Delta L}{L} \]

\[ F = \frac{EA}{L} (\Delta L) \]

\[ U = \frac{1}{2} \frac{EA}{L} (\Delta L)^2 \]

\[ F = k(\delta a) \]

\[ U = \frac{1}{2} k(\delta a)^2 \]
Strain Energy

\[ \frac{F}{A} = E \frac{\Delta L}{L} \]

\[ U_{\text{Strain}} = \frac{1}{2} \frac{EA}{L} (\Delta L)^2 \]
Strain Energy

\[
\frac{F}{A} = E \frac{\Delta L}{L}
\]

\[
U_{\text{Strain}} = \frac{1}{2} \frac{EA}{L} (\Delta L)^2
\]

\[
U_{\text{Strain}} = \frac{1}{2} E \left( \frac{\Delta L}{L_0} \right)^2 AL_0 = \frac{1}{2} E \varepsilon^2 V
\]
Strain Energy

\[ \frac{F}{A} = E \frac{\Delta L}{L} \]

\[ U_{\text{Strain}} = \frac{1}{2} \frac{EA}{L} (\Delta L)^2 \]

\[ U_{\text{Strain}} = \frac{1}{2} E \left( \frac{\Delta L}{L_0} \right)^2 A L_0 = \frac{1}{2} E \varepsilon^2 V \]

\[ \frac{U_{\text{Strain}}}{\text{Volume}} = \frac{1}{2} E \varepsilon^2 \]

\[ U_{\text{Strain}} = \frac{E}{2} \int \varepsilon^2(x, y, z) dx dy dz \]
Bending

$L_0$

$\alpha_0$

$L_0$
Bending

\[ x = R \theta \]

\[ L = L(z) \]
\[ L(0) = L_0 \]

\[ \Delta L(z) = L(z) - L_0 = (R + z) \frac{L_0}{R} - L_0 = \frac{z L_0}{R} \]

\[ \varepsilon = \frac{\Delta L(z)}{L_0} = \frac{z}{R} \]
\begin{align*}
L &= L(z) \\
L(0) &= L_0 \\
\Delta L(z) &= L(z) - L_0 = (R + z) \frac{L_0}{R} - L_0 = \frac{zL_0}{R} \\
\varepsilon &= \frac{\Delta L(z)}{L_0} = \frac{z}{R} \\
U_{\text{Strain}} &= \frac{E}{2} \int \varepsilon^2 (x, y, z) dxdydz
\end{align*}
Bending

\[ x = R \theta \]

\[ \varepsilon = \frac{\Delta L(z)}{L_0} = \frac{z}{R} \]

\[ U_{\text{Strain}} = \frac{E}{2} \int \varepsilon^2 (x, y, z) dx dy dz \]

\[ U_{\text{Bend}} = \frac{E}{2} \int_0^L dx \int_{-h/2}^{h/2} dy \int_{-h/2}^{h/2} \left( \frac{z}{R} \right)^2 dz = \frac{E}{2} \frac{1}{R^2} L \cdot h \cdot \frac{h^3}{8 \cdot 3} \cdot 2 = \frac{E}{2} \left( \frac{h^4}{12} \right) \frac{L}{R^2} \]
Bend Energy

\[ U_{Bend} = \frac{E}{2} \left( \frac{h^4}{12} \right) \frac{L}{R^2} \]
Bend Energy

\[ U_{\text{Bend}} = \frac{E}{2} \left( \frac{h^4}{12} \right) \frac{L}{R^2} \]

\[ U_{\text{Bend}} = \frac{E}{2} \left( \frac{\pi r^4}{4} \right) \frac{L}{R^2} \]
Bend Energy

\[ U_{\text{Bend}} = \frac{E}{2} \left( \frac{h^4}{12} \right) \frac{L}{R^2} \]

\[ U_{\text{Bend}} = \frac{E}{2} \left( \frac{\pi r^4}{4} \right) \frac{L}{R^2} \]

Areal Moment of Inertia

\[ I = \frac{wh^3}{12} \]

\[ I = \frac{\pi r^4}{4} \]

\[ I = \frac{\pi (r^4 - r_0^4)}{4} \]
Bend Energy

\[ U_{\text{Bend}} = \frac{EI}{2} \frac{L}{R^2} \]

Areal Moment of Inertia

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Bend Energy

\[ U_{Bend} = \frac{EI}{2} \frac{L}{R^2} \]

Energy required to bend filament into circular arc of length, \( L \), and radius, \( R \)

\[ U_{Loop} = \frac{YI(2\pi R)}{2R^2} = \frac{\pi EI}{R} \]
Bend Energy

\[ U_{\text{Bend}} = \frac{EI}{2} \frac{L}{R^2} \]

Energy required to bend filament into circular arc of Radius, \( R \), and Length \( L=R \)

\[ U_{\text{Rad}} = \frac{YI(R)}{2R^2} = \frac{EI}{2R} = \frac{EI}{2L} \]
Bend Energy

\[ U_{\text{Bend}} = \frac{EI}{2} \frac{L}{R^2} \]

Energy required to bend filament into circular arc of Radius, \( R \), and Length \( L = R \)

\[ U_{\text{Rad}} = \frac{YI(R)}{2R^2} = \frac{EI}{2R} = \frac{EI}{2L} \]

\[ \frac{1}{2} k_B T = \frac{EI}{2\xi} \]

\[ \xi = \frac{EI}{k_B T} \]
Persistence Length

\[ \xi = \frac{EI}{k_B T} \]

Correlation of Tangent Angles

Oooooh what fun…!

Just wait until Lecture II
Persistence Length DNA

\[ \xi = \frac{EI}{k_B T} \]
Persistence Length DNA

\[ \xi = \frac{EI}{k_B T} \]

\[ \xi \approx 340 \text{nm} \]
**Persistence Length** DNA

\[ \xi = \frac{EI}{k_B T} \]


![Graph showing force vs. extension for DNA with labels: Inextensible wormlike chain, Stretching, Relaxing, Melting hysteresis, Elastic modulus of B-form DNA.]

\[ \sigma = E \varepsilon \]

\[ \sigma = \frac{F}{A_0} = \frac{F}{\pi r^2} \quad \varepsilon = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0} \]

\[ r = 10 \text{ Å} \]

\[ L_0 = 48,500 \text{bp} = 16.4 \mu \text{m} \]

\[ E = \frac{F}{\pi r^2 \frac{L_0}{L - L_0}} = 0.34 \text{GPa} \]
**Persistence Length** DNA

\[ \xi = \frac{EI}{k_B T} \]

\[ I = \frac{\pi R^4}{4} \approx 0.78 \text{nm}^4 \]

\[ E \approx 0.34 \text{GPa} \]

\[ \xi \approx 66 \text{nm} \]
Persistence Length Actin

\[ \xi = \frac{EI}{k_B T} \]
Persistence Length Actin

\[ \xi = \frac{EI}{k_B T} \]

\[ I = \frac{\pi R^4}{4} \approx 0.78 \text{nm}^4 \quad E \approx 1.6 \text{GPa} \]

\[ \xi \approx 47 \mu m \]
**Persistence Length** Microtubules

\[ \xi = \frac{EI}{k_B T} \]
Persistence Length Microtubules

\[ \xi = \frac{EI}{k_BT} \]

\[ R \approx 12.5nm \]
\[ R_0 \approx 10nm \]

\[ I = \frac{\pi (R^4 - R_0^4)}{4} \approx 0.78nm^4 \]
\[ E \approx 1.6GPa \]

\[ \xi \approx 4.5mm \]
**Persistence Length** A length dependence?

\[ U_{\text{Strain}} = \frac{E}{2} \int \varepsilon^2 \, dx \, dy \, dz \]

**Fig. 3.** The persistence length of MTs as a function of their contour lengths. The contour length ranges from 2.6 to 47.5 \( \mu \text{m} \). The fit with Eq. 4 is superimposed on the experimental points.

*PNAS (2006) 103 10248–10253*
Stretching Experiments M13 virus

**Stretching Experiments** Actin

Tsuda *et al.* *PNAS* **93** *(1996)*
Life at Low D

END Lecture I

Thank you.
Life at Low D

Lecture II: Vitrual Lab
Bend Energy

\[ U_{\text{Bend}} = \frac{EI}{2} \frac{L}{R^2} \]

Energy required to bend filament of Length \( L \) into circular arc of Radius, \( R \)
Bend Energy

\[ U_{\text{Bend}} = \frac{EI}{2} \frac{L}{R^2} \]

Energy required to bend filament of Length \( L \) into circular arc of Radius, \( R \)

\[ U_{\text{Bend}} = \sum_{\text{all arcs}} U_{\text{Bend}}^o = \frac{EIL}{2} \int_0^L \frac{1}{R^2(s)} \, ds \]
Bend Energy

\[ U_{\text{Bend}} = \frac{EIL}{2} \int_{0}^{L} \frac{1}{R^2(s)} ds \]

\[ ds = Rd\theta \quad \Rightarrow \quad \frac{1}{R} = \frac{d\theta}{ds} \]

\[ U_{\text{Bend}} = \frac{EIL}{2} \int_{0}^{L} \left( \frac{d\theta}{ds} \right)^2 ds \]
Bend Energy

\[ U_{\text{Bend}} = \frac{EIL}{2} \int_{0}^{L} \left( \frac{d\theta}{ds} \right)^2 ds \]
“Molecular” Model - Estimates from Molecular Interactions

\[ F = k_s (\delta a) \]

\[ \varepsilon = \frac{\delta a}{a_0} \quad \sigma = \frac{F}{a_0^2} = \frac{k_s \delta a}{a_0^2} \quad \Rightarrow \quad \sigma = \frac{k_s}{a_0} \varepsilon \]

<table>
<thead>
<tr>
<th>“Monomer”</th>
<th>( \kappa ) (N/m)</th>
<th>( a ) (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G-Actin</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

\[ k_s^{\text{actin}} = 2 \, \frac{N}{m} \quad a_0^{\text{actin}} = 1 \, \text{Å}^0 \]
**Bending Deformations** (as collection of stretched beams)

\[ E_{Bend}^i = \frac{YIL}{2R_i^2} \]

\[ E_{Bend}^j = \frac{YIL}{2R_j^2} \]

\[ E_{Bend} = \sum_n \frac{YIL}{2R_n^2} \]

\[ E_{Bend} = \frac{YIL}{2} \int_0^L \frac{1}{R^2(s)} ds \]
Bending Moment
Flexural Rigidity

$E_{Bend} = \frac{YIL}{2R^2} = \frac{BL}{2R^2}$

Fix one end
Displace rod by amount $x$

$x = \frac{L^2}{2R}$

Harmonic Oscillator

$E = \frac{2Bx^2}{L^3}$

$E = \frac{k}{2} x^2$

where $k = \frac{4B}{L^3}$

$\langle x^2 \rangle = \frac{k_B T}{k} = \frac{L^3 k_B T}{4B}$

Compare “deflections” for the filament models
Energies and Lengthscales

\[ 1k_B T \approx 4 \times 10^{-21} J \quad 1J = 1N \cdot m \]

**Gigajoule** - 1 billion joules.

Six gigajoules is about the amount of chemical energy in a barrel of oil.

**Terajoule** - 1 trillion joules.

About 60 terajoules were released by the bomb that exploded over Hiroshima.

\[ 1k_B T = 4 \ pN \cdot nm \]
“Molecular” Model

\[ F = k_s(\delta a) \]

\[ \varepsilon = \frac{\delta a}{a_0} \quad \sigma = \frac{F}{a_0^2} \]

\[ \sigma = \frac{k_s}{a_0} \varepsilon \]

\[ E = \frac{k_s}{a_0} \]

\[ k_s^{actin} = 2^N/m \quad a_0^{actin} = 1^{\circ} \AA \]

http://www.uic.edu/classes/phys/phys450/MARKO/N014.html