# The Lemberg Program in International Economics and Finance

See Brandeis International Business School.

Department of

## **Mathematics**

Courses of Study: Minor Major (BA) Combined BA/MA Master of Arts Doctor of Philosophy

## **Objectives**

## Undergraduate Major

As our society becomes more technological, it is increasingly affected by mathematics. Quite sophisticated mathematics is now central to the natural sciences, to ecological issues, to economics, and to our commercial and technical life. A student who takes such general-level courses as MATH 5a, 8a, 10a, 10b, 15a, or 20a will better understand the world and be prepared to act in it.

Mathematics is, at the same time, a subject of the greatest depth and beauty with a history extending from antiquity. The department attempts to make this depth and beauty manifest. The undergraduate major introduces students to some fundamental fields—algebra, real and complex analysis, geometry, and topology—and to the habit of mathematical thought. Mathematics majors may go on to graduate school, scientific research, finance, actuarial science, or mathematics teaching, but many choose the major for its inherent interest.

## **Graduate Program in Mathematics**

The graduate program in mathematics is designed primarily to lead to the PhD. The formal course work gives the student a broad foundation for work in modern pure mathematics. An essential part of the program consists of seminars on a variety of topics of current interest in which mathematicians from Greater Boston often participate. In addition, the Brandeis-Harvard-MIT-Northeastern Mathematics Colloquium gives the student an opportunity to hear the current work of eminent mathematicians from all over the world.

## How to Become a Major

Students who enjoy mathematics are urged to consider majoring in it; Brandeis offers a wide variety of mathematics courses, and majors will have the benefits of small classes and individual faculty attention. To become a major a student should have completed either MATH 15a and 20a, or MATH 22a and b by the end of the sophomore year—these courses are prerequisites to the higher-level offerings. Therefore, it is important for students to start calculus and linear algebra (MATH 10a, 10b, 15a, 20a, or 22a) in the first year.

## How to Be Admitted to the Graduate Program

The general requirements for admission to graduate work in mathematics are the same as those for the Graduate School as a whole. The department has available a variety of fellowships and scholarships for well-qualified students. To be considered for such financial support, the student should submit an application by January 15.

## Faculty

## Ruth Charney, Chair

Geometric group theory. Topology.

### Mark Adler

Analysis. Differential equations. Completely integrable systems.

## Refik Inanc Baykur

Symplectic topology. 4-manifolds.

## Joël Bellaïche

Number theory.

## Mario Bourgoin

Knot theory.

#### Lior Fishman

Diophantine approximation. Geometric measure theory.

## Ira Gessel, Graduate Advising Head

Combinatorics.

#### Ivan Horozov

Number theory.

#### Kiyoshi Igusa

Differential topology. Homological algebra.

## Dmitry Kleinbock, Undergraduate Advising

Dynamical systems. Ergodic theory. Number theory.

#### **Bong Lian**

Representation theory. Calabi-Yau geometry. String theory.

## Alan Mayer (on leave fall 2008)

Classical algebraic geometry and related topics in mathematical physics.

## Susan Parker, Elementary Mathematics Coordinator (on leave spring 2009)

Combinatorics. Elementary mathematics instruction.

### Daniel Ruberman

Geometric topology and gauge theory.

## Gerald Schwarz (on leave 2008-2009)

Algebraic groups. Transformation groups.

## Pierre Van Moerbeke (on leave 2008-2009)

Stochastic processes. Korteweg-deVries equation. Toda lattices.

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## Requirements for the Minor

- A. MATH 22a or 15a; MATH 22b or 20a.
- **B.** Three additional semester courses, either MATH courses numbered 27 or higher or cross-listed courses. Most MATH courses numbered 27 or higher require MATH 23b as a prerequisite.

Students interested in analysis, physics, or applied mathematics are advised to choose additional courses from among MATH 35a, 36a, 36b, 37a, and 45a. Students interested in algebra or computer science are advised to consider MATH 28a, 28b, 30a, 30b, and 38b.

## Requirements for the Major

- A. MATH 22a or 15a; MATH 22b or 20a.
- **B.** MATH 23b or exemption. See item E in "Special Notes Relating to Undergraduates."
- C. MATH 35a, 40a, or 45a.
- D. MATH 28a, 28b, or 30a.
- **E.** Four additional semester courses, either MATH courses numbered 27 or higher or cross-listed courses. A course used to satisfy the requirements for the major must be passed with a grade of C– or higher.

## Honors

A degree with honors requires items A, B, C, and D above, as well as: six additional semester courses, either MATH courses numbered 27 or higher or cross-listed courses, passed with at least a grade of B. At least four of the courses used to satisfy the major requirement must be honors courses. The honors courses are MATH 30a, 30b, 32a, 34a, 38b, 40a, 40b, 45a, and all MATH courses numbered 100 or higher.

## **Teacher Preparation Track**

Students who complete the Brandeis program for Massachusetts High School Teacher Licensure (see the Education Program section in this *Bulletin*) may earn a bachelor's degree in mathematics by satisfying major requirements A, B, C, and D above and the following:

- **F.** MATH 8a (Introduction to Probability and Statistics) or 36a (Probability).
- **G.** Two additional courses, either MATH courses numbered 27 or higher or cross-listed courses.
- H. A computer science course numbered 10 or higher.
- I. Completion of the High School Teacher Licensure Program.

## Combined BA/MA Program

Undergraduate students are eligible for the BA/MA program in mathematics if they have completed MATH 101a and b; 110a; 111a and b; and 121 a and b with a grade of B— or better and demonstrated a reading knowledge of mathematical French, German, or Russian. No more than three of these courses, however, may be counted toward the major. In addition, students must fulfill a minimum of three years' residence on campus. A student must make formal written application for admission to this program on forms available at the Graduate School office. This must be done no later than May 1 preceding his/her final year of study on campus.

## Special Notes Relating to Undergraduates

- **A.** With permission of the undergraduate advising head, courses taken in other Brandeis departments or taken at other universities may be substituted for required mathematics courses. A course used to satisfy the requirements for the major or minor must be passed with a grade of C- or higher.
- **B.** Students planning to take MATH 10a or 10b or to place into MATH 15a or 20a should take the Calculus Placement Exam. This online exam can be found, along with instructions for scoring and interpreting the results, at www.brandeis.edu/registrar/newstudent/testing.html. Students planning to take MATH 22a must take the MATH 22a Placement Exam, which can be found at the same place.

Students with AP Mathematics credit should consult the chart on page 23 of this *Bulletin* to see which Brandeis mathematics courses are equivalent to their AP credit. Note: Students who want to use their AP score to place into an upper-level course must still take the Calculus Placement Exam or the MATH 22a Placement Exam to make sure that their preparation is sufficient. Questions about placement should be directed to the elementary mathematics coordinator or the undergraduate advising head.

- C. The usual calculus sequence is MATH 10a, 10b, and 20a. Students may precede this sequence with MATH 5a. Many students also take MATH 15a (Applied Linear Algebra), which has MATH 5a (or placement out of MATH 5a) as a prerequisite. Students with a strong interest in mathematics and science are encouraged to take MATH 22a,b in place of MATH 15a and MATH 20a.
- **D.** A student may not receive credit for more than one of MATH 15a and 22a; or MATH 20a and 22b. Similarly, a student may not receive credit for all three of MATH 28a, 28b, and 30a.
- **E.** Students should normally take MATH 23b before taking upper-level courses (i.e., those numbered above 23). For many students this means taking MATH 23b concurrently with MATH 15a or MATH 20a or MATH 20a or b. Students may also take MATH 23b concurrently with MATH 35a and MATH 36a, as these do not have MATH 23b as a prerequisite. A student may be exempted from the requirement of taking MATH 23b by satisfactory performance on a placement exam. The placement exam will be given at the beginning of the fall semester.
- **F.** Students interested in graduate school or a more intensive study of mathematics are urged to include all of the following courses in their program:
- 1. MATH 22a and b.
- 2. MATH 30a and b.
- 3. MATH 35a or 40a and b.
- 4. MATH 45a.

A course numbered 100 or higher.

- **G.** The following schedule determines course offerings in mathematics:
- 1. Offered every semester are MATH 5a, 10a and b, 15a, 20a, and 23b.
- 2. Offered once each year are MATH 8a, 30a and b, 35a, 36a and b, 37a, 40a and b, 45a.
- 3. In addition, the following semester courses are usually offered every second year according to the following schedule:
- a. even-odd years (e.g., 2008-2009): MATH 3a, 28a, 32a, 39a, and 47a.
- b. odd-even years (e.g., 2009-2010): MATH 28b, 34a, 38b, and 56a.

**H.** The number of cross-listed courses used to satisfy the requirements for the major, the honors, or teacher preparation track must not exceed two; for the minor, the limit is one.

## Requirements for the Degree of Master of Arts

**A.** One year academic residency as a full-time student.

**B.** Successful completion of an approved schedule of courses: MATH 101a and b, MATH 110a, MATH 111a and b, and MATH 121a and b.

C. Proficiency in reading French, German, or Russian.

## Requirements for the Degree of Doctor of Philosophy

#### Program of Study

The normal first year of study consists of MATH 101a and b, 111a and b, and MATH 121a and b. With the permission of the graduate adviser, a student with superior preparation may omit one or more of these courses and elect higher-level courses instead. In this case the student must take an examination in the equivalent material during the first two weeks of the course. The second year's work will normally consist of MATH 110a and higher-level courses in addition to preparation for the qualifying examinations described below and participation in the second-year seminar. Upon completion of the qualifying examinations, the student will choose a dissertation adviser and begin work on a thesis. This should be accompanied by advanced courses and seminars.

### **Teaching Requirements**

An important part of the doctoral program is participation, as a teaching fellow, in a structured program of undergraduate teaching. During the spring semester of the first year, every student takes part in our teaching apprenticeship program to learn basic classroom teaching skills. All graduate students are then expected to teach a section of calculus or precalculus for at least three semesters, usually beginning in the second year of study. Teaching fellows must also enroll every fall semester in the Teaching Practicum, in which their teaching is evaluated and discussed.

## Residence Requirement

The minimum academic residence requirement is three years.

#### Language Requirement

Proficiency in reading one of French, German, or Russian, and one other language (besides English) determined with the consent of the adviser.

## **Qualifying Examination**

The qualifying examination consists of two parts: a major examination and a minor examination. Both are normally completed by the end of the third year. For the major examination, the student will choose a limited area of mathematics (e.g., differential topology, several complex variables, or ring theory) and a major examiner from among the faculty. Together they will plan a program of study and a subsequent examination in that material. The aim of this study is to prepare the student for research toward the PhD. The minor examination will be more limited in scope and less advanced in content. Its subject matter should be significantly different from that of the major examination.

#### Dissertation and Defense

The doctoral degree will be awarded only after the submission and acceptance of an approved dissertation and the successful defense of that dissertation.

## Courses of Instruction

# (1-99) Primarily for Undergraduate Students

# MATH 1a Introduction to Mathematical Concepts

sn]

Mathematical reasoning where it would be expected, and elsewhere. A variety of short topics involving games and puzzles, number theory, combinatorics, and topology. Usually offered every third year. Staff

# MATH 3a Mathematics for Elementary and Middle School Teachers

[sn]

An in-depth exploration of the fundamental ideas underlying the mathematics taught in elementary and middle school. Emphasis is on problem solving, experimenting with mathematical ideas, and articulating mathematical reasoning. Usually offered every second year.

Ms. Charney (spring)

## MATH 5a Precalculus Mathematics

Does not satisfy the School of Science requirement.

Brief review of algebra followed by the study of functions. Emphasis on exponential, logarithmic, and trigonometric functions. The course's goal is to prepare students for MATH 10a. The decision to take this course should be guided by the results of the mathematics placement exam. Usually offered every semester in multiple sections.

Staff (fall and spring)

# MATH 8a Introduction to Probability and Statistics

[ qr sn ]

Discrete probability spaces, random variables, expectation, variance, approximation by the normal curve, sample mean and variance, and confidence intervals. Does not require calculus; only high school algebra and graphing of functions. Usually offered every year. Mr. Bourgoin [fall]

## MATH 10a Techniques of Calculus (a)

sn

Prerequisite: A satisfactory grade of C- or higher in MATH 5a or placement by examination.

Introduction to differential (and some integral) calculus of one variable, with emphasis on techniques and applications. Usually offered every semester in multiple sections.

Ms. Parker and Staff (fall), Staff (spring)

## MATH 10b Techniques of Calculus (b)

Prerequisite: A satisfactory grade of C- or higher in MATH 10a or placement by examination. Continuation of 10a. Students may not take MATH 10a and MATH 10b simultaneously.

Introduction to integral calculus of one variable with emphasis on techniques and applications. Usually offered every semester in multiple sections.

Mr. Bourgoin (fall), Staff (spring)

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## MATH 15a Applied Linear Algebra

sn]

Prerequisites: MATH 5a and permission of the instructor, placement by examination, or any mathematics course numbered 10 or above. Students may take MATH 15a or 22a for credit, but not both.

Matrices, determinants, linear equations, vector spaces, eigenvalues, quadratic forms, linear programming. Emphasis on techniques and applications. Usually offered every semester.

Staff (fall and spring)

# MATH 20a Techniques of Calculus: Calculus of Several Variables

sn

Prerequisites: MATH 10a and b. Students may take MATH 20a or 22b for credit, but not both.

Among the topics treated are vectors and vector-valued functions, partial derivatives and multiple integrals, extremum problems, line and surface integrals, Green's and Stokes's theorems. Emphasis on techniques and applications. Usually offered every semester.

Mr. Horozov (fall), Staff (spring)

# MATH 22a Linear Algebra and Intermediate Calculus, Part I

sn ]

Prerequisite: MATH 22 placement exam and permission of the instructor. Students may take MATH 15a or 22a for credit, but not both.

MATH 22a and b cover linear algebra and calculus of several variables. The material is similar to that of MATH 15a and MATH 20b, but with a more theoretical emphasis and with more attention to proofs. Usually offered every year.

Mr. Ruberman (fall)

# MATH 22b Linear Algebra and Intermediate Calculus, Part II

sn]

Prerequisite: MATH 22a or permission of the instructor. Students may take MATH 20a or 22b for credit, but not both. See MATH 22a for course description. Usually offered every year. Mr. Mayer (spring)

## MATH 23b Introduction to Proofs

[ sn wi ]

Prerequisites: MATH 15a, 20a, or 22a, or permission of the instructor.
Emphasizes the analysis and writing of proofs. Various techniques of proof are introduced and illustrated with topics chosen from set theory, calculus, algebra, and geometry. Usually offered every semester.

Mr. Gessel (fall), Mr. Lian (spring)

#### MATH 28a Introduction to Groups

[ sn ]

Prerequisites: MATH 23b and either MATH 15a or 22a, or permission of the instructor. Groups. Lagrange's theorem. Modulo n addition and multiplication. Matrix groups and permutation groups. Homomorphisms, normal subgroups, cosets, and factor groups. Usually offered every second year. Mr. Horozov (spring)

## MATH 28b Introduction to Rings and Fields

sn ]

Prerequisites: MATH 23b and either MATH 15a, 22a, or permission of the instructor. Fields. Z/p and other finite fields. Commutative rings. Polynomial rings and subrings of C. Euclidean rings. The quotient ring A/(f). Polynomials over Z. Usually offered every second year. Staff

## MATH 30a Introduction to Algebra, Part I

[sn]

Prerequisite: MATH 23b and MATH 22a, or permission of the instructor.

An introduction to the basic notions of modern algebra-rings, fields, and linear algebra. Usually offered every year.

Mr. Bellaiche (fall)

## MATH 30b Introduction to Algebra, Part II

sn

Prerequisite: MATH 30a or permission of the instructor.

A continuation of MATH 30a, culminating in Galois theory. Usually offered every year. Mr. Lian (spring)

## MATH 32a Differential Geometry

sn]

Prerequisites: MATH 23b and either MATH 22b or permission of the instructor. Results in the classical differential geometry of curves and surfaces are studied theoretically and also implemented as computer algorithms. Static images and animations of geometrical objects are illustrated using the mathematical visualization program 3D-XplorMath. Computer projects involving MathLab and Mathematica are important components of the course, and for those without prior experience in using these programming systems, appropriate training is provided. Usually offered every second year. Staff (spring)

## MATH 34a Introduction to Topology

sn

Prerequisites: MATH 23b and either MATH 22a and b or permission of the instructor. An introduction to point set topology, covering spaces, and the fundamental group. Usually offered every second year. Staff

#### MATH 35a Advanced Calculus

sn

Prerequisites: MATH 15a or 22a and MATH 20a or 22b.

Infinite series: convergence tests, power series, and Fourier series. Improper integrals: convergence tests, the gamma function, Fourier and Laplace transforms. Complex numbers. Usually offered every year.

Staff (spring)

## MATH 36a Probability

qr sn ]

Prerequisite: MATH 20a or 22b.
Sample spaces and probability measures, elementary combinatorial examples.
Random variables, expectations, variance, characteristic, and distribution functions. Independence and correlation. Chebychev's inequality and the weak law of large numbers. Central limit theorem. Markov and Poisson processes. Usually offered every year.

Mr. Kleinbock (fall)

## MATH 36b Mathematical Statistics

qr sn

Prerequisite: MATH 36a or permission of the instructor.

Probability distributions, estimators, hypothesis testing, data analysis. Theorems will be proved and applied to real data. Topics include maximum likelihood estimators, the information inequality, chisquare test, and analysis of variance. Usually offered every year. Mr. Igusa (spring)

## MATH 37a Differential Equations

[ sn ]

Prerequisites: MATH 15a or 22a and MATH 20a or 22b.

A first course in ordinary differential equations. Study of general techniques, with a view to solving specific problems such as the brachistochrone problem, the hanging chain problem, the motion of the planets, the vibrating string, Gauss's hypergeometric equation, the Volterra predator-prey model, isoperimetric problems, and the Abel mechanical problem. Usually offered every year. Mr. Baykur (fall)

## MATH 38b Number Theory

[ sn ]

Prerequisites: MATH 23b and either MATH 22a or permission of the instructor.
Congruences, finite fields, the Gaussian integers, and other rings of numbers.
Quadratic reciprocity. Such topics as quadratic forms or elliptic curves will be covered as time permits. Usually offered every second year.
Staff

## MATH 39a Introduction to Combinatorics

sn | Prerequisites: COSI 29a or MATH 23b.

Trerequisites: COSI 29a or MATH 23b.
Topics include graph theory (trees, planarity, coloring, Eulerian and Hamiltonian cycles), combinatorial optimization (network flows, matching theory), enumeration (permutations and combinations, generating functions, inclusion-exclusion), and extremal combinatorics (pigeonhole principle, Ramsey's theorem). Usually offered every second year.
Staff (spring)

#### MATH 40a Introduction to Real Analysis, Part I

sn]

Prerequisites: MATH 23b and MATH 22a and b or permission of the instructor. MATH 40a and b give a rigorous introduction to metric space topology, continuity, derivatives, and Riemann and Lebesgue integrals. Usually offered every year.

Mr. Fishman (fall)

#### MATH 40b Introduction to Real Analysis, Part II

sn]

Prerequisite: MATH 40a or permission of the instructor.

See MATH 40a for course description. Usually offered every year. Mr. Mayer (spring)

# MATH 45a Introduction to Complex Analysis

sn ]

Prerequisites: MATH 15a or 22a and MATH 20a or 22b, and MATH 23b or permission of the instructor.

An introduction to functions of a complex variable. Topics include analytic functions, line integrals, power series, residues, conformal mappings. Usually offered every year

Mr. Mayer (spring)

# MATH 47a Introduction to Mathematical Research

sn wi

Prerequisite: MATH 23b or permission of the instructor.

Students work on research projects that involve generating data, making conjectures, and proving theorems, and present their results orally and in writing. Introduces applications of computers in mathematical research: symbolic computation, typesetting, and literature search. Usually offered every second year. Mr. Igusa (fall)

## MATH 56a Introduction to Stochastic Processes and Models

sn

Prerequisites: MATH 15a, 20a, and 36a. Basic definitions and properties of finite and infinite Markov chains in discrete and continuous time, recurrent and transient states, convergence to equilibrium, Martingales, Wiener processes and stochastic integrals with applications to biology, economics, and physics. Usually offered every second year.

#### MATH 98a Independent Research

Usually offered every year. Staff

#### MATH 98b Independent Research

Usually offered every year.

# (100-199) For Both Undergraduate and Graduate Students

Undergraduate students should consult with the instructor regarding the required background for each course.

### MATH 101a Algebra I

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Groups, rings, modules, Galois theory, affine rings, and rings of algebraic numbers. Multilinear algebra. The Wedderburn theorems. Other topics as time permits. Usually offered every year.
Mr. Bellaiche (fall)

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## MATH 101b Algebra II

sn

Continuation of MATH 101a. Usually offered every year. Mr. Horozov (spring)

## MATH 110a Geometric Analysis

[ sn ]

Manifolds, tensor bundles, vector fields, and differential forms. Frobenius theorem. Integration, Stokes's theorem, and de Rham's theorem. Usually offered every year.

Mr. Adler (fall)

## MATH 110b Differential Geometry

sn ]

Riemannian metrics, parallel transport, geodesics, curvature. Introduction to Lie groups and Lie algebras, vector bundles and principal bundles. Usually offered every second year.

Staff

## MATH 111a Real Analysis

[ sn ]

Measure and integration. Lp spaces, Banach spaces, Hilbert spaces. Radon-Nikodym, Riesz representation, and Fubini theorems. Fourier transforms. Usually offered every year.

Mr. Kleinbock (fall)

#### MATH 111b Complex Analysis

[sn]

The Cauchy integral theorem, calculus of residues, and maximum modulus principle. Harmonic functions. The Riemann mapping theorem and conformal mappings. Other topics as time permits. Usually offered every year.

Mr. Adler (spring)

## MATH 121a Topology I

sn

Fundamental group, covering spaces. Cell complexes, homology and cohomology theory, with applications. Usually offered every year.

Mr. Baykur (fall)

## MATH 121b Topology II

sn

Continuation of MATH 121a. Manifolds and orientation, cup and cap products, Poincare duality. Other topics as time permits. Usually offered every year. Mr. Igusa (fall)

#### MATH 150a Combinatorics

sn

Emphasis on enumerative combinatorics. Generating functions and their applications to counting graphs, paths, permutations, and partitions. Bijective counting, combinatorial identities, Lagrange inversion and Mobius inversion. Usually offered every second year.

Mr. Gessel

## MATH 150b Topics in Combinatorics

sn]

Possible topics include symmetric functions, graph theory, extremal combinatorics, combinatorial optimization, coding theory. Usually offered every second year.

Mr. Gessel (fall)

# (200 and above) Primarily for Graduate Students

All graduate-level courses will have organizational meetings the first week of classes.

## MATH 200a Second-Year Seminar

A course for second-year students in the PhD program designed to provide exposure to current research and practice in giving seminar talks. Students read recent journal articles and preprints and present the material. Usually offered every year. Mr. Gessel

#### MATH 201a Topics in Algebra

Introduction to a field of algebra. Possible topics include representation theory, vertex algebras, algebraic groups. Usually offered every year.

Staff

#### MATH 202a Algebraic Geometry I

Varieties and schemes. Cohomology theory. Curves and surfaces. Usually offered every second year.

Mr. Horozov (fall)

## MATH 202b Algebraic Geometry II

Continuation of MATH 202a. Usually offered every second year.
Staff

## MATH 203a Number Theory

Basic algebraic number theory (number fields, ramification theory, class groups, Dirichlet unit theorem), zeta and L-functions (Riemann zeta function, Dirichlet L-functions, primes in arithmetic progressions, prime number theorem). Usually offered every second year. Staff

## MATH 203b Topics in Number Theory

Possible topics include class field theory, cyclotomic fields, modular forms, analytic number theory, ergodic number theory. Usually offered every year.

Mr. Bellaiche (spring)

## MATH 204a T.A. Practicum

Teaching elementary mathematics courses is a subtle and difficult art involving many skills besides those that make mathematicians good at proving theorems. This course focuses on the development and support of teaching skills. The main feature is individual observation of the graduate student by the practicum teacher, who provides written criticism of and consultation on classroom teaching practices. Usually offered every year. Ms. Parker (fall)

## MATH 205b Commutative Algebra

Associated primes, primary decomposition. Filtrations, completions, graded rings. Dimension theory, Hilbert functions. Regular sequences, depth, regular local rings. Other topics as time permits. Usually offered every second year. Staff

## MATH 211a Topics in Differential Geometry and Analysis I

Possible topics include complex manifolds, elliptic operators, index theory, random matrix theory, integrable systems, dynamical systems, ergodic theory. Usually offered every year.

Mr. Lian (fall)

#### MATH 212b Functional Analysis

Banach and Hilbert spaces, linear operators, operator topologies, Banach algebras. Convexity and fixed point theorems, integration on locally compact groups. Spectral theory. Other topics as time permits. Usually offered every second year. Mr. Kleinbock (spring)

#### MATH 221a Topology III

Vector bundles and characteristic classes. Elementary homotopy theory and obstruction theory. Cobordism and transversality; other topics as time permits. Usually offered every year. Mr. Ruberman (fall)

## MATH 221b Topics in Topology

Topics in topology and geometry. In recent years, topics have included knot theory, symplectic and contact topology, gauge theory, and three-dimensional topology. Usually offered every year. Staff

## MATH 223a Lie Algebras

Theorems of Engel and Lie. Semisimple Lie algebras, Cartan's criterion. Universal enveloping algebras, PBW theorem, Serre's construction. Representation theory. Other topics as time permits. Usually offered every second year. Staff

## MATH 224b Lie Groups

Basic theory of Lie groups and Lie algebras. Homogeneous spaces. Haar measure. Compact Lie groups, representation theory, Peter-Weyl theorem, differential slice theorem. Complex reductive groups. Other topics as time permits. Usually offered every second year. Mr. Ruberman (spring)

#### MATH 250a Complex Algebraic Geometry I

Riemann surfaces, Riemann-Roch theorems, Jacobians. Complex manifolds, Hodge decomposition theorem, cohomology of sheaves, Serre duality. Vector bundles and Chern classes. Other topics as time permits. Usually offered every second year.

## MATH 250b Complex Algebraic Geometry

Continuation of MATH 250a. Usually offered every second year.
Staff

## MATH 299a Readings in Mathematics

MATH 301a Further Topics in Algebra Staff

# MATH 302a Topics in Algebraic Geometry Staff

## MATH 311a Further Topics in Analysis Staff

MATH 321a Further Topics in Topology Staff

# MATH 326a Topics in Mathematics Staff

## MATH 399a Readings in Mathematics Staff

## MATH 401d Research

Independent research for the PhD degree. Specific sections for individual faculty members as requested. Staff

## **Cross-Listed Courses**

The following courses are approved for the program. Not all are given in any one year. Please consult the *Schedule of Classes* each semester.

#### BIOL 51a

Biostatistics

#### COSI 30a

Introduction to the Theory of Computation

#### ECON 184b

**Econometrics** 

## ECON 185a

Econometrics with Linear Algebra

## PHIL 106b

Mathematical Logic

## PHYS 110a

Mathematical Physics

## Courses of Related Interest

### PHIL 38b

Philosophy of Mathematics