Schrödinger Cats, Maxwell’s Demon and Quantum Error Correction

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Quantum Error Correction

N qubits have errors N times faster. Maxwell demon must overcome this factor of N – and not introduce errors of its own! (or at least not uncorrectable errors)
Full Steane Code – Arbitrary Errors

$$|0_L\rangle = \frac{1}{\sqrt{8}} \left[ |000000\rangle + |101010\rangle + |011011\rangle + |110010\rangle \right]$$

$$|1_L\rangle = \frac{1}{\sqrt{8}} \left[ |111111\rangle + |010101\rangle + |100110\rangle + |001100\rangle \right]$$

$$+ |000111\rangle + |101101\rangle + |011100\rangle + |110100\rangle + |110100\rangle$$

Single round of error correction

6 ancillae

7 qubits
All previous attempts to overcome the factor of $N$ and reach the ‘break even’ point of QEC have failed.

Current industrial approach (IBM, Google, Intel, Rigetti):

Scale up, then error correct

- Large, complex:
  - Non-universal (Clifford gates only)
  - Measurement via many wires
  - Difficult process tomography
- Large part count
- Fixed encoding

‘Surface Code’
(readout wires not shown)

O’Brien et al. arXiv:1703.04136 predict ‘break-even’ will be difficult even at the 50 qubit scale.
All previous attempts to overcome the factor of $N$ and reach the ‘break even’ point of QEC have failed.

We need a simpler and better idea...

‘Error correct and then scale up!’

Don’t use material objects as qubits.

Use microwave photon states stored in high-Q SC resonators.
Surface Code
(readout wires not shown)

- Large, complex:
  - Non-universal (Clifford gates only)
  - Measurement via many wires
  - Difficult process tomography
- Large part count
- Fixed encoding

Cat Code Photonic Qubit hardware shortcut
(readout wire shown)

- Precision:
  - Universal control (all possible gates)
  - Measurement via single wire
  - Easy process tomography
  - Long-lived cavities
  - Fault-tolerant QEC
- Reduced part count
- Flexible encoding
"Hardware-Efficient Bosonic Encoding"


Replace ‘Logical’ qubit with this:

- Cavity has long lifetime (~ms)
- Single dominant error channel photon loss: \( \Gamma = \kappa \langle \hat{n} \rangle \)
  makes QEC easier

earlier ideas: Gottesman, Kitaev & Preskill, PRA 64, 012310 (2001)
Chuang, Leung, Yamamoto, PRA 56, 1114 (1997)
Photonic Code States

Can we find novel (multi-photon) code words that can store quantum information even if some photons are lost?

Ancilla transmon coupled to resonator gives us universal control to make ‘any’ code word states we want.

\[ |\Psi\rangle = \psi_0 |0_L\rangle + \psi_1 |1_L\rangle \]

Logical code words (superpositions of photon Fock states)
Encoding qubits in cavity photon states

Minimal encoding cannot correct errors but has minimal loss rate:

\[ |0_L\rangle = |0\rangle \quad 0 \text{ photons} \]
\[ |1_L\rangle = |1\rangle \quad 1 \text{ photon} \]

\[ \dot{n} = -\kappa n \]

We will use more complicated states with more photons (e.g. Schrödinger cat states)

More photons means higher loss (error) rate

This is the analog of $N$ physical qubits forming a logical qubit. QEC Maxwell demon has to overcome the higher error rate.
Quick review of microwave resonators and photonic states
Coherent state $|\alpha\rangle$ is closest thing to a classical sinusoidal RF signal

$$\psi(\Phi) \equiv \langle \Phi | \alpha \rangle = \psi_0 (\Phi - \alpha)$$

Can displace in both position and momentum

$$\alpha = |\alpha| e^{i\theta}$$
Coherent state = displaced vacuum

\[ |\alpha\rangle = e^{[\alpha a^\dagger - \alpha^* a]} |0\rangle \]
\[ = e^{-\frac{1}{2}|\alpha|^2} \alpha e^{a a^\dagger} |0\rangle \]
\[ = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \]

Poisson distribution of photon number

\[ \overline{n} = |\alpha|^2 \]

This is the only kind of state that can be created with a classical drive applied to a cavity.
Dispersive coupling of two-level ancilla to high-Q cavity yields universal control

\[ H = \omega_c a^\dagger a + \frac{\omega_q}{2} \sigma^z + \chi \sigma^z a^\dagger a \]

\[ \omega_c \neq \omega_q \]

[2χ ⊥ 3,000 (κ, γ)]
Quantum optics at the single-photon level

- **Universal control** enables: photon state engineering

Goal: arbitrary photon Fock state superpositions

\[ |\psi\rangle = a_0 |0\rangle + a_1 |1\rangle + a_2 |2\rangle + a_3 |3\rangle + \ldots \]

Use the coupling between the cavity (harmonic oscillator) and the two-level qubit (anharmonic oscillator) to achieve this goal.
Previous State of the Art for Complex Oscillator States

Expt'l. Wigner tomography: Leibfried et al., 1996 ion traps (NIST – Wineland group)

Rydberg atom cavity QED
Haroche/Raimond, 2008 Rydberg (ENS)

Phase qubit circuit QED
Hofheinz et al., 2009
(UCSB – Martinis/Cleland)

~ 10 photons

\[ Q \]

\[ \Phi \]
Dispersive Hamiltonian

\[ H = \omega_c a^\dagger a + \frac{\omega_q}{2} \sigma^z + \chi \sigma^z a^\dagger a \]

- resonator
- qubit
- dispersive coupling

**cavity frequency** = \( \omega_r + \chi \sigma^z \)

`strong-dispersive` limit

\[ 2\chi \sim 2 \times 10^3 \kappa \]
Strong-Dispersive Limit yields a powerful toolbox

Cavity frequency depends on qubit state

Microwave pulse at this frequency excites cavity only if qubit is in ground state

Microwave pulse at this frequency excites cavity only if qubit is in excited state

Engineer’s tool #1:
Conditional displacement of cavity
Reinterpret dispersive term:
- quantized light shift of qubit frequency

$$H = \omega_c a^\dagger a + \frac{\omega_q}{2} \sigma^z + \chi \sigma^z a^\dagger a$$

resonator    qubit    dispersive coupling
Microwave photon number distribution in a coherent state (measured via quantized light shift of qubit transition frequency)

\[ H = \omega_c a^\dagger a + \frac{\omega_q}{2} \sigma^z + \chi \sigma^z a^\dagger a \]

\[ 2\chi \cong 3,000 (\kappa, \gamma) \]


Microwaves are particles!
Engineer’s tool #2:
Conditional flip of qubit if exactly $n$ photons

$$H = \omega_c a^\dagger a + \frac{\omega_q}{2} \sigma^z + \chi \sigma^z a^\dagger a$$

resonator  qubit  dispersive coupling

Reinterpret dispersive term:
- quantized light shift of qubit frequency

$$\frac{\omega_q + 2 \chi a^\dagger a}{2} \sigma^z$$
strong dispersive coupling

\[ V_{\text{DISPERSE}} \approx \chi a^\dagger a \sigma^z \]

Qubit Spectroscopy

Coherent state in the cavity

Conditional bit flip \( \pi_n \)
Strong Dispersive Coupling Gives Powerful Tool Set

Cavity-conditioned bit flip $\pi_n$

Qubit-conditioned cavity displacement $D_\alpha^g$

- multi-qubit geometric entangling phase gates (Paik et al.)
- Schrödinger cats are now ‘easy’ (Kirchmair et al.)

Photon Schrödinger cat states on demand

experiment
  G. Kirchmair
  B. Vlastakis

theory
  M. Mirrahimi
  Z. Leghtas
Paradoxically, we will use code words made of ‘delicate’ Schrödinger cat states of cavity photons.

\[ \psi_+ = \frac{1}{\sqrt{2}} \{ |\alpha\rangle + |1-\alpha\rangle \} \]

‘even eat’ only even \( n \)’s

\[ \psi_- = \frac{1}{\sqrt{2}} \{ |\alpha\rangle - |1-\alpha\rangle \} \]

‘odd eat’ only odd \( n \)’s

Superposition of two different ‘macroscopic’ states

\[ \text{“size” = “distance}^2 \text{”} = |2\alpha|^2 = 4\pi \]

(normalization is only approximate)
Parity of Cat States

\[ P = e^{\pi a^\dagger a} = (-1)^n \rightarrow P = \sum_n p_n (-1)^n \]

Coherent state: \[ |\psi\rangle = |\alpha = 2\rangle \]
Mean photon number: 4

Even parity cat state: \[ |\psi\rangle = |\alpha\rangle + |\alpha\rangle \]
Only photon numbers: 0, 2, 4, …
\[ \hat{P} |\psi\rangle = + |\psi\rangle \]

Odd parity cat state: \[ |\psi\rangle = |\alpha\rangle - |\alpha\rangle \]
Only photon numbers: 1, 3, 5, …
\[ \hat{P} |\psi\rangle = - |\psi\rangle \]
Key enabling technology: ability to make nearly ideal measurement of photon number parity (without measuring photon number!)

\[ \hat{P} = (-1)^{a^\dagger a} = \sum_{n=0}^{\infty} |n\rangle (-1)^n \langle n| \]

We learn whether \( n \) is even or odd without learning the value of \( n \).

(analogous to measuring \( Z_1 Z_2 \) without measuring \( Z_1, Z_2 \))

Measurement is 99.8% QND.
(Can be repeated hundreds of times.)

If we can measure parity, we can perform complete state tomography (measure Wigner function)
Measuring Photon Number Parity

- use quantized light shift of qubit frequency

\[
\frac{\omega_q + 2\chi a^\dagger a}{2} \sigma^z
\]

\[
e^{-i\frac{2\chi\hat{n}t}{2} \sigma^z} = e^{-i\frac{\pi\hat{n}}{2} \sigma^z}
\]

\[
\hat{n} = 1, 3, 5, \ldots \quad \hat{n} = 0, 2, 4, \ldots
\]


Sun, Petrenko et al., *Nature* 511, 444 (2014) 99.8% QND(!)
Using photon number parity \( \hat{P} = (-1)^{\hat{n}} \) to do cavity state tomography

Wigner function:
--quasi-probability distribution in phase space

equivalent to the full density matrix for state tomography.
Wigner Function = “Displaced Parity”

Full state tomography on large dim. Hilbert space can be done very simply over a single input-output wire.

Simple Recipe:
1. Apply microwave tone to displace oscillator in phase space.
2. Measure mean parity.

Handy identity (Luterbach and Davidovitch):

\[ W(\beta) = \langle \Psi | D(+\beta) \hat{P} D(-\beta) | \Psi \rangle \]

\[ \hat{P} = (-1)^{\hat{N}} = \text{parity} \]
State Tomography:
Wigner Function of a Cat State

Interference fringes prove cat is coherent (even for sizes > 100 photons)

\[
\frac{1}{\sqrt{2}} \left[ | -\alpha \rangle + | +\alpha \rangle \right]
\]

Even parity vacunm noise Fringes prove this is a coherent cat, not a mixture
Using Schrödinger cat states to store and correct quantum information

“I have good news and bad news”

Courtesy of Mitra Farmand
Encode information in two orthogonal even-parity logical code “words”

\[ |\Psi\rangle = \psi_0 |0_L\rangle + \psi_1 |1_L\rangle \]

\[ |0_L\rangle = |\alpha\rangle + |\alpha\rangle \]

\[ |1_L\rangle = |i\alpha\rangle + |i\alpha\rangle \]

\[ a|\alpha\rangle = |\alpha\rangle \]

Magic property: coherent states are invariant under photon loss!

\[ a|0_L\rangle = \alpha\{|\alpha\rangle - |-\alpha\rangle\} \]

Photon loss flips the parity which is the error syndrome we can measure (99.8% QND).

Store a qubit as a superposition of two cats of same parity

\[ \hat{P} |\Psi\rangle = (+1) |\Psi\rangle \]
Coherent states are eigenstates of photon destruction operator. \( a\ket{\alpha} = \alpha \ket{\alpha} \)

Effect of photon loss on code words:

\[
a \ket{0_L} = a \left( \ket{\alpha} + \ket{-\alpha} \right) \rightarrow \left( \ket{\alpha} - \ket{-\alpha} \right) \quad \text{(if } \alpha \text{ real)}
\]

\[
a^2 \ket{0_L} = a^2 \left( \ket{\alpha} + \ket{-\alpha} \right) \rightarrow \left( \ket{\alpha} + \ket{-\alpha} \right) = \ket{0_L}
\]

\[
a \ket{1_L} = a \left( \ket{i\alpha} + \ket{-i\alpha} \right) \rightarrow i \left( \ket{i\alpha} - \ket{-i\alpha} \right)
\]

\[
a^2 \ket{1_L} = a^2 \left( \ket{i\alpha} + \ket{-i\alpha} \right) = (i)^2 \left( \ket{i\alpha} + \ket{-i\alpha} \right) = -\ket{1_L}
\]

After loss of 4 photons cycle repeats:

\[
a^4 \left( \psi_0 \ket{0_L} + \psi_1 \ket{1_L} \right) \rightarrow \left( \psi_0 \ket{0_L} + \psi_1 \ket{1_L} \right)
\]

We can recover the state if we know: (via monitoring parity jumps) \( N_{\text{Loss}} \mod 4 \)
2016: First true Error Correction Engine that works

- Commercial FPGA with custom software developed at Yale
- Single system performs all measurement, control, & feedback (latency ~200 nanoseconds)
- ~15% of the latency is the time it takes signals to move at the speed of light from the quantum computer to the controller and back!

MAXWELL’S DEMON

A prototype quantum computer being prepared for cooling close to absolute zero.

Schoelkopf-Devoret lab
Experiment:

‘Extending the lifetime of a quantum bit with error correction in superconducting circuits,’


Theory:

‘cat codes’

Implementing a Full QEC System: Debugger View


(This is all real, raw data.)
Process Fidelity: Uncorrected Transmon

\[ \tau \approx 15 \mu s \]
System’s Best Component

\[ |\psi\rangle = \psi_g |n = 0\rangle + \psi_e |n = 1\rangle \]

\[ \tau \approx 290 \mu s \]

\[ \tau \approx 15 \mu s \]
Process Fidelity: Cats without QEC

\[ \alpha = \sqrt{2} \]

\[ \tau \approx 290 \mu s \]

\[ \tau \approx 130 \mu s \]

\[ \tau \approx 15 \mu s \]
Process Fidelity: Cats with QEC

\[ \alpha = \sqrt{2} \]

QEC – NO POST-SELECTION.

\[ \tau \approx 290 \mu s \]
\[ \tau \approx 320 \mu s \]
\[ \tau \approx 130 \mu s \]
\[ \tau \approx 15 \mu s \]
Only High-Confidence Trajectories

\[ \alpha = \sqrt{2} \]

Exclude results with heralded errors

Still keep ~80% of data

\[ \tau \approx 560 \mu s \]
\[ \tau \approx 290 \mu s \]
\[ \tau \approx 320 \mu s \]
\[ \tau \approx 130 \mu s \]
\[ \tau \approx 15 \mu s \]
Cavity is not just a quantum memory, it is a qubit.

Universal Gate Set on a Logical Qubit Encoded in an Oscillator (‘cat code’)

Heeres et al., Nature Communications 8, 94 (2017)
Encoding qubits in cavity photon states: ‘kitten code’

aka ‘binomial code’

\[ |0_L\rangle = \frac{|0\rangle + |4\rangle}{\sqrt{2}} \quad 0,4 \text{ photons} \]
\[ |1_L\rangle = |2\rangle \quad 2 \text{ photons} \]

initial state: \[ |\psi\rangle = \alpha |0_L\rangle + \beta |1_L\rangle \]

single photon loss: \[ a |\psi\rangle = \sqrt{2}[\alpha |3\rangle + \beta |1\rangle] \]

‘kitten code’ can correct single photon loss

General binomial code corrects L losses, G gains and D dephasing events:

Demonstration of quantum error correction and universal gate set on a binomial bosonic logical qubit

L. Hu,1, * Y. Ma,1, * W. Cai,1 X. Mu,1 Y. Xu,1 W. Wang,1 Y. Wu,2 H. Wang,1 Y. P. Song,1 C.-L. Zou,3, † S. M. Girvin,4 L.-M. Duan,1, 2 and L. Sun1, ‡

Kitten code QEC: arXiv:1805.09072
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Kitten code QEC approaching breakeven:
arXiv:1805.09072

(a)

![Graph showing fidelity over time with different encoding methods and error rates](image)

- uncorrected Fock 0, 1 encoding
- corrected binomial code for $t_w=17.895$ μs
- uncorrected binomial code
- uncorrected transmon

- $\tau=216\pm2$ μs
- $\tau=71\pm2$ μs
- $\tau=200\pm1$ μs
- $\tau=38\pm1$ μs

Process fidelity $F_x(t)$ vs. Time (μs)
Repeated gates on logical qubit

Logical qubit Ramsey fringes with QEC

Kitten code QEC: arXiv:1805.09072
We are on the way!

“Age of Qu. Error Correction.”
“Age of Quantum Feedback”
“Age of Measurement”
“Age of Entanglement”
“Age of Coherence”

Achieved goal of reaching “break-even” point for error correction with cat code and (almost) with kitten code. Now need to surpass by 10x or more.

M. Devoret and RS, Science (2013)
In (microwave) light there is truth.....

‘Circuit QED’
Extra Slides
We can recover the state if we know:
(via monitoring parity jumps)

\[ N_{\text{Loss}} \mod 4 \]

Amplitude damping is deterministic
(independent of the number of parity jumps!)

\[
|W(t)\rangle = |e^{-\kappa t/2} \alpha\rangle \pm |-e^{-\kappa t/2} \alpha\rangle
\]

Maxwell Demon takes this into account ‘in software.’
Cat in Two Boxes
(two-legged cat only)

Theoretical proposal by Paris group:

Qubit measures joint parity!

\[ P_{12} = P_1 P_2 = e^{i\pi(\hat{n}_1 + \hat{n}_2)} \]

\[ |\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}} \left[ |+\alpha\rangle |+\alpha\rangle \pm |-\alpha\rangle |-\alpha\rangle \right] \]
Cat in Two Boxes

Experiment by Yale group: *Science* 352, 1087 (2016)

Qubit measures *joint* parity!

\[ P_{12} = P_1 P_2 = e^{i\pi(\hat{n}_1 + \hat{n}_2)} \]

- Universal controllability
- 3-level qubit can measure

\[ P_1, P_2, \text{ and } P_{12} \]
Two-cavities: 4-dimensional phase space and Wigner functions.

Theory

Entanglement of two logical cat states

$$|\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}} \left[ |+\alpha\rangle|+\alpha\rangle \pm |-\alpha\rangle|-\alpha\rangle \right]$$

9 sigma violation of Bell inequality

Experiment

Two-cavities: 4-dimensional phase space and Wigner functions.
Entanglement of Two Logical Cat-Qubits

CHSH: (Milman et al.: evaluate Wigner at 4 points in 4D phase space)

\[ B = W(\beta'_1, \beta'_2) + W(\beta_1, \beta'_2) + W(\beta'_1, \beta_2) - W(\beta_1, \beta_2) \]

\[ W(\beta'_1, \beta'_2) W(\beta_1, \beta'_2) W(\beta'_1, \beta_2) W(\beta_1, \beta_2) \]

\[ B = 2.18 \pm 0.02 \]

CHSH Bell: \[ 2 \leq B \leq 2\sqrt{2} \]